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Probing transverse quark polarization in deep-inelastic leptonproduction

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Abstract

The azimuthal dependence of hadrons produced in lepton scattering off a polarized hadron probes the quark transverse-spin distributions. In the calculation of the asymmetries, transverse momenta of quarks in the distribution and fragmentation functions must be incorporated. In addition to the $\sin(\phi + \phi_S)$ asymmetry for transversely polarized hadrons, known as the Collins effect, we find a $\sin(3\phi - \phi_S)$ asymmetry. Furthermore, we find a $\sin 2\phi$ angular dependence for longitudinally polarized hadrons.

In hard scattering processes one has the possibility of measuring specific matrix elements of quark and gluon fields. In leading order they can be readily interpreted as quark and gluon densities including also the spin (helicity and transverse spin) densities. Inclusive measurements in deep-inelastic lepton-hadron scattering (DIS) enable the extraction of the unpolarized distribution $f_1(x)$ and one of two spin distributions, namely the helicity distribution $g_1(x)$. The other (transverse) spin distribution $h_1(x)$, cannot be measured in inclusive DIS because of its chiral structure. It has been suggested to measure this in Drell-Yan scattering [1] or in semi-inclusive measurements in lepton-hadron scattering [2]. One of the ways in which semi-inclusive measurements could be used involves transverse momentum dependence. In Ref. [3], Collins shows how the semi-inclusive deep-inelastic process $e + p^\uparrow \rightarrow e + h + X$, where p^\uparrow denotes a

transversely polarized hadron (spin vector S_T^i), enables one to probe the quark transverse spin through a leading asymmetry depending on the azimuthal angle of the outgoing hadron's momentum and that of the target hadron's spin vector, the so-called Collins effect. In Ref. [4], however, we found that terms proportional to $k_T^i(k_T \cdot S_T/M^2)$, where k_T is the quark transverse momentum with respect to the hadron, in the relevant matrix element are neither forbidden by the symmetries of QCD, nor suppressed by powers of $1/Q$. In this letter we present *all* leading contributions that enter when transverse momentum dependence is considered, treating also the case where the initial hadron is longitudinally polarized.

We consider the process $\ell + H \rightarrow \ell' + h + X$, where ℓ and ℓ' are the incoming and scattered leptons (considered massless) with momenta l and l' , H is the incoming hadron with momentum P (with $P^2 = M^2$) and spin vector S (with $P \cdot S = 0$ and $S^2 = -1$), h is the produced hadron with momentum P_h (with $P_h^2 = M_h^2$). The cross section for this process can be written

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as a product of a leptonic and a hadronic tensor

$$\frac{d\sigma}{dx_B dy dz d^2 P_{h\perp}} = \frac{\pi y \alpha^2}{2zQ^4} 2M\mathcal{W}^{\mu\nu} L_{\mu\nu}, \quad (1)$$

where we consider the situation in which the transverse momentum $P_{h\perp}^2$ is assumed to be of $\mathcal{O}(M^2)$. We have used the scalar variables

$$x_B = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_h}{P \cdot q}. \quad (2)$$

Here $q = l - l'$ is the momentum of the exchanged virtual photon (we will limit ourselves to electromagnetic interactions), which is spacelike ($-q^2 = Q^2$). We assume that Q^2 becomes large as compared to the hadronic scale, say M^2 , but x_B and z remain constant, well away from their endpoints 0 and 1. It is convenient to introduce the vector $\tilde{P}^\mu \equiv P^\mu - (P \cdot q/q^2) q^\mu$ which is orthogonal to q , leading to the timelike unit vector $\hat{l}^\mu \equiv 2x_B \tilde{P}^\mu/Q$ (with $\hat{l}^2 = 1$). Together with the spacelike unit vector $\hat{q}^\mu \equiv q^\mu/Q$ (with $\hat{q}^2 = -1$), we can define tensors in the space orthogonal to P and q ,

$$g_\perp^{\mu\nu} \equiv g^{\mu\nu} + \hat{q}^\mu \hat{q}^\nu - \hat{l}^\mu \hat{l}^\nu \\ = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} - \frac{\tilde{P}^\mu \tilde{P}^\nu}{\tilde{P}^2}, \quad (3)$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} \hat{l}_\rho \hat{q}_\sigma = \frac{2x_B}{Q^2} \epsilon^{\mu\nu\rho\sigma} P_\rho q_\sigma. \quad (4)$$

