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# **Appendix A**

# **Appendix: Proofs**

This appendix contains the proofs of Chapter 2 (*Incentive-Compatible Sovereign Debt*), Chapter 3 (*Collective Pension Funds*), and Chapter 4 (*Sand in the Wheels of Capitalism*).

# A.1 Proofs of Chapter 2

# **Proof of Proposition 2.3.3:**

*Proof.* Let  $(O_1, I_d)$  be an optimal contract and let *D* be the constant value of  $O_1$  when  $I_d(y) = 0$ . Consider a new contract  $(\tilde{O}_1, \tilde{I}_d)$  given by

$$\tilde{I}_d(y) = \begin{cases} 0 & \text{if } \tilde{D} \le \min\{\gamma y + B, y\} \\ 1 & \text{if } \tilde{D} > \min\{\gamma y + B, y\} \end{cases}$$

and

$$\tilde{O}_1(y) = \begin{cases} \tilde{D} & \text{if } \tilde{I}_d(y) = 0\\ \gamma y & \text{if } \tilde{I}_d(y) = 1 \end{cases}$$

and suppose first that  $\tilde{D} = D$ . If  $\tilde{I}_d(y) = I_d(y)$ , then the construction of  $\tilde{O}_1$  implies that  $\tilde{O}_1(y) \ge O_1(y)$ . If  $\tilde{I}_d(y) < I_d(y)$ , i.e. if  $\tilde{I}_d(y) = 0$  and  $I_d(y) = 1$ , then it follows

from proposition 2.3.2 that

$$O_1(\mathbf{y}) \le D \le \tilde{O}_1(\mathbf{y})$$

Furthermore, we can rule out  $\tilde{I}_d(y) > I_d(y)$ . To see this, suppose that *y* is such that  $\tilde{I}_d(y) = 1$  and  $I_d(y) = 0$ . Then we know that  $O_1(y) = D$ , but this cannot be, as we also know that  $\gamma y + B < D$  from  $\tilde{I}_d(y) = 1$ , and an optimal contract must be repudiation proof. This proves that  $\tilde{O}_1(y) \ge O_1(y)$  if  $\tilde{D} = D$ .

Now, one can choose  $\tilde{D} \leq D$  such that the investor participation constraint is still satisfied. By construction, the resulting contract  $(\tilde{O}_1, \tilde{I}_d)$  satisfies truthful revelation and is repudiation-proof-like any sovereign debt contract. As  $\tilde{I}_d(y) \leq I_d(y)$ , it must be optimal.

Since both  $(O_1, I_d)$  and  $(\tilde{O}_1, \tilde{I}_d)$  are optimal contracts, we have

$$E(I_d - \tilde{I_d})B = 0$$

Consider the state observation function  $I_d(y)$ . For all states  $y \in \left[0, \frac{D-B}{\gamma}\right)$  we must have  $I_d(y) = 1$ , since  $I_d(y) = 0$  would mean that  $O_1(y) = D$  which contradicts repudiation-proofness. There may be more states for which  $I_d(y) = 1$ , as we only know that  $I_d(y) \ge \tilde{I}_d(y)$ . Let  $T_2$  denote the set of those states, so  $T_2 = \left\{ y \ge \frac{D-B}{\gamma} \middle| I_d(y) = 1 \right\}$ . We see that

$$EI_d = \int_{0}^{\frac{D-B}{\gamma}} 1f(y)dy + \int_{T_2} 1f(y)dy$$

furthermore we have

$$E\tilde{I_d} = \int_{0}^{\frac{D-B}{\gamma}} 1f(y)dy$$

Now since  $\tilde{D} \leq D$  and B > 0, it follows that (i)  $D = \tilde{D}$  and (ii)  $T_2$  has probability

mass zero; hence we see that  $I_d = \tilde{I}_d$  almost surely. It follows that, as  $EO_1 = E\tilde{O}_1$ , we must also have that  $O_1 = \tilde{O}_1$  almost surely, and I conclude that the optimal contract is a sovereign debt contract.

