Mean-field phase diagram of two-dimensional electrons with disorder in a weak magnetic field

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We study two-dimensional interacting electrons in a weak perpendicular magnetic field with the filling factor $\nu \gg 1$ and in the presence of a quenched disorder. In the framework of the Hartree-Fock approximation, we obtain the mean-field phase diagram for the partially filled highest Landau level. We find that the charge-density-wave (CDW) state can exist if the Landau level broadening $1/2\tau$ does not exceed the critical value $1/2\tau_c = 4T_0/\pi$, where T_0 is the critical temperature of the CDW formation in the clean case. Our analysis of weak crystallization corrections to the mean-field results shows that these corrections are of the order of $(1/\nu)^{2/3} \ll 1$ and therefore can be neglected.

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I. INTRODUCTION

A two-dimensional electron gas (2DEG) in a perpendicular magnetic field was a subject of intensive studies, both theoretical and experimental, for several decades. The behavior of the system in a strong magnetic field where only the lowest Landau level is occupied has been investigated in great details.[1] Several attempts[2,3] were made in order to incorporate the case with larger filling factors $\nu > 1$ into the theory. Usually in these approaches, the ratio of the characteristic Coulomb energy (at distances of the order of the magnetic length) to the cyclotron energy has been assumed to be small. However, in a weak magnetic field this is not the case, and the characteristic Coulomb energy exceeds the cyclotron energy. An attempt to investigate the situation with large Coulomb energy was made in Ref. 4.

The progress in understanding the clean 2DEG in a weak magnetic field was achieved by Aleiner and Glazman.[5] They have derived the low-energy effective theory on the partially filled highest Landau level by using the small parameter $1/\nu \ll 1$. By treating the effective interaction within the Hartree-Fock approximation, Koulakov, Fogler, and Shklovskii[6] predicted a unidirectional charge-density-wave (CDW) state (stripe phase) for the half filled highest Landau level at zero temperature and in the absence of disorder. Moessner and Chalker[8] extended the ideas of Fukuyama, Platzmann, and Anderson[7] to the case of a partially filled highest Landau level and showed the existence of the mean-field CDW state on the half-filled Landau level below some temperature T_0.

Recently, the existence of compressible states near half filling with anisotropic transport properties was demonstrated experimentally for high Landau levels.[9,10] This stimulates an extensive study of the clean 2DEG in a weak magnetic field and pinning of stripes by disorder.[11]

In the clean case, the properties of the CDW states can be described on the basis of the low-energy effective theory for smooth "elastic" deformations.[11] Recently, attempts were made to derive such a theory microscopically starting from the mean-field solution.[12,13] The effects of a quenched disorder on the unidirectional CDW state (stripe phase) were investigated in the framework of the phenomenological elasticity theory,[14] and a rich variety of different regimes, which depend on the strength of disorder, were found. However, to identify the phenomenological parameters of the theory, a successive microscopic theory should be developed.

At present, a thorough microscopic analysis of the effects of disorder on the mean-field transition from the liquid state to the CDW one, as well as on the phase diagram of the mean-field CDW states, is absent. The main objective of the present paper is to investigate these effects on the existence of the mean-field CDW states in 2DEG in a weak perpendicular magnetic field H (filling factor $\nu \gg 1$). For the considered case of a large number of the occupied Landau levels, the mean-field analysis is legitimate because the fluctuations of the order parameter are strongly suppressed.[11] On the other hand, the mean-field approach cannot be applied to the critical region in the direct vicinity of the phase transition. This region is, however, small and does not lead to any significant uncertainty in our results for critical temperatures of the transitions.

We assume the presence in the system of a weak quenched disorder, i.e., the elastic collisions time satisfies the condition $\tau_0 \gg \omega_H^{-1}$, where $\omega_H = eH/m$ is the cyclotron frequency, e the electron charge, and m the effective electron mass (we use the units with $\hbar = 1$, $c = 1$, and $k_B = 1$). Under this condition, the Landau-level broadening $1/2\tau$, which is of the order of $\sqrt{\omega_H \tau_0}/\tau_0$, is much less than the spacing ω_H between them. This case can be realized in high mobility samples which were used for experimental studies of the anisotropic magnetotransport.[9,10,15] Keeping in mind that the relation $T_0 \sim 1/\tau$ usually holds, one expects a much more pronounced influence of the quenched disorder on the properties of electrons on the partially filled highest Landau level even for a small level broadening $1/2\tau \ll \omega_H$.

One of the main results of our paper is that at zero temperature the mean-field CDW state is destroyed when the Landau-level broadening exceeds the critical value $1/2\tau_c$

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$=4T_0/\pi$. At nonzero temperatures the quenched disorder leads to the decrease of the temperature of the CDW instability as compared to the clean case. The physical reason is that the scattering on impurities breaks the CDW correlations, and therefore results in the destruction of the coherent CDW state. This is somewhat similar to the suppression of the critical temperature in conventional superconductors by magnetic impurities[16] or in anisotropic superconductors by nonmagnetic impurities.[17]

The paper is organized as follows. In Sec. II we introduce the formalism that allows us to evaluate the free energy of the CDW state in the presence of disorder. In Sec. III we investigate the instability of the liquid state towards the formation of the CDW state, and present the mean-field phase diagrams at the half filling and arbitrary temperature, and at zero temperature and arbitrary filling. The weak crystallization corrections to the mean-field solution are presented in Sec. IV. Section V contains the comparison of the theory with the recent experimental and numerical results. We end with conclusions in Sec. VI.

II. FREE ENERGY OF THE CDW STATES

We consider two-dimensional interacting electrons in the presence of a weak quenched disorder and a weak perpendicular magnetic field. The parameter that characterizes the strength of the Coulomb interaction is $r_s=\sqrt{2}e^2/\varepsilon v_F$ with v_F being the Fermi velocity and ε the dielectric constant of a media. We assume that the Coulomb interaction between the electrons is weak, $r_s \ll 1$, and the magnetic field obeys the condition $Nr_s \gg 1$, where $N=[\nu/2]$ is the integer part of $\nu/2$. In this case it is possible to construct an effective-field theory for the electrons on the highest partially filled Landau level by integrating out all other degrees of freedom.[5,18] We also assume that the electrons on the partially filled highest Landau level are spin polarized. This assumption is based on the calculations[3,19] that show the existence of fractional states, composite fermions, and skyrmions only on the lowest and the first excited Landau levels, as well as on the experimental observations.

In order to study the transition from the liquid state to the CDW one we employ the Landau expansion of the free energy in powers of the CDW order parameter $\Delta(\mathbf{q}_j)$, where the vectors $\mathbf{q}_j$ that characterize the CDW state have the same length[7] $q_j=Q$. We perform the expansion up to the fourth order in the CDW order parameter under the assumption $Nr_s^2 \gg 1$. In this case the Hartree-Fock approximation is well justified[8] because the corrections are small in the parameter $a_B/l_H=1/Nr_s^2 \ll 1$, where $a_B=\varepsilon/me^2$ is the Bohr radius and $l_H=1/\sqrt{m\omega_H}$ the magnetic length.