Azimuthal angles will be fixed with respect to the lepton scattering plane. The x -axis is derived from the lepton momentum which can be written as

$$l^\mu = \frac{(2-y)Q}{2y} \hat{l}^\mu + \frac{Q}{2} \hat{q}^\mu + l_\perp^\mu, \quad (5)$$

with $l_\perp^2 = -l_\perp'^2 = Q^2(1-y)/y^2$. We define the unit vector in the x -direction to be $\hat{x}^\mu \equiv l_\perp^\mu/|l_\perp|$. The (unpolarized) lepton tensor $L^{\mu\nu} = 2l^\mu l'^\nu + 2l'^\mu l^\nu - Q^2 g^{\mu\nu}$ can be written as

$$L^{\mu\nu} = \frac{4Q^2}{y^2} \left[-\frac{1}{2} (1-y + \frac{1}{2}y^2) g_\perp^{\mu\nu} \right. \\ \left. + (1-y) (\hat{x}^\mu \hat{x}^\nu + \frac{1}{2}g_\perp^{\mu\nu}) \right. \\ \left. + (1-y)^{\frac{1}{2}} (1 - \frac{1}{2}y) (\hat{x}^\mu \hat{l}^\nu + \hat{x}^\nu \hat{l}^\mu) \right. \\ \left. + (1-y) \hat{l}^\mu \hat{l}^\nu \right]. \quad (6)$$

The four tensor structures are mutually orthogonal.

The interesting physics is in the hadron tensor which, in leading order in $1/Q$, is given by

$$2M\mathcal{W}^{\mu\nu} = 2e^2 \int d^4k d^4k' \delta^4(k+q-k') \\ \times \text{Tr} [\Phi(k) \gamma^\mu \Delta(k') \gamma^\nu]. \quad (7)$$

We consider for the moment only the quark contribution for one flavor. Furthermore we limit ourselves to the symmetric part because the unpolarized lepton tensor is symmetric. One has on the distribution side (involving the target hadron) [5]

$$\Phi_{\alpha\beta}(k) = \int \frac{d^4x}{(2\pi)^4} e^{ik \cdot x} \\ \times \langle PS | \bar{\psi}_\beta(0) \mathcal{G}(0, x) \psi_\alpha(x) | PS \rangle, \quad (8)$$

while on the fragmentation side (involving the produced hadron) [6]

$$\Delta_{\alpha\beta}(k') = \sum_X \frac{1}{2} \int \frac{d^4x}{(2\pi)^4} e^{ik' \cdot x} \\ \times \langle 0 | \mathcal{G}(0, x) \psi_\alpha(x) | P_h, X \rangle \langle P_h, X | \bar{\psi}_\beta(0) | 0 \rangle. \quad (9)$$

In the first equation a color-summation is understood, in the second a color-average. A link operator $\mathcal{G}(0, x) = \mathcal{P} \exp[-ig \int_0^x ds^\mu A_\mu(s)]$ is inserted to make the definitions color gauge-invariant. With an appropriate choice of the path structure in the link operator used in the above definitions and an appropriate choice of gauge, the above expression for the hadronic tensor $\mathcal{W}^{\mu\nu}$ corresponds to the Born graph in a diagrammatic expansion [4].