# A.2 **Proofs of Chapter 3**

# A.2.1 Proof of Proposition 3.2.1

The dynamic investment problem to solve is<sup>1</sup>

$$\max_{\alpha_t,\alpha_{t+1}} Eu(\tilde{b})$$

such that

$$w_{t} = 0$$
  

$$w_{t+1} = \alpha_{t}(1+\tilde{r}_{t}) + (y - \alpha_{t})$$
  

$$w_{t+2} = \alpha_{t+1}(1+\tilde{r}_{t+1}) + (w_{t+1} + y - \alpha_{t+1})$$
  

$$\tilde{b} = w_{t+2}$$

First, we rewrite this problem in recursive form

$$v_t(w_t) = \max_{\alpha_t} E v_{t+1} \left( (w_t + y) + \alpha \tilde{r} \right)$$
(A.2.1)

where  $v_t$ , the remaining-value function, is a function of the financial reserve,  $w_t$ . We know that  $v_{t+2}(w) = u(w) = \frac{w^{1-\phi}}{1-\phi}$ , as individuals consume their financial reserve in retirement. Note that  $v_t(0)$  is the expected utility in retirement of a young individual at time *t* who invests optimally throughout his life. Optimal investment,  $\alpha_t$ , is a function of the single state variable,  $w_t$ .

We consider the trial solution function  $v_{t+1}(w_{t+1}) = \gamma_{t+1} \frac{(w_{t+1}+h_{t+1})^{1-\phi}}{1-\phi}$ , where  $\gamma_{t+1} > 0$  is a scalar and  $h_{t+1}$  is the human capital reserve of an individual. Our trial

<sup>&</sup>lt;sup>1</sup>We drop the superscripts to save on notation.

solution implies that

$$v_{t+1}((w_t + y) + \alpha_t \tilde{r}) = \gamma_{t+1} \frac{((w_t + y) + \alpha_t \tilde{r} + h_{t+1})^{1-\phi}}{1-\phi}$$
$$= \gamma_{t+1} \frac{(w_t + h_t + \alpha_t \tilde{r})^{1-\phi}}{1-\phi}$$

so that the first-order condition reads as

$$\gamma_{t+1} E \tilde{r} \left( w_t + h_t + \alpha_t \tilde{r} \right)^{-\phi} = 0$$

Solving for optimal investment in the risky asset yields

$$\alpha_t(w_t) = a^* \left( w_t + h_t \right)$$

where

$$a^* := \frac{\left(\frac{-r^l(1-p)}{r^h p}\right)^{\frac{1}{\phi}} - 1}{r^l - r^h \left(\frac{-r^l(1-p)}{r^h p}\right)^{\frac{1}{\phi}}}$$
(A.2.2)

Finally, it follows from (A.2.1) that

$$v_t(w_t) = \gamma_{t+1} E \frac{(w_t + h_t + a^* (w_t + h_t) \tilde{r})^{1-\phi}}{1-\phi}$$
  
=  $\delta \gamma_{t+1} \frac{(w_t + h_t)^{1-\phi}}{1-\phi}$ 

where

$$\delta := E \left( 1 + a^* \tilde{r} \right)^{1 - \phi}$$

so that our trial solution is correct with  $\gamma_t = \delta \gamma_{t+1}$ . Note that

$$\delta \approx 1 + (1 - \phi)a^*\mu > 1$$

where we've used a first-order approximation. We conclude that, conditional on

an optimal investment strategy, expected lifetime utility of the young at time t is

$$v_t(0) = \delta^2 u(h_t)$$
$$= \delta^2 u(2y)$$

# A.2.2 Proof of Proposition 3.2.2

Incomplete markets give rise to additional constraints,  $\alpha_t = 0$  and  $\alpha_{t+1} \le w_{t+1}$ . To obtain a lower bound for the welfare loss due to credit constraints, we assume that the middle-aged do not face constraints, only the young do. Then the new investment problem is