A. Formalism

The thermodynamical potential of the spin-polarized 2DEG projected on the Nth Landau level in the presence of the random potential $V_{dis}(\mathbf{r})$ and the magnetic field is given by

$$\Omega=-\frac{T}{N_r}\int \mathcal{D}[\overline{\psi},\psi]\int \mathcal{D}[V_{dis}]\mathcal{P}[V_{dis}]\exp(\mathcal{S}[\overline{\psi},\psi,V_{dis}]), \tag{1}$$

where the action $\mathcal{S}[\overline{\psi},\psi,V_{dis}]$ in the Matsubara representation has the form

$$\mathcal{S}=\int_r \sum_{\omega_n}^{\alpha}\Bigg\{\overline{\psi^{\alpha}_{\omega_n}}(\mathbf{r})[i\omega_n+\mu-\mathcal{H}_0-V_{dis}(\mathbf{r})]\psi^{\alpha}_{\omega_n}(\mathbf{r}) - \frac{T}{2}\sum_{\omega_m,\nu_l}\int_{\mathbf{r}'}\overline{\psi^{\alpha}_{\omega_n}}(\mathbf{r})\psi^{\alpha}_{\omega_n-\nu_l}(\mathbf{r})U_0(\mathbf{r},\mathbf{r}')\overline{\psi^{\alpha}_{\omega_m}}(\mathbf{r}) \times\psi^{\alpha}_{\omega_m+\nu_l}(\mathbf{r}')\Bigg\}. \tag{2}$$

Here $\psi^{\alpha}_{\omega_n}(\mathbf{r})$ and $\overline{\psi^{\alpha}_{\omega_n}}(\mathbf{r})$ are the annihilation and creation operators of an electron on the Nth Landau level, T the temperature, μ the chemical potential, $\omega_n=\pi T(2n+1)$ the Matsubara fermionic frequency, and $\nu_n=2\pi Tn$ the Bosonic one. The free Hamiltonian $\mathcal{H}_0$ for 2D electrons with mass m in the perpendicular magnetic field $H=\epsilon_{ab}\partial_a A_b$ is $\mathcal{H}_0=(-i\nabla-e\vec{A})^2/(2m)$. The screened electron-electron interaction $U_0(\mathbf{r})$ on the Nth Landau level takes into account the effects of interactions with electrons on the other levels, and has the form (see Ref. 5 for the clean case and Ref. 18 for the weakly disordered case)

$$U_0(q)=\frac{2\pi e^2}{\varepsilon q}\frac{1}{1+\frac{2}{qa_B}\left(1-\frac{\pi}{6\omega_H\tau}\right)[1-\mathcal{J}_0^2(qR_c)]}, \tag{3}$$

where $R_c=l_H\sqrt{\nu}$ is the cyclotron radius on the Nth Landau level and $\mathcal{J}_0(x)$ the Bessel function of the first kind. The range of the screened electron-electron interaction (3) is determined by the Bohr radius a_B. We also assume the Gaussian distribution for the random potential $V_{dis}(\mathbf{r})$

$$\mathcal{P}[V_{dis}(\vec{r})]=\frac{1}{\sqrt{\pi g}}\exp\left(-\frac{1}{g}\int_r V_{dis}^2(\mathbf{r})\right), \tag{4}$$

where $g=1/\pi\rho\tau_0$, ρ is the thermodynamical density of states. This distribution corresponds to a short-range random potential with the correlation length smaller than the magnetic field length l_H. In order to average over the disorder we introduce N_r replicated copies of the system labeled by the replica indices $\alpha=1,\ldots,N_r$.

B. Hartree-Fock decoupling and the average over disorder

The CDW ground state is characterized by the order parameter $\Delta(\mathbf{q})$ that is related to the electron density

$$\langle\rho(\mathbf{q})\rangle=L_xL_yn_LF_N(q)\Delta(\mathbf{q}). \tag{5}$$

Here L_xL_y is the area of the 2DEG, $n_L=1/2\pi l_H^2$ the number of states on one Landau level, and the form factor $F_N(q)$ is

$$F_N(q)=L_N\left(\frac{q^2l_H^2}{2}\right)\exp\left(-\frac{q^2l_H^2}{4}\right), \tag{6}$$

where $L_N(x)$ is the Laguerre polynomial. For the case $N \gg 1$, one can use the following asymptotic expression for the form factor (6):

$$F_N(q)=\mathcal{J}_0(qR_c), \quad qR_c \ll \frac{R_c^2}{l_H^2}=\nu. \tag{7}$$

After the Hartree-Fock decoupling[20] of the interaction term in the action (2) we obtain

$$\mathcal{S}=-\frac{N_r\Omega_\Delta}{T}+\int_r\sum_{\omega_n}^{\alpha}\overline{\psi^\alpha_{\omega_n}}(\mathbf{r})[i\omega_n+\mu-\mathcal{H}_0-V_{dis}(\mathbf{r})+\lambda(\mathbf{r})]\psi^\alpha_{\omega_n}(\mathbf{r}), \tag{8}$$

$$\Omega_\Delta=\frac{n_L(L_xL_y)^2}{2}\int_q U(q)\Delta(\mathbf{q})\Delta(-\mathbf{q}), \tag{9}$$

where the potential $\lambda(\mathbf{r})$ results from the perturbation of the uniform electron density by the charge-density wave, and is connected with the CDW order parameter as follows:

$$\lambda(\mathbf{q})=L_xL_yU(q)F_N^{-1}(q)\Delta(\mathbf{q}), \tag{10}$$

and $U(q)=-n_LU_{HF}(q)$ with the Hartree-Fock potential $U_{HF}(q)$ given by

$$U_{HF}(q)=U_0(q)F_N^2(q)-\int_p\frac{e^{-i\mathbf{q}\mathbf{p}l_H^2}}{n_L}U_0(\mathbf{p})F_N^2(\mathbf{p}). \tag{11}$$

The averaging over the random potential $V_{dis}(\mathbf{r})$ in Eq. (1) is straightforward and results in the following quartic term:

$$\frac{g}{2}\int_r\sum_{\omega_n\omega_m}^{\alpha\beta}\overline{\psi^\alpha_{\omega_n}}(\mathbf{r})\psi^\alpha_{\omega_n}(\mathbf{r})\overline{\psi^\beta_{\omega_m}}(\mathbf{r})\psi^\beta_{\omega_m}(\mathbf{r}) \tag{12}$$

in the action. This term can be decoupled by means of the Hubbard-Stratonovich transformation[21] with the Hermitian matrix field variables[22,23] $Q^{\alpha\beta}_{nm}(\vec{r})$

$$\int\mathcal{D}[Q]\exp\int_r\left[-\frac{1}{2g}\mathrm{tr}Q^2(\vec{r})+i\psi^\dagger(\vec{r})Q(\vec{r})\psi(\vec{r})\right], \tag{13}$$

where the symbol tr denotes the matrix trace over the Matsubara and replica indices. The measure for the functional integral over the matrix field Q is defined as: the integral (13) equals unity when the Fermionic fields $\psi^\dagger$ and ψ vanish. Note also that in Eq. (13) we introduce the matrix notations according to

$$\psi^\dagger(\cdots)\psi=\sum_{\omega_n,\omega_m}^{\alpha,\beta}\overline{\psi^\alpha_{\omega_n}}(\cdots)^{\alpha\beta}_{nm}\psi^\beta_{\omega_m}. \tag{14}$$

After making all these steps, the action becomes

$$\mathcal{S}=-\frac{N_r\Omega_\Delta}{T}-\frac{1}{2g}\int_r\mathrm{tr}Q^2+\int_r\psi^\dagger(\vec{r})(i\omega+\mu-\mathcal{H}_0+\lambda+iQ)\psi(\vec{r}), \tag{15}$$

where ω is the frequency matrix $(\omega)^{\alpha\beta}_{nm}=\omega_n\delta_{nm}\delta^{\alpha\beta}$.

C. Saddle point in the Q field

The Q matrix field can be naturally split into the transverse V and the longitudinal P components as follows: $Q=V^{-1}PV$. The longitudinal component P has the block-diagonal structure in the Matsubara space, $P^{\alpha\beta}_{nm}\propto\Theta(nm)$, where $\Theta(x)$ is the Heaviside step function, and corresponds to massive modes. The transverse component V is a unitary rotation and describes massless (diffusive) modes (see Refs. 24 and 25 for details).