In order to analyse the hadronic tensor it is convenient to work in a frame in which the hadrons H and h are collinear (see also [7]). We can define the rank-two tensors

$$g_T^{\mu\nu} \equiv g^{\mu\nu} - n_+^\mu n_-^\nu - n_-^\mu n_+^\nu, \quad (10)$$

$$\epsilon_T^{\mu\nu} \equiv \epsilon^{\mu\nu}{}_{\rho\sigma} n_+^\rho n_-^\sigma, \quad (11)$$

using the null vectors, defined implicitly by

$$P = \frac{Q}{x_B \sqrt{2}} n_+ + \frac{M^2 x_B}{Q \sqrt{2}} n_-, \quad (12)$$

$$P_h = \frac{z Q}{\sqrt{2}} n_- + \frac{M_h^2}{z Q \sqrt{2}} n_+, \quad (13)$$

satisfying $n_+ \cdot n_- = 1$. In analogy to the Drell-Yan process [4], we define a ‘transverse’ vector as $a_T^\mu \equiv g_T^{\mu\nu} a_\nu$, having only transverse components \mathbf{a}_T in the above-mentioned collinear frames. Note that in general the photon momentum q has non-zero transverse components \mathbf{q}_T . Also, the hadron spin vector is decomposed according to

$$S = \lambda \frac{Q}{x_B M \sqrt{2}} n_+ - \lambda \frac{x_B M}{Q \sqrt{2}} n_- + S_T, \quad (14)$$

with λ the helicity and S_T the transverse spin, satisfying $\lambda^2 + S_T^2 = -S^2 = 1$.

A ‘perpendicular’ vector we define as $a_\perp^\mu \equiv g_\perp^{\mu\nu} a_\nu = a_T^\mu - (\mathbf{a}_T \cdot \mathbf{q}_T / Q^2) q^\mu$. These have only two non-zero components \mathbf{a}_\perp in the frames where the hadron and the virtual photon are collinear. So in general the outgoing hadron has non-zero $\mathbf{P}_{h\perp}$, specifically, $\mathbf{P}_{h\perp} \approx -z \mathbf{q}_T$. The use of ‘ \approx ’ means ‘up to corrections of (relative) order $1/Q^2$ ’. The target hadron has neither T - nor \perp -components. In short, a transverse tensor is perpendicular to both P and P_h , whereas a perpendicular tensor is perpendicular to both P and q . The two types are connected by boosts of order $|\mathbf{P}_{h\perp}|/Q$, so that at leading order they may be freely interchanged. Only at $\mathcal{O}(1/Q)$ the differences become important [8].

After this kinematical intermezzo, we return to the calculation of the hadronic tensor (7). Assuming quark momenta in hadrons to be limited, i.e., in the expressions for Φ and Δ the quantities k^2 , $k \cdot P$, k'^2 , and $k' \cdot P_h$, are of hadronic scale, one infers that $k^+ \gg k'^+$ and $k^- \ll k'^-$, so that

$$\delta^4(k + q - k') \approx \delta(k^+ + q^+) \delta(q^- - k'^-) \times \delta^2(\mathbf{k}_T - \mathbf{k}'_T - \mathbf{q}_T), \quad (15)$$

such that $k^+/P^+ \approx -q^+/P^+ \approx x_B$ and $P_h^-/k'^- \approx P_h^-/q^- \approx z$. Another consequence of Eq. (15) is that one becomes sensitive to the integrals $\int dk^- \Phi(k)$ and $\int dk'^+ \Delta(k')$. Only specific Dirac projections will contribute in leading order. For the distributions we need the matrix elements ($i = 1, 2$)

$$\frac{1}{2} \int dk^- \text{Tr} [\gamma^+ \Phi(k)] = f_1(x, \mathbf{k}_T^2), \quad (16)$$

$$\begin{aligned} \frac{1}{2} \int dk^- \text{Tr} [i\sigma^{i+} \gamma_5 \Phi(k)] &= h_{1T}(x, \mathbf{k}_T^2) S_T^i \\ &+ \left[h_{1L}^\perp(x, \mathbf{k}_T^2) \lambda + h_{1T}^\perp(x, \mathbf{k}_T^2) \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} \right] \frac{k_T^i}{M}, \end{aligned} \quad (17)$$

where $x = k^+/P^+ \approx x_B$. Performing the k_T -integration, only the distribution functions $f_1(x, \mathbf{k}_T^2)$ and $h_1(x, \mathbf{k}_T^2) \equiv h_{1T}(x, \mathbf{k}_T^2) + (\mathbf{k}_T^2/2M^2) h_{1T}^\perp(x, \mathbf{k}_T^2)$, contribute to the distributions $f_1(x)$ and $h_1(x)$, respectively.