$$\max_{\alpha_{t+1}} Eu\left(\tilde{b}\right)$$

such that

$$w_{t} = 0$$
  

$$w_{t+1} = y$$
  

$$w_{t+2} = \alpha_{t+1} (1 + \tilde{r}_{t+1}) + (w_{t+1} + y - \alpha_{t+1})$$
  

$$\tilde{b} = w_{t+2}$$

It is easy to see that the middle aged will invest  $a^*(2y)$  so that

$$w_{t+2} = 2y(1+a^*\tilde{r}_{t+1})$$

and

$$Eu(\tilde{b}) = Eu(2y(1+a^*\tilde{r}_{t+1}))$$
  
=  $E(1+a^*\tilde{r}_{t+1})^{1-\phi}u(2y)$   
=  $\delta$   $u(2y)$ 

With proposition 4.3.2 we see that the lower bound for the welfare loss due to credit constraints is

$$\left(\delta^2 - \delta\right) u(2y)$$

# A.3 Proofs of Chapter 4

# A.3.1 Cobb-Douglas Production

We derive the steady state equilibria for a Cobb-Douglas production economy, where F is given by

$$F(K,L) = K^{\alpha}L^{\beta} \tag{A.3.1}$$

with  $0 < \alpha + \beta < 1$ . Steady-state capital allocations for  $c \in [0, \overline{c}]$  are

$$K^{Y*} = \left(\frac{\alpha\theta^Y}{r^*(c)+c}\right)^{\frac{1}{1-\alpha}}; \quad K^{O*} = \left(\frac{\alpha\theta^O}{r^*(c)}\right)^{\frac{1}{1-\alpha}}$$
(A.3.2)

with  $r^*(c)$  given by capital market clearing condition

$$\left(\frac{\alpha\theta^{Y}}{r^{*}(c)+c}\right)^{\frac{1}{1-\alpha}} + \left(\frac{\alpha\theta^{O}}{r^{*}(c)}\right)^{\frac{1}{1-\alpha}} = \bar{K}$$

The boundary value  $\bar{c}$  follows from (4.3.21) and is given by

$$\bar{c} = \alpha \frac{\theta^{Y} - \theta^{O}}{\left(\frac{1}{2}\bar{K}\right)^{1-\alpha}}$$

For  $c > \overline{c}$  we have steady state capital allocations

$$K^{Y*} = \left(\frac{\alpha\theta^Y}{r^*(c)+c}\right)^{\frac{1}{1-\alpha}}; K^{O*} = \frac{1}{2}\bar{K}$$

where capital market clearing condition

$$\left(\frac{\alpha\theta^Y}{r^*(c)+c}\right)^{\frac{1}{1-\alpha}} = \frac{1}{2}\bar{K}$$

allows us to obtain the equilibrium interest rate explicitly,

$$r^*(c) = \frac{\alpha \theta^Y}{\left(\frac{1}{2}\bar{K}\right)^{1-\alpha}} - c$$

Next we determine what steady state equilibria can be politically supported.

# A.3.2 Proof of lemma 4.4.2

Consider the equilibrium interest rate in period *t*. For  $c_t < \bar{c}_t$ , we have  $\hat{K}_t^O \ge \bar{K}_t^O$  so that old dirms scale down and the interest rate is given by

$$\left(\frac{\alpha\theta^{Y}}{r_{t}+c_{t}}\right)^{\frac{1}{1-\alpha}}+\left(\frac{\alpha\theta^{O}}{r_{t}}\right)^{\frac{1}{1-\alpha}}=\bar{K}$$