This decomposition of the variable Q into P and V is motivated by the saddle-point structure of the action (15) at zero temperature ($\omega_n\to 0$) and in the absence of the potential $\lambda(\mathbf{r})$. The corresponding saddle-point solution has the form $Q_{sp}=V^{-1}P_{sp}V$, where the matrix P_{sp} is

$$(P_{sp})^{\alpha\beta}_{nm}=P^n_{sp}\delta_{nm}\delta^{\alpha\beta} \tag{16}$$

with P^n_{sp} obeying the equation

$$\pi\rho\tau_0P^n_{sp}=iG^n_0(\mathbf{r},\mathbf{r}). \tag{17}$$

This equation is equivalent to the self-consistent Born approximation equation.[26] The Green function $G^n_0(\mathbf{r},\mathbf{r}')$ is determined as

$$G^n_0(\mathbf{r},\mathbf{r}')=\sum_k\phi^*_{Nk}(\mathbf{r})G_0(\omega_n)\phi_{Nk}(\mathbf{r}'), \tag{18}$$

$$G_0(\omega_n)=[i\omega_n+\mu-\epsilon_N+iP^n_{sp}]^{-1}, \tag{19}$$

where $\epsilon_N=\omega_H(N+1/2)$ and $\phi_{Nk}(\mathbf{r})$ are the eigenvalues and eigenfunctions of the Hamiltonian $\mathcal{H}_0$, and k denotes pseudomomentum.

In the case of a small disorder $\omega_H\tau_0\gg 1$ the solution of Eq. (17) has the form[26]

$$P^n_{sp}=\frac{\mathrm{sgn}\,\omega_n}{2\tau},\tau=\pi\sqrt{\frac{\rho}{m}}\frac{\tau_0}{\sqrt{\omega_H\tau_0}}. \tag{20}$$

The fluctuations of the V field are responsible for the localization corrections to the conductivity (in the weak localization regime they correspond to the maximally crossed diagrams). However, in the considered case, these corrections are of the order of $\ln N/N\ll 1$ and therefore can be neglected. For this reason we simply put $V=1$.

The presence of the potential λ results in a shift of the saddle-point value (20) due to the coupling to the fluctuations $\delta P=P-P_{sp}$ of the P field. The corresponding effective action for the δP field follows from Eq. (9) after integrating out fermions:

$$\mathcal{S}[\delta P,\lambda]=\int_r \mathrm{tr}\ln G_0^{-1}-\frac{N_r\Omega_\Delta}{T}-\frac{1}{2g}\int_r \mathrm{tr}(P_{sp}+\delta P)^2 + \int_r \mathrm{tr}\ln[1+(i\delta P+\lambda)G_0^{-1}]. \tag{21}$$

As a result, the thermodynamical potential can be written as

$$\Omega=-\frac{T}{N_r}\ln\int \mathcal{D}[\delta P]I[\delta P]\exp\mathcal{S}[\delta P,\lambda], \tag{22}$$

where, following Ref. 24, the integration measure $I[\delta P]$ is

$$\ln I[\delta P]=-\frac{1}{(\pi\rho)^2}\int\sum_{nm}^{\alpha\beta}[1-\Theta(nm)]\delta P_{nn}^{\alpha\alpha}\delta P_{mm}^{\beta\beta}. \tag{23}$$

The quadratic in δP part of the action (21) together with the contribution (23) from the integration measure determine the propagator of the δP fields (see Ref. 18 for details),

$$\langle\delta P_{m_1m_2}^{\alpha\beta}(\mathbf{q})\delta P_{m_3m_4}^{\gamma\delta}(-\mathbf{q})\rangle =\frac{g\delta_{m_1m_4}\delta_{m_2m_3}\delta^{\alpha\delta}\delta^{\beta\gamma}}{1+g\pi_0^{m_1}(m_3-m_1;q)}-\frac{2[1-\Theta(m_1m_3)]}{(\pi\rho)^2} \times\frac{g\delta_{m_1m_2}\delta^{\alpha\beta}}{1+g\pi_0^{m_1}(0;q)}\frac{g\delta_{m_3m_4}\delta^{\delta\gamma}}{1+g\pi_0^{m_3}(0;q)}, \tag{24}$$

where the bare polarization operator $\pi_0^m(n;q)$ is

$$\pi_0^m(n;q)=-n_LG_0(\omega_m+\nu_n)G_0(\omega_m)F_N^2(q). \tag{25}$$

D. Thermodynamical potential

To find the expansion of the thermodynamical potential Ω in powers of the CDW order parameter $\Delta(\mathbf{q})$, it is convenient to introduce a new variable $\widetilde{\delta P}=\delta P+i\lambda$ and expand tr ln in the action (21) in powers of this new field $\widetilde{\delta P}$. Then the thermodynamical potential can be written in the form

$$\Omega=\Omega_0+\Omega_\Delta+\delta\Omega, \tag{26}$$

where

$$\Omega_0(\mu)=\int_r \mathrm{tr}\ln G_0^{-1}-\frac{1}{2g}\int_r \mathrm{tr}\,P_{sp}^2 \tag{27}$$

is the mean-field thermodynamical potential of the homogeneous state, and

$$\delta\Omega=-\frac{T}{N_r}\ln\int\mathcal{D}[\widetilde{\delta P}]\exp\tilde{\mathcal{S}}[\widetilde{\delta P},\lambda] \tag{28}$$

takes into account the fluctuations of the massive longitudinal field $\widetilde{\delta P}$ and their interaction with the CDW order parameter (potential λ). The action $\tilde{\mathcal{S}}[\widetilde{\delta P},\lambda]$ has the form

$$\tilde{\mathcal{S}}=\mathcal{S}^{(2)}[\lambda]+\mathcal{S}_{int}[\widetilde{\delta P},\lambda]+\mathcal{S}^{(2)}[\widetilde{\delta P}]+\sum_{n=3}^{\infty}\mathcal{S}^{(n)}[\widetilde{\delta P}], \tag{29}$$

with

$$\mathcal{S}^{(2)}[\lambda]=\frac{N_r}{2g}\sum_{\omega_n}\int_r\lambda(\mathbf{r})\lambda(\mathbf{r}), \tag{30}$$

$$\mathcal{S}_{int}[\widetilde{\delta P},\lambda]=-\frac{i}{g}\int_r\lambda(\mathbf{r})\mathrm{tr}\,\widetilde{\delta P}(\mathbf{r}), \tag{31}$$

and

$$\mathcal{S}^{(n)}[\widetilde{\delta P}]=\frac{(-i)^n}{n}\mathrm{tr}\prod_{j=1}^{n}\int_{r_j}\widetilde{\delta P}(\mathbf{r}_j)G_0(\mathbf{r}_j\mathbf{r}_{j+1}), \tag{32}$$

where $\mathbf{r}_{n+1}=\mathbf{r}_1$. Note that the terms in the action (29), that are proportional to N_r^2, are omitted because they do not contribute to $\delta\Omega$ in the replica limit $N_r\to 0$. Another important observation is that the propagator of the $\widetilde{\delta P}$ fields is the same as for the δP fields (24).

By using Eqs. (28)–(32) we can write

$$\delta\Omega=-\frac{T}{2g}\sum_{\omega_n}\int_r\lambda(\mathbf{r})\lambda(\mathbf{r})-\frac{T}{N_r}\ln\langle\exp\tilde{\mathcal{S}}_{int}\rangle, \tag{33}$$

where $\langle\cdots\rangle$ denotes the average over $\widetilde{\delta P}$ with respect to the action $\tilde{\mathcal{S}}[\widetilde{\delta P},0]$. This equation allows us to find the contributions to the thermodynamical potential Ω up to any order of the CDW order parameter $\Delta(\mathbf{q})=F_N(q)U(q)^{-1}\lambda(\mathbf{q})/L_xL_y$.

In this paper we will work only with the expansion up to the fourth-order term (the Landau expansion). This implies that our consideration is valid only close to the transition point where the value of the order parameter is small and one can truncate the series (28) after several first terms. It should be mentioned, however, that we should avoid a direct vicinity of the phase transition (the critical region, for more details see Sec. IV) where the fluctuations of the order parameter break the mean-field approach.