Similarly, the leading parts for the (fragmentation) matrix elements in Eq. (9) for the case that one sums over the polarization of the produced hadron are ($i, j = 1, 2$)

$$\frac{1}{2z} \int dk'^+ \text{Tr} [\gamma^- \Delta(k')] = D_1(z, z^2 \mathbf{k}_T'^2), \quad (18)$$

$$\begin{aligned} \frac{1}{2z} \int dk'^+ \text{Tr} [i\sigma^{i-} \gamma_5 \Delta(k')] \\ = \frac{\epsilon_T^{ij} k_{Tj}'}{M_h} H_1^\perp(z, z^2 \mathbf{k}_T'^2), \end{aligned} \quad (19)$$

where $z = P_h^-/k^-$ is the longitudinal momentum fraction of the produced hadron, and $\mathbf{p}_T = -z \mathbf{k}'_T$ is its transverse momentum with respect to the fragmenting quark. The normalization of D_1 is given by the momentum sum rule $\sum_h \int_0^1 dz \int d^2 \mathbf{p}_T z D_1(z, \mathbf{p}_T^2) = 1$. Hermiticity and parity invariance have been used in deriving the above (real) parametrizations. Interestingly, a structure corresponding to H_1^\perp in the distribution part will be excluded because of time-reversal invariance. The fragmentation functions are not invariant under the time-reversal operation because the states $|P_h, X\rangle$ are out-states. That H_1^\perp can well be nonzero in QCD has been made plausible in Refs. [3,9] by applying simple models. The function also appears in the cross section for scattering of polarized leptons off unpolarized hadrons, but in that case it is suppressed by a factor $1/Q$ [10]. Note that we use somewhat different functions as in Ref. [3], the connection being $z D_1(z, z^2 \mathbf{k}_T'^2) = \hat{D}(z, k_T')$ and $\epsilon_T^{ij} k_{Tj}' z H_1^\perp(z, z^2 \mathbf{k}_T'^2) / M_h = \Delta \hat{D}(z, k_T', e_T)$, where e_T is a unit vector in the i -direction.

By means of Fierz transformation of the two γ -matrices in Eq. (7), one finds that the projection D_1 in (18) selects from the distribution side the function f_1 (called \hat{f} in Ref. [3]). On the other hand, H_1^\perp

comes with the matrix element in Eq. (17). In the parametrization of this matrix element three functions come in. In Ref. [3] only the function h_{1T} (called \hat{f}_T) was considered. We will consider here the additional function h_{1T}^\perp for transversely polarized hadrons and h_{1L}^\perp for longitudinally polarized hadrons. The rest of the calculation is a matter of inserting the above parametrizations into the (Fierz transformed) trace in Eq. (7). The result is

$$\begin{aligned}
2M \mathcal{W}^{\mu\nu} = & 2e^2 \int d^2 k_T d^2 p_T \delta^2(\mathbf{p}_T + z \mathbf{k}_T + z \mathbf{q}_T) \\
& \times \left\{ f_1(x, k_T^2) z D_1(z, p_T^2) [-g_T^{\mu\nu}] \right. \\
& + \frac{h_{1T}(x, k_T^2) H_1^\perp(z, p_T^2)}{M_h} \\
& \times \left[S_T^{\{\mu} \epsilon_T^{\nu\} \rho} p_{T\rho} - (\epsilon_T^{\rho\sigma} S_{T\rho} p_{T\sigma}) g_T^{\mu\nu} \right] \\
& + \frac{k_T \cdot S_T}{M} \frac{h_{1T}^\perp(x, k_T^2) H_1^\perp(z, p_T^2)}{M M_h} \\
& \times \left[k_T^{\{\mu} \epsilon_T^{\nu\} \rho} p_{T\rho} - (\epsilon_T^{\rho\sigma} k_{T\rho} p_{T\sigma}) g_T^{\mu\nu} \right] \\
& + \lambda \frac{h_{1L}^\perp(x, k_T^2) H_1^\perp(z, p_T^2)}{M M_h} \\
& \left. \times \left[k_T^{\{\mu} \epsilon_T^{\nu\} \rho} p_{T\rho} - (\epsilon_T^{\rho\sigma} k_{T\rho} p_{T\sigma}) g_T^{\mu\nu} \right] \right\}, \quad (20)
\end{aligned}$$