Implicit differentation yields

$$-\frac{1}{1-\alpha}\frac{K_{t}^{Y}}{r_{t}+c_{t}}\left(\frac{dr_{t}}{dc_{t}}+1\right)-\frac{1}{1-\alpha}\frac{K_{t}^{O}}{r_{t}}\frac{dr_{t}}{dc_{t}}=0$$
(A.3.3)

which rewrites as

$$\frac{dr_t}{dc_t} = -\frac{K_t^Y}{K_t^Y + K_t^O(\frac{r_t + c_t}{r_t})}$$

so that  $\frac{dr_t}{dc_t} \varepsilon(-1,0)$ . For  $c_t \ge \bar{c}_t$ , we have  $K_t^O = \hat{K}_t^O$  and the interest rate is given by

$$\left(\frac{\alpha\theta^{Y}}{r_{t}+c_{t}}\right)^{\frac{1}{1-\alpha}} = \bar{K} - \hat{K}_{t}^{O}$$
$$r_{t} = \frac{\alpha\theta^{Y}}{\left(\bar{K} - \hat{K}_{t}^{O}\right)^{1-\alpha}} - c_{t}$$

and we see that  $\frac{dr_t}{dc_t} = -1$ . Now for (i), capital in the Y-sector is given by

$$K_t^Y = \left(\frac{\alpha \theta^Y}{r_t + c_t}\right)^{\frac{1}{1-\alpha}}$$

Taking the derivative with respect to  $c_t$  gives

$$\frac{dK_t^Y}{dc_t} = \frac{-1}{1-\alpha} \frac{K_t^Y}{r_t + c_t} \left(\frac{dr_t}{dc_t} + 1\right)$$

so that  $\frac{dK_t^Y}{dc_t} < 0$  for  $c_t \in [0, \bar{c}_t)$  and  $\frac{dK_t^Y}{dc_t} = 0$  for  $c \ge \bar{c}_t$ . Wages in the Y-sector are

$$w_t^Y = \theta^Y \beta \left( K_t^Y \right)^{\alpha}$$

Taking the derivative with respect to  $c_t$  yields

$$\frac{dw_t^Y}{dc_t} = -\frac{\beta}{1-\alpha} K_t^Y \left(\frac{dr_t}{dc_t} + 1\right)$$
(A.3.4)

so that  $\frac{dw_t^Y}{dc_t} < 0$  for  $c_t \in [0, \bar{c}_t]$ ;  $\frac{dw_t^Y}{dc_t} = 0$  for  $c_t > \bar{c}_t$ . For (ii), capital in the O-sector is given by

$$K_t^O = \left(\frac{\alpha \theta^O}{r_t}\right)^{\frac{1}{1-\alpha}}$$

for  $c_t < \bar{c}_t$ ; and

$$K_t^O = \hat{K}^O$$

for  $c_t \ge \bar{c}_t$ . Hence  $\frac{dK_t^O}{dc_t} > 0$  for  $c_t \in [0, \bar{c}_t)$  and  $\frac{dK_t^O}{dc_t} = 0$  for  $c_t \ge \bar{c}_t$ . Wages in the O-sector are

$$w_t^O = \theta^O \beta \left( K_t^O \right)^C$$

Taking the derivative with respect to  $c_t$  gives

$$\frac{dw_t^O}{dc_t} = -\frac{\beta}{1-\alpha} K_t^O \frac{dr_t}{dc_t}$$
(A.3.5)

so that  $\frac{dw^O}{dc_t} > 0$  for  $c_t \varepsilon[0, \bar{c}_t)$  and  $\frac{dw^O_t}{dc_t} = 0$  for  $c_t \ge \bar{c}_t$ . For (iii), first consider profits. Let  $\pi_t^Y$  and  $\pi_t^O$  denote profits in the Y- and O-sector respectively. Then