1. Second-order contribution

The second-order contribution to the thermodynamical potential $\delta\Omega$ is

$$\delta\Omega^{(2)}=-\frac{T}{2g}\sum_{\omega_n}\int_r\lambda(\mathbf{r})\lambda(\mathbf{r})-\frac{T}{2N_r}\langle S_{int}^2\rangle_0, \tag{34}$$

where $\langle\cdots\rangle_0$ stands for the average over $\widetilde{\delta P}$ with respect to the action $\tilde{\mathcal{S}}^{(2)}[\widetilde{\delta P}]$. We replace the average over the full action $\tilde{\mathcal{S}}[\widetilde{\delta P},0]$ by the average over the quadratic part $\tilde{\mathcal{S}}^{(2)}[\widetilde{\delta P}]$ only because the higher order in $\widetilde{\delta P}$ terms lead to the contributions that are proportional to N_r^2, and therefore vanish in the replica limit $N_r\to 0$.

With the help of Eqs. (10), (24), and (31), we obtain

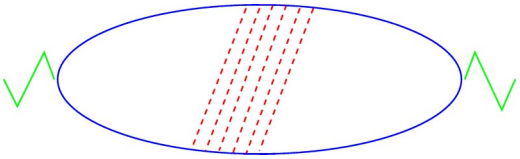

FIG. 1. Second-order contribution to the thermodynamic potential. Solid line denotes electron Green function, dashes are impurity lines, and vertexes are $\lambda(\mathbf{r})$.

$$\frac{\delta\Omega^{(2)}}{L_x^2 L_y^2} = n_L \frac{T}{2} \sum_{\omega_n} \int_q \frac{U^2(q) G_0^2(\omega_n)}{1+g\pi_0^{\omega_n}(0,q)} \Delta(\mathbf{q})\Delta(-\mathbf{q}). \quad (35)$$

The corresponding diagram in the usual ''cross technique'' is shown in Fig. 1.

2. Third-order contribution

The contribution of the third power of the CDW order parameter to the thermodynamical potential $\delta\Omega^{(3)}$ can be written as

$$\delta\Omega^{(3)} = -\frac{T}{3!N_r}\langle S_{int}^3\rangle_{\overline{\delta P}}^{(c)} = -\frac{T}{3!N_r}\langle S_{int}^3 S^{(3)}\rangle_0^{(c)}, \quad (36)$$

where the superscript (c) indicates that only connected diagrams are taken into account. Here we omit again the terms that vanish in the replica limit $N_r \to 0$. After performing the averaging over $\overline{\delta P}$ with the help of Eqs. (10), (24), (31), and (32), we obtain

$$\frac{\delta\Omega^{(3)}}{L_x^3 L_y^3} = (2\pi)^2 n_L \frac{T}{3} \sum_{\omega_n} \prod_{j=1}^{3} \left[\int_{q_j} \frac{U(q_j)\Delta(\mathbf{q}_j) G_0(\omega_n)}{1+g\pi_0^{\omega_n}(0,q_j)} \right] \times \delta(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3) \exp\frac{i}{2}(q_1^x q_2^y - q_1^y q_2^x). \quad (37)$$

The contribution $\delta\Omega^{(3)}$ corresponds to the diagram in Fig. 2.

3. Fourth-order contribution

The fourth-order contribution $\delta\Omega^{(4)}$ is

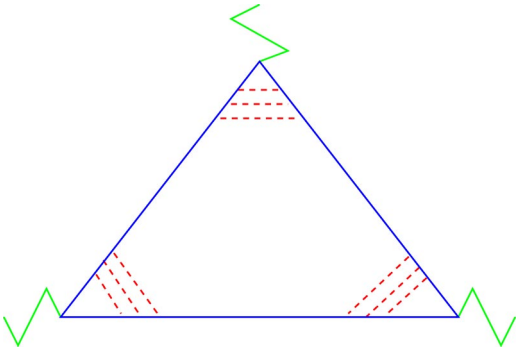

FIG. 2. The third-order contribution to the thermodynamical potential.

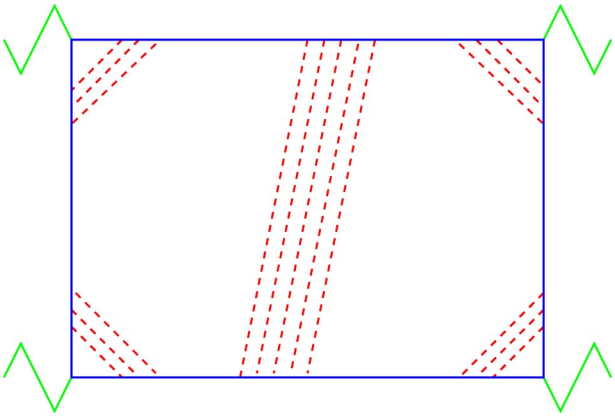

FIG. 3. The fourth-order contribution to the thermodynamical potential.

$$\delta\Omega^{(4)} = -\frac{T}{4!N_r}\langle S_{int}^4\rangle^{(c)} = -\frac{T}{4!N_r}\left\langle S_{int}^4 \left[S^{(4)} + \frac{1}{2}(S^{(3)})^2 \right] \right\rangle_0^{(c)}, \quad (38)$$

where again only terms which is proportional to N_r are kept. By using Eqs. (10), (24), (31), and (32), we find

$$\frac{\delta\Omega^{(4)}}{L_x^4 L_y^4} = (2\pi)^2 n_L \frac{T}{4} \sum_{\omega_n} \prod_{j=1}^{4} \left[\int_{q_j} \frac{U(q_j)\Delta(\mathbf{q}_j) G_0(\omega_n)}{1+g\pi_0^{\omega_n}(0,q_j)} \right] \times \delta(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3+\mathbf{q}_4) \frac{1-g\pi_0^{\omega_n}(0,|\mathbf{q}_1+\mathbf{q}_2|)}{1+g\pi_0^{\omega_n}(0,|\mathbf{q}_1+\mathbf{q}_2|)} \times \exp\frac{i}{2}(q_1^x q_2^y - q_1^y q_2^x) \exp\frac{i}{2}(q_3^x q_4^y - q_3^y q_4^x). \quad (39)$$

In the usual cross technique the contribution $\delta\Omega^{(4)}$ corresponds to the diagram shown in Fig. 3.

E. Free energy

The free energy of the CDW state can be written in the form

$$\mathcal{F} = \mathcal{F}_0 + \Omega(\mu) - \Omega_0(\mu_0) + (\mu-\mu_0)N_e, \quad (40)$$

where $\mathcal{F}_0$ is the free energy of the normal (homogeneous) state, N_e the total number of electrons, μ and μ_0 the chemical potentials of the CDW state and the normal state, respectively.

In order to find the free energy of the CDW state to the fourth order in the CDW order parameter we expand $\Omega_0(\mu_0)$ around the point μ to the second order in $\mu-\mu_0$. This results in

$$\mathcal{F} = \mathcal{F}_0 + \Omega(\mu) - \Omega_0(\mu) - \frac{1}{2}(\mu-\mu_0)^2 \frac{\partial^2\Omega_0}{\partial^2\mu_0}. \quad (41)$$

The difference $\mu-\mu_0$ of the chemical potentials in the CDW and the normal states is

$$\mu-\mu_0=\frac{\partial\delta\Omega}{\partial N_e}\left(\frac{\partial N_e}{\partial\mu}\right)^{-1}, \tag{42}$$

and from Eq. (26) we obtain

$$\mathcal{F}=\mathcal{F}_0+\Omega_\Delta+\delta\Omega+\frac{1}{2}\left(\frac{\partial\delta\Omega^{(2)}}{\partial\mu}\right)^2\left(\frac{\partial N_e}{\partial\mu}\right)^{-1}. \tag{43}$$

With the expression (35) for $\delta\Omega^{(2)}$ this gives

$$\mathcal{F}=\mathcal{F}_0+\Omega_\Delta+\delta\Omega+\frac{n_L(L_xL_y)^3}{2}\left[T\sum_{\omega_n}\int_q\frac{U^2(q)G_0^3(\omega_n)}{[1+g\pi_0^{\omega_n}(0,q)]^2}\Delta(\mathbf{q})\times\Delta(-\mathbf{q})\right]^2\left[T\sum_{\omega_n}G_0^2(\omega_n)\right]^{-1}. \tag{44}$$

F. Free energy of the CDW states

1. Triangular CDW state

The CDW order parameter for the triangular lattice symmetry (bubble phase) can be written in the form[7]

$$\Delta(\mathbf{q})=\frac{(2\pi)^2}{L_xL_y}\Delta(Q)\sum_{j=1}^{3}[\delta(\mathbf{q}-\mathbf{Q}_j)+\delta(\mathbf{q}+\mathbf{Q}_j)], \tag{45}$$

where the vectors $\mathbf{Q}_j$ have the angle $2\pi/3$ between each other and obey the condition $\mathbf{Q}_1+\mathbf{Q}_2+\mathbf{Q}_3=0$.