where $\{\mu\nu\}$ indicates symmetrization of the indices. The way to project the Lorentz indices of the convolution variables k_T and p_T on external momenta was discussed in Appendix A of Ref. [4] for the Drell-Yan process. The resulting leading-order hadronic tensor is most conveniently expressed in terms of four structure functions constructed from the perpendicular vectors and tensors defined earlier,

$$\begin{aligned}
\mathcal{W}^{\mu\nu} = & -\mathcal{W}_T g_\perp^{\mu\nu} - \mathcal{U}^T \\
& \times \left[S_\perp^{\{\mu} \epsilon_\perp^{\nu\} \rho} \hat{h}_\rho - (\epsilon_\perp^{\rho\sigma} S_{\perp\rho} \hat{h}_\sigma) g_\perp^{\mu\nu} \right] \\
& + \mathcal{V}^T [(\hat{h} \cdot S_\perp) \hat{h}^{\{\mu} \epsilon_\perp^{\nu\} \rho} \hat{h}_\rho \\
& + (\epsilon_\perp^{\rho\sigma} S_{\perp\rho} \hat{h}_\sigma) (2\hat{h}^\mu \hat{h}^\nu + g_\perp^{\mu\nu})] \\
& - \mathcal{V}^L \lambda \hat{h}^{\{\mu} \epsilon_\perp^{\nu\} \rho} \hat{h}_\rho, \quad (21)
\end{aligned}$$

where $\hat{h} = P_{h\perp}/|P_{h\perp}|$. We define the convolution product (reinstating the sum over quark and antiquark flavors)

$$\begin{aligned}
I[fD] \equiv & \sum_a e_a^2 \int d^2 k_T d^2 p_T \delta^2(\mathbf{p}_T + z \mathbf{k}_T + z \mathbf{q}_T) \\
& \times f^a(x_B, k_T^2) D^a(z, p_T^2) \\
\approx & \sum_a e_a^2 \int d^2 k_T f^a(x_B, k_T^2) \\
& \times D^a(z, (P_{h\perp} - z k_T)^2), \quad (22)
\end{aligned}$$

where the use of transverse two-vectors is most appropriate because of the definitions of the fragmentation functions. For the second line we used $\mathbf{q}_T \approx -\mathbf{P}_{h\perp}/z$. The structure functions now read

$$M \mathcal{W}_T = z I[f_1 D_1], \quad (23)$$

$$M \mathcal{U}^T = -I \left[(\hat{h} \cdot p_T) \frac{h_1 H_1^\perp}{M_h} \right], \quad (24)$$

$$\begin{aligned}
M \mathcal{V}^T = & -I \left[(4(k_T \cdot \hat{h})^2 (p_T \cdot \hat{h}) - 2(k_T \cdot \hat{h})(k_T \cdot p_T) \right. \\
& \left. - (p_T \cdot \hat{h}) k_T^2) \frac{h_{1T}^\perp H_1^\perp}{2 M^2 M_h} \right], \quad (25)
\end{aligned}$$

$$\begin{aligned}
M \mathcal{V}^L = & -I \left[(2(k_T \cdot \hat{h})(p_T \cdot \hat{h}) \right. \\
& \left. - k_T \cdot p_T) \frac{h_{1L}^\perp H_1^\perp}{M M_h} \right]. \quad (26)
\end{aligned}$$