$$\pi_t^Y = \theta^Y \left( K_t^Y \right)^\alpha - (r_t + c_t) K_t^Y - w_t^Y$$

and

$$\pi_t^O = (\theta^O)(K_t^O)^\alpha - r_t K_t^O - w_t^O$$

We take the derivative of  $\pi_t^Y$  with respect to  $c_t$  and obtain

$$\frac{d\pi_t^Y}{dc_t} = \frac{\alpha + \beta - 1}{1 - \alpha} \left( \frac{dr_t}{dc_t} + 1 \right) K_t^Y$$

so that  $\frac{d\pi_t^Y}{dc_t} < 0$  for  $c_t \in [0, \bar{c}_t)$  and  $\frac{d\pi_t^Y}{dc_t} = 0$  for  $c \ge \bar{c}_t$ . Similarly, for O-sector profits, we get

$$\frac{d\pi_t^O}{dc_t} = \frac{\alpha + \beta - 1}{1 - \alpha} \frac{dr_t}{dc_t} K_t^O$$

so that  $\frac{d\pi_t^O}{dc_t} > 0$  for  $c_t \in [0, \bar{c}_t)$  and  $\frac{d\pi_t^O}{dc_t} > 0$  for  $c_t \ge \bar{c}_t$ . Turning to total profits,  $\Pi_t = \pi_t^Y + \pi_t^O$ , we have

$$\frac{d\Pi_t}{dc_t} = \frac{\alpha + \beta - 1}{1 - \alpha} \left[ \left( \frac{dr_t}{dc_t} + 1 \right) K_t^Y + \frac{dr_t}{dc_t} K_t^O \right]$$
(A.3.6)

and we see that  $\Pi_t$  is decreasing in  $c_t$  iff

$$\frac{dr_t}{dc_t}\bar{K} + K_t^Y \ge 0$$

Recall that for  $c_t \in [0, \bar{c}_t)$  we have

$$\frac{dr_t}{dc_t} = -\frac{K_t^Y}{K_t^Y + K_t^O(\frac{r_t + c_t}{r_t})}$$

so that  $-\frac{dr_t}{dc_t}\bar{K} \leq K_t^Y$  and total profits are nonincreasing in  $c_t$ . For  $c_t \geq \bar{c}_t$ , we have  $\frac{dr_t}{dc_t} = -1$  so that total profits are increasing in  $c_t$ . This result is due to the fact that  $r_t$  declines in  $c_t$  while the allocation of capital does not change in this range of capital market frictions. Hence the cost of capital goes down for O-firms, stays the same for Y-firms, as production in both sectors remains the same. While profits may increase in  $c_t$ , capital income cannot. Recall that capital income is given by

$$s_t = \frac{r_t \bar{K} + \Pi_t}{\eta}$$

For  $c_t \in [0, \bar{c}_t)$ , we have  $\frac{d\Pi_t}{dc_t} \leq 0$  so that  $\frac{ds_t}{dc_t} \leq 0$ . For  $c_t \geq \bar{c}_t$  we have

$$\frac{ds_t}{dc_t} = \frac{1}{\eta} \left( -\bar{K} + \frac{1 - \alpha - \beta}{1 - \alpha} K_t^O \right)$$

so that  $\frac{ds_t}{dc_t} < 0$ . Now, old capitalists income is given by

$$w_t^O + s_t$$

Let  $c_t \in [0, \bar{c}_t)$ , then we have

$$\frac{d(w_t^O + s_t)}{dc_t} = -\frac{\beta}{1 - \alpha} K_t^O \frac{dr_t}{dc_t} + \frac{1}{\eta} \left( \frac{\beta}{1 - \alpha} \frac{dr_t}{dc_t} \bar{K} + \frac{\alpha + \beta - 1}{1 - \alpha} K_t^Y \right)$$

$$< -\frac{\beta}{1 - \alpha} K_t^O \frac{dr_t}{dc_t} + \frac{\beta}{1 - \alpha} \frac{dr_t}{dc_t} \bar{K}$$

$$< 0$$

Next, for  $c_t \ge \bar{c}_t$ ,  $\frac{dw_t^O}{dc_t} = 0$  and  $\frac{ds_t}{dc_t} < 0$  which shows part (iii).