By using Eqs. (35), (37), (39), and (44), we obtain the following expression for the free energy of the triangular CDW state:

$$\mathcal{F}^t=\mathcal{F}_0+4\frac{L_xL_y}{2\pi l_H^2}T_0(Q)[a_2\Delta^2+a_3\Delta^3+a_4\Delta^4]. \tag{46}$$

Here the three coefficients a_1, a_2, and a_3 of the Landau expansion are as follows:

$$a_2=3\left(1-\frac{T_0(Q)}{\pi^2T}\sum_n\frac{1}{\xi_n^2+\gamma^2(Q)}\right), \tag{47}$$

where

$$\xi_n=n+\frac{1}{2}+\frac{1}{4\pi T\tau}-i\frac{\mu_N}{2\pi T},\quad \gamma(Q)=\frac{F_N(Q)}{4\pi T\tau} \tag{48}$$

with $\mu_N=\mu-\epsilon_N$ being the chemical potential measured from the Nth Landau level and

$$T_0(Q)=U(Q)/4, \tag{49}$$

$$a_3=i8\frac{T_0^2(Q)}{\pi^3T^2}\cos\left(\frac{\sqrt{3}Q^2}{4}\right)\sum_n\frac{\xi_n^3}{[\xi_n^2+\gamma^2(Q)]^3}, \tag{50}$$

and

$$a_4=\frac{24T_0^3(Q)}{\pi^4T^3}\left\{\frac{1}{2}\sum_n\frac{\xi_n^4}{[\xi_n^2+\gamma^2(Q)]^4}\left[3D_n(0)+\left(1+\cos\frac{\sqrt{3}Q^2}{2}\right)[D_n(Q)+D_n(\sqrt{3}Q)]+\frac{1}{2}D_n(2Q)\right]+3\left(\sum_n\frac{\xi_n}{[\xi_n^2+\gamma^2(Q)]^2}\right)^2\left(\sum_n\xi_n^{-2}\right)^{-1}\right\}, \tag{51}$$

with

$$D_n(Q)=\frac{\xi_n^2-\gamma^2(Q)}{\xi_n^2+\gamma^2(Q)} \tag{52}$$

2. Unidirectional CDW state

The CDW order parameter of the unidirectional state (stripe phase) is[6–8]

$$\Delta(\mathbf{q})=\frac{(2\pi)^2}{L_xL_y}\Delta(Q)[\delta(\mathbf{q}-\mathbf{Q})+\delta(\mathbf{q}-\mathbf{Q})], \tag{53}$$

where the vector $\mathbf{Q}$ is oriented along the spontaneously chosen direction, and from Eqs. (35), (37), (39), and (44), the free energy of the unidirectional CDW state reads

$$\mathcal{F}^u=\mathcal{F}_0+4\frac{L_xL_y}{2\pi l_H^2}T_0(Q)(b_2\Delta^2+b_4\Delta^4). \tag{54}$$

Here the coefficients b_2 and b_4 of the Landau expansion are

$$b_2=\frac{a_2}{3}=\left(1-\frac{T_0(Q)}{\pi^2T}\sum_n\frac{1}{\xi_n^2+\gamma^2(Q)}\right) \tag{55}$$

and

$$b_4=\frac{4T_0^3(Q)}{\pi^4T^3}\left[\sum_n\frac{\xi_n^4}{[\xi_n^2+\gamma^2(Q)]^4}\left(D_n(0)+\frac{1}{2}D_n(2Q)\right)+2\left(\sum_n\frac{\xi_n}{[\xi_n^2+\gamma^2(Q)]^2}\right)^2\left(\sum_n\xi_n^{-2}\right)^{-1}\right]. \tag{56}$$

Expressions (46)–(52) for the free energy of the triangular CDW state and Eqs. (54)–(56) for the free energy of the unidirectional CDW state are the key results of the present paper. They generalize the results from Refs. 7 and 8 for the clean case to the weakly disordered case, and coincide with them in the limit $1/\tau\to 0$.

III. MEAN-FIELD PHASE DIAGRAM

A. Instability line

The vanishing of the coefficient in front of the quadratic term in the Landau expansion of the free energy signals about the instability of the normal state towards the formation of the CDW. This instability corresponds to the second-order phase transition from the homogeneous state to the

CDW state. As usual, the specific parameters of the forming CDW state are determined by the high-order terms in the Landau expansion.

From Eqs. (47) and (55) we obtain the following equation:

$$\frac{T}{T_0(Q)}=\frac{1}{\pi^2}\sum_n \frac{1}{\xi_n^2+\gamma^2(Q)} \tag{57}$$

for the instability line. The solution $T(Q)$ of this equation depends on the modulus Q of the vector that characterizes the CDW state. The temperature T_2 of the second-order phase transition corresponds to the maximal value of $T(Q)$:

$$T_2=\max_Q T(Q) \tag{58}$$

and the corresponding value Q_0, $T_2=T(Q_0)$, determines the period of the CDW state. The Hartree-Fock potential (11) has minima at those vectors Q_k for which the form factor $F_N(Q_k)$ vanishes. In the clean case this corresponds to $Q_0=\min Q_k=r_0/R_c$, where $r_0\approx 2.4$ is the first zero of the Bessel function of the first kind.[6] It can be seen from Eq. (57) that a weak disorder does not shift the vector Q_0 (see Appendix). Thus the equation for the temperature of the second-order phase transition into the CDW state reads

$$\frac{T}{T_0}=\frac{2}{\pi^2}\mathrm{Re}\,\psi'\left(\frac{1}{2}+\frac{1}{4\pi T\tau}+i\frac{\mu_N}{2\pi T}\right), \tag{59}$$

where $\psi'(z)$ is the derivative of digamma function, Re the real part, and $T_0\equiv T_0(Q_0)$ is the temperature of the transition in the clean case. According to Ref. 6,

$$T_0=\frac{r_s\omega_H}{4\pi\sqrt{2}}\left[\ln\left(1+\frac{c}{r_s}\right)-\frac{c}{\sqrt{2}+r_s}\right], \tag{60}$$

where $c=1/(\sqrt{2}r_0)\approx 0.3$, and the transition temperature T_0 is determined by the characteristic Coulomb energy $e^2/R_c=r_s\omega_H/\sqrt{2}\ll\omega_H$.

Equation (59) contains the chemical potential μ_N measured from the Nth Landau level that, together with the temperature T and the broadening of Landau levels $1/2\tau$, determines the filling factor $\nu_N=\nu-2N$ of the partially filled highest Landau level. However, in order to find this relation one needs to know the density of states in the system. This question about the density of states is a very subtle[27] and beyond the scope of the present paper. For this we use the chemical potential μ_N rather than filling factor ν_N.

Equation (59) can be solved analytically in the two extreme cases: when temperature T is closed to the temperature T_0 of instability in the absence of disorder, and when the temperature T is close to zero.