Note that in \mathcal{U}^T precisely the combination $h_1 = h_{1T} + (k_T^2/2M^2) h_{1T}^\perp$ appears. The dot products can easily be converted into squares k_T^2 , p_T^2 , and $P_{h\perp}^2$, using the delta function in the convolution integrals. So the structure functions depend on the variables x_B , z , and $P_{h\perp}^2$. Finally, the leading-order cross section is obtained by contracting Eq. (21) with the leptonic tensor, Eq. (6),

$$\begin{aligned}
\frac{d\sigma}{dx_B dy dz d^2 P_{h\perp}} = & \frac{4\pi\alpha^2}{yzQ^2} [\mathcal{W}_T (1-y + \frac{1}{2}y^2) \\
& + \mathcal{U}^T |\mathcal{S}_T| (1-y) \sin(\phi + \phi_S) \\
& + \mathcal{V}^T |\mathcal{S}_T| (1-y) \sin(3\phi - \phi_S) \\
& + \mathcal{V}^L \lambda (1-y) \sin 2\phi], \quad (27)
\end{aligned}$$

where $\cos \phi = -\hat{x} \cdot \hat{h}$ and $\sin \phi = \epsilon_\perp^{\mu\nu} \hat{x}_\mu \hat{h}_\nu$, and likewise for the azimuth of the spin vector, $|\mathcal{S}_\perp| \cos \phi_S = -\hat{x} \cdot S_\perp$ and $|\mathcal{S}_\perp| \sin \phi_S = \epsilon_\perp^{\mu\nu} \hat{x}_\mu S_{\perp\nu}$ and we used $|\mathcal{S}_\perp| \approx |\mathcal{S}_T|$. Our analysis shows that the $\sin(\phi + \phi_S)$ asymmetry in the process $e + p^\dagger \rightarrow e + h +$

X , found in Ref. [3] probes the transverse spin distribution $h_1(x, \mathbf{k}_T^2)$, which upon integration over \mathbf{k}_T gives $h_1(x)$. One must be aware, however, of an additional $\sin(3\phi - \phi_S)$ azimuthal dependence for transversely polarized hadrons. The corresponding structure function, \mathcal{V}^T , probes the new distribution function $h_{1T}^+(x, \mathbf{k}_T^2)$. For longitudinally polarized hadrons there is a $\sin 2\phi$ asymmetry multiplying the structure function \mathcal{V}^L , which probes the distribution function $h_{1L}^+(x, \mathbf{k}_T^2)$. These three distribution functions determine the transverse spin density in a polarized hadron including the dependence on quark transverse momenta as given in Eq. (17). They are all convoluted with the same fragmentation function $H_1^\perp(z, \mathbf{p}_T^2)$.

Although the structure functions contain complicated convolutions in transverse momentum space, they simplify considerably if one assumes a Gaussian transverse momentum dependence. That is,

$$f(x, \mathbf{k}_T^2) = f(x, 0) \exp(-R_H^2 \mathbf{k}_T^2), \quad (28)$$

$$\begin{aligned} D(z, \mathbf{p}_T^2) &= D(z, 0) \exp(-R_h^2 \mathbf{k}_T'^2) \\ &= D(z, 0) \exp(-R_h^2 \mathbf{p}_T^2 / z^2), \end{aligned} \quad (29)$$

for a general distribution and fragmentation function, respectively. The structure function \mathcal{W}_T of Eq. (23), for instance, with this Gaussian ansatz becomes

where the convolutions have the same simple form as in Eq. (30). Note that we considered only two transverse radii, one for the target and one for the measured hadron, but not one for every distribution separately.

On completion of this work, we became aware of Ref. [11] treating semi-inclusive deep-inelastic scattering in a similar fashion. The author finds similar asymmetries in addition to the $\sin(\phi + \phi_S)$ asymmetry. He assumes Gaussian \mathbf{k}_T -behavior for the distribution and fragmentation functions throughout the paper. Our expressions for the structure functions, Eqs. (23)–(26), are valid for any \mathbf{k}_T -dependence.

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