# A.3.3 Proof of Lemma 4.4.7

To prove lemma 4.4.7, we first prove two auxiliary lemmas that give the economic equilibrium in periods t and t + 1. Since we consider out-of steady-state dynamics we assume that the equilibrium at time t - 1 is given by steady state allocations for some arbitrary  $c \in [0, \overline{c}]$ . The first lemma describes the economic equilibrium after a downward change in policy:

Let the economic equilibrium at time t - 1 be given by steady state values for some  $c \le \overline{c}$ . Consider a downward change in policy  $c_t = c_{t+1} \le c$ , then

$$K_t^Y = \left(\frac{\alpha \theta^Y}{r_t^* + c_t}\right)^{\frac{1}{1-\alpha}}$$

and

$$K_{t+1}^{O} = \left(\frac{\alpha \theta^{O}}{r_{t}^{*}}\right)^{\frac{1}{1-\alpha}}$$

with  $r_t^*$  given by  $K_t^Y + K_t^O = \overline{K}$ , furthermore  $K_{t+1}^Y = K_t^Y$  and  $K_{t+1}^O = K_t^O$ .

Since  $c_t \leq c$ , we have  $\bar{K}^O(c) \geq \bar{K}^O(c_t)$ , where  $\bar{K}^O$  is the equilibrium cut-off value function defined after (4.3.14). It follows that

$$\hat{K}_t^O = K_{t-1}^Y \ge \bar{K}^O(c_t)$$

so that  $K_t^Y$  and  $K_t^O$  are as posed. Note that because  $c_t \leq c$ , we have  $K_t^O < K_{t-1}^O$  by

lemma 4.4.1 and hence  $K_t^Y > K_{t-1}^Y$  by market clearing. We must verify that the same allocation obtains in period t + 1. Moving forward one period we have

$$\hat{K}_{t+1}^{O} = K_{t}^{Y} > \bar{K}^{O}(c_{t+1})$$

so that again old firms scale down, the same interest rate obtains (i.e.  $r_t^* = r_{t+1}^*$ ), and allocations are as posed.

Lemma A.3.3 shows that a downward change in policy results in steady state values that correspond to a lower reallocation cost. Informally, we can say that changing policy downward moves the economy to a new steady state corresponding to the new value of the friction  $c_t$ . The same need not be true for an upward policy change as the next lemma shows.

Let the economic equilibrium at time t - 1 be given by steady state values for some  $c \le \overline{c}$ . Consider an upward change in policy  $c_t = c_{t+1} > c$ , then

(i) if  $c_t \leq \bar{c}$  we have

$$K_t^Y = \left(\frac{\alpha \theta^Y}{r_t^* + c_t}\right)^{\frac{1}{1-\alpha}} \quad ; \quad K_t^O = \left(\frac{\alpha \theta^O}{r_t^*}\right)^{\frac{1}{1-\alpha}}$$

with  $r_t^*$  given by  $K_t^Y + K_t^O = \overline{K}$ , furthermore  $K_{t+1}^Y = K_t^Y$  and  $K_{t+1}^O = K_t^O$ ; (ii) if  $\overline{c} < c_t \le \overline{c}_t$  we have

$$K_t^Y = \left(\frac{\alpha\theta^Y}{r_t^* + c_t}\right)^{\frac{1}{1-\alpha}} \quad ; \quad K_t^O = \left(\frac{\alpha\theta^O}{r_t^*}\right)^{\frac{1}{1-\alpha}}$$

with  $r_t^*$  given by  $K_t^Y + K_t^O = \bar{K}$ , furthermore  $K_{t+1}^Y = K_t^O$  and  $K_{t+1}^O = K_t^Y$ ; and (iii) if  $c_t > \bar{c}_t$  we have

$$K_t^Y = \bar{K} - \hat{K}_t^O \quad ; \quad K_t^O = \hat{K}_t^O$$

furthermore  $K_{t+1}^O = K_t^Y$  and  $K_{t+1}^Y = K_t^O$ .