In the first case, the broadening of the Landau level $1/2\tau$ and the chemical potential μ_N are small compared to the temperature T_0 of the instability in the clean case, and therefore the leading order expansion in powers of $1/T_0\tau$ and μ_N/T_0 is legitimate. It appears that the presence of disorder decreases the temperature of instability linearly:

$$\frac{T}{T_0}=1-\frac{7\zeta(3)}{\pi^3T_0\tau}-\frac{\mu_N^2}{4T_0^2},\quad \frac{1}{2\tau},\mu_N\ll T_0. \tag{61}$$

In the opposite case $T\ll T_0$, Eq. (59) reduces to

$$\frac{T}{T_0}=\frac{4\sqrt{3}}{\pi^2}\sqrt{1-\frac{\pi}{8T_0\tau}-\frac{\pi^2\mu_N^2}{16T_0^2}},\quad \sqrt{1-\frac{\pi}{8T_0\tau}}\ll 1,\frac{\mu_N}{T_0}\ll 1. \tag{62}$$

We see from Eq. (62) that the second-order phase transition into the CDW state can occur only if the broadening $1/2\tau$ of the Landau level is smaller than the critical value $1/2\tau_c(\mu_N)$, which depends on the chemical potential μ_N. The maximal value of $1/\tau_c(\mu_N)$ is reached at $\mu_N=0$ and equals

$$\frac{1}{\tau_c}=\frac{8T_0}{\pi}. \tag{63}$$

Equations (61) and (62) shows that, provided the broadening of the Landau level $1/2\tau$ is smaller than the critical one, $1/2\tau<1/2\tau_c$, the region of existence of the CDW state on the phase diagram in the disordered case is smaller as compared to the clean case.

B. Half filled Landau level ($\nu_N=1/2$)

We now consider the case of the half filled Nth Landau level ($\nu_N=1/2$), that is related to the recent experiments.[9] In this case the chemical potential is zero, $\mu_N=0$, provided the density of states is symmetric around the center of the Nth Landau level. As follows from Eq. (59), the temperature of the second-order phase transition for this case can be found from the equation

$$\frac{T}{T_0}=\frac{2}{\pi^2}\zeta\left(2,\frac{1}{2}+\frac{1}{4\pi T\tau}\right), \tag{64}$$

where $\zeta(2,z)=\Sigma_{m=0}^{\infty}(m+z)^{-2}$ is the generalized Riemann zeta function. The analytical solutions of this equation in the cases of high and low temperatures can be obtained from Eqs. (61) and (62) by putting μ_N to zero. The entire behavior of the solution (spinodal line), obtained numerically from Eq. (64), is shown in Fig. 4.

We mention that at $\nu_N=1/2$ the coefficient a_3 vanishes due to the particle-hole symmetry. This means that the transition from the normal state into the CDW state is of the second order for both cases of unidirectional and triangular lattice symmetry. Therefore, to find the structure of the CDW state, one has to take into account the fourth-order terms in the Landau expansion. In the vicinity of the spinodal line, it follows from Eqs. (51) and (56) with $\mu_N=0$ that

$$a_4=\frac{12T_0^3}{\pi^4T^3}[-7\zeta(4,u)+12\Phi_0(u)+6\Phi_2(u)+8\Phi_{\sqrt{3}}(u)] \tag{65}$$

and

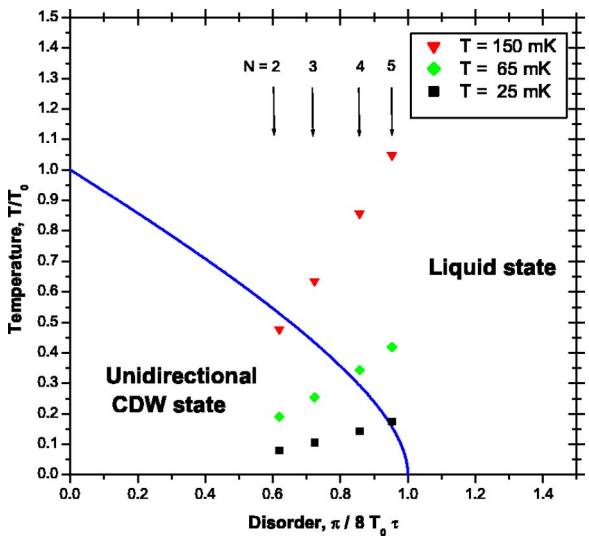

FIG. 4. Phase diagram at $\nu_N=1/2$. The spinodal line obtained from Eq. (64) is shown by the solid line. The triangles and rhombi are the experimental data after Ref. 9.

$$b_4=\frac{2T_0^3}{\pi^4T^3}[-3\zeta(4,u)+4\Phi_0(u)+2\Phi_2(u)], \tag{66}$$

where we introduce the new variable $u=1/2+1/4\pi T\tau$ and the new function

$$\Phi_a\left(\frac{1}{2}+z\right)=\frac{1}{z^2\mathcal{J}_0^2(ar_0)}\left[\zeta\left(2,\frac{1}{2}+z\right)-\frac{1}{z\mathcal{J}_0(ar_0)}\operatorname{Im}\psi\left(\frac{1}{2}+z+iz\mathcal{J}_0(ar_0)\right)\right] \tag{67}$$

with Im being the imaginary part. With these expressions we can minimize $\mathcal{F}^{t,u}$ with respect to the order parameter Δ and find that the unidirectional CDW state has lower free energy. Therefore, at a half filled Landau level, $\nu_N=1/2$, and $1/\tau<1/\tau_c$, the transition takes place from the liquid state into the unidirectional CDW state. We will compare these results with the experimental ones in Sec. V A.

C. Phase diagram at zero temperature

In this section we analyze the zero-temperature phase diagram in the case where the Landau level broadening is close to its critical value $1/2\tau_c=4T_0/\pi$. Under these conditions, the CDW order parameter Δ is small, and one can use the Landau expansions (46) and (54) at zero temperature. The coefficients of these expansions are

$$a_2=3\left(1-\frac{8T_0\tau}{\pi}H_1(\mu_N\tau)\right),\quad a_3=2\pi\left(\frac{8T_0\tau}{\pi}\right)^2H_2(\mu_N\tau), \tag{68}$$

and

$$a_4=3\pi^2\left(\frac{8T_0\tau}{\pi}\right)^3H_3(\mu_N\tau,QR_c) \tag{69}$$

for the triangular CDW state, and

$$b_2=\frac{a_2}{3},\quad b_4=\frac{\pi^2}{2}\left(\frac{8T_0\tau}{\pi}\right)^3H_4(\mu_N\tau,QR_c) \tag{70}$$

for the unidirectional CDW state. Here we introduce four functions $H_i(z)$ as

$$H_1(z)=\frac{1}{1+4z^2},\quad H_2(z)=4zH_1(z), \tag{71}$$

$$H_3(z,r)=\frac{1}{2}\frac{108z^2-5}{(1+4z^2)^3}+3R_0(z,r)+2R_1(z,r)+\frac{1}{2}R_2(z,r)+2R_{\sqrt{3}}(z,r), \tag{72}$$

and

$$H_4(z,r)=\frac{28z^2-1}{(1+4z^2)^3}+2R_0(z,r)+R_2(z,r), \tag{73}$$

where

$$R_a(z,r)=\frac{2H_1(z)}{\mathcal{J}_0^2(ar)}-\frac{1}{\mathcal{J}_0^3(ar)}\arctan\frac{2\mathcal{J}_0(ar)}{1+4z^2-\mathcal{J}_0^2(ar)}. \tag{74}$$

As a result, the instability line obeys the equation

$$\frac{\pi}{8T_0\tau}=H_1(\mu_N\tau), \tag{75}$$

which is the zero temperature limit of Eq. (59). In the case of a small chemical potential $\mu_N\ll T_0$, the solution of Eq. (75),

$$\frac{\pi}{8T_0\tau}=1-\frac{\pi^2}{16}\frac{\mu_N^2}{T_0^2},\quad \mu_N\ll T_0, \tag{76}$$

follows from Eq. (62) at $T=0$.