For (i): suppose  $c < c_t \le \overline{c}$ . Then also  $c_t \le \overline{c}_t$ , where  $\overline{c}_t$  is given by (4.4.2).

Hence we have

$$K_{t-1}^Y = \hat{K}_t^O \ge \bar{K}^O(c_t)$$

by the monotonicity of  $\bar{K}^O$ . It follows that old firms scale down and  $K_t^Y$  and  $K_t^O$  are as posed. Consider period t + 1. Since  $c_t = c_{t+1} \leq \bar{c}$ , we have

$$K_t^Y(c_t) = \hat{K}_{t+1}^O \ge \bar{K}^O(c_{t+1})$$

by the definition of  $\bar{c}$ , given by (4.3.21), and the monotonicity in of  $K^Y$  and  $\bar{K}^O$  in  $c_t$ . Hence  $K_{t+1}^Y$  and  $K_{t+1}^O$  are as posed.

For (ii): suppose  $\bar{c} < c_t \leq \bar{c}_t$ . Then we have

$$\hat{K}_t^O \ge \bar{K}^O(c_t)$$

so that old firms scale down and allocations are as posed. Moving forward one period it follows from  $\bar{c} < c_t = c_{t+1}$  that

$$K_t^Y(c_t) < \bar{K}^O(c_{t+1})$$

Hence old firms do not adjust capital and  $K_{t+1}^O = K_t^Y$ . By market clearing then  $K_{t+1}^Y = K_t^O$ .

For (iii): suppose  $\bar{c}_t < c_t$ , then  $\hat{K}_t^O < \bar{K}^O(c_t)$  so that old firms do not adjust capital. It follows that  $K_t^O = \hat{K}_t^O$  and, by market clearing,  $K_t^Y = \bar{K} - \hat{K}^O$ . In period t + 1, since  $\bar{c} < c_t = c_{t+1}$  we have  $\hat{K}_{t+1}^O < \bar{K}^O(c_{t+1})$  and so  $K_{t+1}^O = K_t^Y$  and  $K_{t+1}^Y = K_t^O$ .

Lemma A.3.3 shows that a small upward change in policy  $(c_t \le \bar{c})$  results in steady state allocations that correspond to a higher friction. Now, consider lifetime utility  $U_t^{YW}$  of the young worker at time *t*. Lemma A.3.3 also implies that young workers will not vote for a higher friction than  $\bar{c}$ . To see this note that  $K_t^Y$  and  $K_{t+1}^O$  are strictly decreasing in  $c_t$  for  $\bar{c} < c_t \le \bar{c}_t$ ; they are constant in  $c_t$  for  $c_t > \bar{c}_t$ . Hence young workers strictly prefer  $c_t = \bar{c}$  over any  $c_t > \bar{c}$ .

With the auxiliary lemmas, we can now proof lemma 4.4.7. The choice of a

persistent friction,  $c_t = c_{t+1} \in [0, \infty]$ , uniquely determines the equilibrium interest rates,  $r_t^*$  and  $r_{t+1}^*$ , and hence the economic equilibrium at time t and t+1 (cf lemma 4.3.1). Let  $c_t^{YW}$  denote the preferred policy of the YW class. We have shown that  $c_t^{YW} \varepsilon[0, \bar{c}]$  and that, if this policy is set, the economy attains steady state values corresponding to the friction  $c_t^{YW}$ . It follows that  $c_t^{YW} = c^{YW}$ , where  $c^{YW}$  is given by lemma 4.4.5. By sincere voting we have  $a_t^{YW} = c^{YW}$ , which concludes the proof.