The line of the first-order transition from the liquid to the triangular CDW state is determined by the following equation:

$$\frac{\pi}{8T_0\tau}=H_1(\mu_N\tau)+\frac{H_2^2(\mu_N\tau)}{9H_3(\mu_N\tau,QR_c)}. \tag{77}$$

As before, the maximum of the solutions $1/\tau(Q)$ of Eq. (77) with respect to Q should be found. It appears that the maximum is not exactly at $Q=Q_0$ as in the clean case,[8] but at some shifted value $Q_0+\delta Q$ with the shift $\delta Q=-0.003(\mu_N/T_0)^2R_c^{-1}$ for small chemical potential $\mu_N\ll T_0$. The existence of the shift is a feature of the disordered case. Below in the limit $\mu_N\ll T_0$, we will neglect this shift. In this case Eq. (77) can be written as

$$\frac{\pi}{8T_0\tau}=1-0.45\frac{\mu_N^2}{T_0^2},\quad \mu_N\ll T_0. \tag{78}$$

By comparing the free energies of the triangular and the unidirectional CDW states, we can find the line of the first-order transition between them

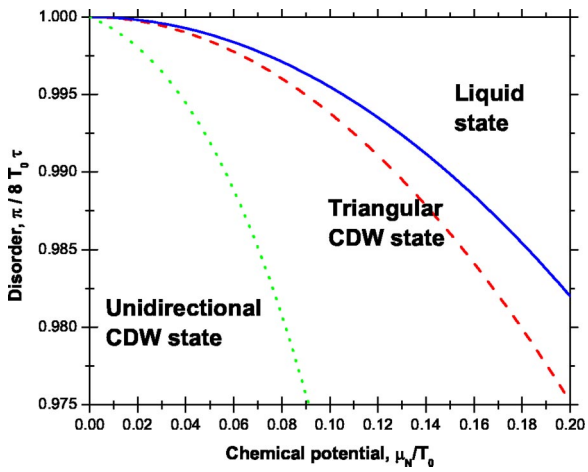

FIG. 5. Phase diagram at zero temperature near $\nu_N = 1/2$. The solid line is obtained from Eq. (77), the dashes are the instability line (75) and the dots are obtained from Eq. (79).

$$\frac{\pi}{8T_0\tau} = H_1 - \frac{H_2^2[2H_3+H_4][3H_4+\sqrt{H_4^2+2H_3H_4}]}{2H_3(2H_3-3H_4)^2}. \tag{79}$$

For the case $\mu_N\tau \ll 1$, Eq. (79) can be simplified as

$$\frac{\pi}{8T_0\tau} = 1 - 2.84\frac{\mu_N^2}{T_0^2}, \quad \mu_N \ll T_0. \tag{80}$$

For other values of μ_N/T_0, Eqs. (75), (77), and (79) were solved numerically. The results are shown in Fig. 5. These results show that for a fixed value of the chemical potential $\mu_N \neq 0$ and decreasing disorder parameter $\pi/8T_0\tau$, the electron system undergoes first the transition from the liquid state into the triangular CDW state and then into the unidirectional CDW state. We should mention, however, that this statement is proven only for small values of the chemical potential $\mu_N \ll T_0$ and for values of the disorder parameter $\pi/8T_0\tau$ close to unity. Outside of this region, the CDW order parameter is not small, and hence we go beyond the applicability of the Landau expansion. Nevertheless, the topology of the phase diagram should remain the same.

IV. WEAK CRYSTALLIZATION CORRECTIONS

The CDW order parameter $\Delta(\mathbf{r})$ introduced in Eq. (5) can be thought of as a saddle-point solution for the plasmon field that appears in the Hubbard-Stratonovich transformation of the electron-electron interaction in the action (2). The Landau expansions (46) and (54) for the free energy of the CDW states were derived under the assumption that one can neglect the fluctuations of the CDW order parameter. This is legitimate for $N \gg 1$ and not very close to the transition (outside the critical region). However, when one approaches the instability line, the fluctuations of the CDW order parameter increase. To analyze the effects of the order-parameter fluctuation, we introduce, following the original ideas of Brazovski,[28] the fluctuations of the CDW order parameter $\Delta(\mathbf{r}) \to \Delta(\mathbf{r}) + \delta(\mathbf{r})$ in the Landau expansion of the free energy and average over the fluctuations $\delta(\mathbf{r})$. We present below the results of the corresponding analysis only for the most interesting case of the half filled Landau level.

We find that the transition from the liquid to the unidirectional CDW state becomes of the first order, and takes place at the lower temperature that can be found from the following equation [see Eq. (64) for comparison]:

$$\frac{T}{T_0} = \frac{2}{\pi^2}\zeta\left(2,\frac{1}{2}+\frac{1}{4\pi T\tau}\right) - g\left(\frac{1}{4\pi T\tau}\right)N^{-2/3}, \tag{81}$$

Here function $g(z)$ is defined as

$$g(z) = 3\left[\frac{3\pi r_0}{16}\right]^{2/3}\left[\frac{\lambda_0^2(z)}{f(z)}\right]^{2/3}\left[\frac{2\lambda_0(z)+\lambda_2(z)}{4\lambda_0(z)-\lambda_2(z)}\right]^{1/3}, \tag{82}$$

where we introduce the following three functions:

$$f(z) = \frac{2}{\pi^2}\left[\beta_1\zeta\left(2,\frac{1}{2}+z\right) + \beta_2 z^2\zeta\left(4,\frac{1}{2}+z\right)\right], \tag{83}$$

$$\lambda_a(z) = \frac{2}{\pi^4}\left[-\zeta\left(4,\frac{1}{2}+z\right) + 2\Phi_a\left(4,\frac{1}{2}+z\right)\right], \tag{84}$$

The constants β_i are given by

$$\beta_1 = \frac{T_0'(Q_0)}{T_0(Q_0)} \approx 2.58, \quad \beta_2 = [\mathcal{J}_0'(Q_0)]^2 \approx 0.27, \tag{85}$$

and function Φ_a is defined by Eq. (68).

We mention that the function $\pi^2 g(z)/2\zeta(2,1/2+z)$ decreases monotonically from the value 0.35 at $z=0$ to zero at $z \to \infty$. Therefore we obtain the following inequality for the shift δT of the mean-field transition temperature T:

$$\frac{\delta T}{T} \leqslant 3\left(\frac{\pi r_0}{16\sqrt{\beta_1}}\right)^{2/3} N^{-2/3}, \quad N \gg 1, \tag{86}$$

the equality corresponds to the clean case.

The appearance of a noninteger power in Eq. (81) results from the fact that the momentum dependence of the correlation function for the order-parameter fluctuations contains $(Q-Q_0)^2$ rather than Q^2 (see Ref. 28).

Equation (81) was derived under the assumption that the main contribution in the momentum space comes from the region $Q \approx Q_0$. This assumption is justified under the following condition:[28]

$$\frac{g\left(\frac{1}{4\pi T\tau}\right)}{r_0^2 f\left(\frac{1}{4\pi T\tau}\right)} \ll N^{2/3}. \tag{87}$$

The combination of functions in the left-hand side of inequality (87) decreases monotonically from 0.023 to 0 while z increases from zero to infinity and hence the condition (87) holds.

According to Eq. (86), the fluctuations reduce the transition temperature by the amount of the order of $N^{-2/3} \ll 1$ and therefore in the considered case of the weak magnetic field ($N \gg 1$) their effects can be neglected. These results indicate that the critical region for the considered transition is indeed small, and the mean-field approach gives a good approximation for $N \gg 1$.

As the Landau level index N decreases, the fluctuations of the order parameter around the mean-field solution become important, see Eq. (86). In this regime, detailed microscopical calculations are hardly possible due to the absence of a successive microscopic theory. One can, however, make predictions for the effect of disorder on the phase diagram on the basis of a phenomenological elastic theory[14] described by an effective Hamiltonian for the fluctuations of the displacement field associated with the translational symmetry breaking. The parameters of the Hamiltonian are phenomenological constants, which, for the reasons discussed above, cannot be calculated microscopically. Our microscopic approach is therefore a complementary one, that is valid in the limit of small fluctuations of the order parameter, i.e., $N \gg 1$.

V. DISCUSSIONS

A. Comparison with experimental results

Now we discuss the possible applications of our theory to the recent experiments. Although our mean-field theory was derived for the case of a large number of the occupied Landau level $N \gg 1$, and neglects corrections of the order of $1/N$, while experimentally one has $N=2,3,4$, we, however, expect that Eq. (59) gives a good estimation for the temperature of the transition from the liquid to the CDW state, even for $N=2,3,4$. We have complementary assurance that it can really be the case because Eq. (59) can be obtained without introducing the CDW order parameter and considering the mean-field theory but as the equation that determines the temperature $T(Q)$ at which the two-particle vertex function at wave vector Q diverges.[29]

We restrict ourselves by discussion of the experiments without an in-plane magnetic field.[9,10] The theory for the half filled highest Landau level contains two physical parameters: the temperature T_0 and the broadening of the Landau level $1/2\tau$. According to Eq. (60), in the absence of a disorder and in a wide range of the parameter r_s, the transition temperature T_0 can be estimated as

$$T_0 \simeq 0.008\omega_H, \quad 0.1 < r_s < 1. \tag{88}$$

We can estimate the broadening $1/2\tau$ of the Landau level from the mobility μ_0 at zero magnetic field. With the help of Eq. (20), we obtain

$$\frac{1}{\tau} \simeq \frac{\sqrt{2N}}{\pi}\sqrt{\frac{e}{\mu_0 n_e}}\omega_H, \tag{89}$$

where n_e is the density of the two-dimensional electron gas.

The results obtained in Sec. III B impose the restriction on the value of the sample mobility μ_0 at zero magnetic field. In order to observe the CDW states at the partially filled Landau level with index N the mobility at zero magnetic field should satisfy the condition $\mu_0 > 0.55 \times 10^3 eN/n_e$. For typical values of the electron density of the two-dimensional electron gas, $n_e \simeq 10^{11}$ cm^{-2}, we can obtain the following estimate: $\mu_0 > N \times 10^6$ cm^2/V s.

In the experiments of Lilly *et al.*,[9] the samples were relatively clean, $\mu_0 > 9 \times 10^6$ cm^2/V s, and the electron density $n_e \approx 2.67 \times 10^{11}$ cm^{-2}. At the temperature $T=150$ mK, they detected an anisotropy in the resistance only in the half filled $N=2$ Landau level, whereas for higher Landau levels, $N>2$, the resistance remains isotropic. As the temperature decreases below 150 mK, the anisotropy in the resistance at half filling appears in higher Landau levels. At the temperature $T=25$ mK, substantial anisotropy was already observed in the half filled $N=2,3,4$ Landau levels. We plot the experimental data from Ref. 9 in Fig. 4 by using Eqs. (88) and (89). We also assume that Eq. (88) remains valid even for $N=2,3,4$, and 5. As it can be seen, the behavior of the transition line is in the agreement with the experimental data: the anisotropy in electronic transport at $T=150$ mK exists only for $N=2$ Landau level, while at $T=25$ mK it exists for $N=2,3$, and 4, but not for $N=5$, where the disorder induced transition from the CDW state into the isotropic liquid state takes place. It should be mentioned that when the transition temperature be independent on the disorder, the anisotropy in the resistance at $T=25$ mK should remain up to $N=12$, as can be seen from Eqs. (88) and (89). Therefore the role of disorder is very important.

The quantitative agreement of our theory with the experimental data is surprisingly good. In our approach, we treat the disorder as a short-range random potential with the correlation length much smaller than the magnetic length l_H. In the experimental samples, however, the disorder potential has long-range correlations. Therefore our theory can provide in this case only a qualitative picture. A thorough study of the effects of a long-range disorder on the phase diagram cannot be performed within the considered self-consistent Born approximation scheme and goes beyond the scopes of the present paper.

B. Comparison with numerical results

The problem of the formation of the CDW state on the second Landau level with $\nu_N = 1/2$ at zero temperature in the presence of a quenched disorder was studied numerically in Ref. 30. The system of 12 electrons interacting via the Coulomb interaction $U(q) = 2\pi e^2/q$ in the presence of the quenched disorder was projected on the second Landau level ($N=2$). The effects of interactions with electrons on the other Landau level was not taken into account. The system was diagonalized numerically. It was found that the CDW state transforms into the liquid state as the dimensionless disorder strength $\omega_H \sqrt{n_L/2\pi\rho\tau_0}$ exceeds 0.12.

In order to be able to compare the results of the presented above mean-field theory with the numerical results, we perform the evaluation of the temperature T_0 in the case for which the numerical results were obtained (instead screened interaction (3) we use $U(q) = 2\pi e^2/q$). Under this circumstances our theory gives the value 0.14.

The small discrepancy may be attributed to two factors: on the one hand, the finite number of electrons in numerical calculations and, on the one hand, unsufficiency of the Hartree-Fock approximation for the problem with the Coulomb interaction $U(q)=2\pi e^2/q$. In the latter case one should take into account the diagrams beyond the Hartree-Fock theory. Nevertheless, the comparison demonstrates that such corrections are small.

We emphasize that our theory which takes into account the screening of electron-electron interaction by electrons on the other Landau levels gives much smaller value 0.01 of the dimensionless disorder strength for the transition from the liquid state to the CDW state.

VI. CONCLUSIONS

For the system of a two-dimensional interacting electrons in the presence of a weak disorder and a weak magnetic field, we investigated the effect of disorder on the existence of the mean-field CDW states in the framework of the Hartree-Fock approximation. In the considered case of large filling factors $\nu \gg 1$, we obtained that the mean-field CDW instability exists if the disorder is rather weak, $1/\tau \leqslant 8T_0/\pi$. We found that at half filling the unidirectional CDW state appears, and the presence of disorder does not change the vector of the CDW. Near half filling, the unidirectional CDW state is energetically more favorable than the triangular one. We obtained that the weak crystallization corrections to the mean-field result are of the order of $(1/\nu)^{2/3} \ll 1$ and thus can be neglected. We discussed the applications of our theory to the recent experimental and numerical results.

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APPENDIX: INSTABILITY VECTOR

In this appendix we prove that the weak disorder does not change the vector at which the instability towards the formation of the CDW state grows. Let us consider the solution $T+\delta T$ of Eq. (57) for the vector $Q=Q_0+\delta Q$, where $\delta Q \ll Q_0$. We will now show that the shift δT is always negative, and hence the maximal instability temperature corresponds to the vector $Q=Q_0$, as it is in the clean case.

For a small deviation δQ we can write

$$T_0(Q)=T_0[1-\beta_1(\delta Q R_c)^2], \quad \mathcal{J}_0^2(QR_c)=\beta_2(\delta Q R_c)^2. \tag{A1}$$

The shift δT results in the substitution

$$\xi_n \to \xi_n - \frac{\delta T}{4\pi T^2 \tau} - i\frac{\delta T \mu}{2\pi T^2}, \tag{A2}$$

in Eq. (57), and we obtain

$$\frac{\delta T}{T} = -(\delta Q R_c)^2)\frac{\beta_1 g_2 + \beta_2 z^2 g_4}{g_2 - 2(z g_3 + y \overline{g_3})}, \tag{A3}$$

where $z=1/4\pi T\tau$ and $y=\mu/2\pi T$. Here we introduce the four functions $g_a(z,y)$ and $\overline{g_a(z,y)}$,

$$g_a(z,y)=\mathrm{Re}\sum_{n=0}^{\infty} \xi_n^{-a}, \quad \overline{g_a(z,y)}=\mathrm{Im}\sum_{n=0}^{\infty} \xi_n^{-a}. \tag{A4}$$

It can be easily seen that the right-hand side of Eq. (A3) is negative for all possible values of z and y.

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