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ASTRONOMY AND ASTROPHYSICS

# Modelling Population II cataclysmic variables

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**Abstract.** We study the long–term evolution of cataclysmic variables (CVs) as a function of the companion star metallicity. It is shown that CVs with secondaries of low metallicity (Z=$10^{-4}$) which belong to population II (PopII CVs) pass through a detached phase with a smaller period width, a shorter minimum period, and display a slightly higher mass transfer rate, resulting in shorter evolutionary timescales compared to CVs with a solar chemical composition of the secondary. A population synthesis model for PopII CVs with a star formation period lasting for $10^9$yr after the onset of Galaxy formation shows that most PopII CVs have already evolved beyond the minimum period. As these systems are faint, they are difficult to observe. Extracting a magnitude–limited sample from the computed intrinsic models and taking into account the sample's incompleteness towards fainter apparent magnitudes we derive a period distribution both for PopII CVs and the more common population I CVs. From a differential comparison of the models we calculate the probability that a CV at a given vector distance **r** from the Sun is member of the population II. Based on our results, we find that a reasonable observable sample of PopII CVs can only be found for $z \gtrsim 2000$pc.

**Key words:** novae, cataclysmic variables – stars: evolution – stars: Population II – binaries: close

## 1. Introduction

Cataclysmic variables (CVs) are short–period binary systems, consisting of a white dwarf (WD) primary and a low–mass companion (the secondary) with late spectral type. The secondary fills its critical Roche volume and loses mass through the inner Lagrangian point, thereby feeding an accretion disk orbiting the primary unless the WD has a strong magnetic field. For a certain range of the mass transfer rate the accretion disk is thermally unstable and undergoes dwarf nova outbursts, a limit cycle with alternating phases of high and low mass accretion rate $\dot{M}_{\rm acc}$ through the disk (see e.g. Cannizzo 1993 or Osaki 1996 for a review). In outburst (high $\dot{M}_{\rm acc}$) accretion dominates all other sources of luminosity, and CVs can reach an absolute magnitude of $\sim 3...5^{\rm mag}$ (Warner 1987). In quiescence CVs are usually fainter by $\sim 4^{\rm mag}$ (Warner 1987), but recently Howell & Szkody (1990) suggested the existence of a class of dwarf novae with very large ("tremendous") outburst amplitudes (TOADs), possibly due to a low mass accretion rate during quiescence (Van Paradijs 1985; Howell, Szkody & Cannizzo 1995; Sproats, Howell & Mason 1996). Frequently it is this variability and the corresponding larger brightness in outburst that leads to the detection of a system. On the other hand, if the mass transfer rate is above a critical limit the accretion disk has constant brightness and the system belongs to the novalike variable subgroup, resembling dwarf novae in outburst. For an extended review of observational properties of CVs we refer to Warner's (1995) book .

The increase in sensitivity of photoelectric devices in recent years made it possible to compile light curves of faint objects even with small telescopes. Howell & Szkody (1990) studied faint CVs at high galactic latitudes (HGL CVs) and proposed to identify them with population II CVs (PopII CVs). PopII CVs are defined as (old) CVs which formed from low metallicity zero age main sequence (ZAMS) binaries in the early phase of Galactic evolution. As CV progenitor binaries do not undergo a supernova explosion the kinematic and spatial distribution of PopII CVs are expected to be similar to PopII single stars, i.e. spread out to a large distance from the Galactic plane with large $\gamma$–velocities.

Here we study PopII CVs from a theoretical point of view and attempt to derive a population model and to predict the observable period distribution for these systems, following a technique which has been successfully applied to PopI CVs (e.g. Kolb 1996 for a review). A comparison between these models should serve as a basis for a more critical evaluation of collective properties of observed suspected PopII CVs.

In Sec. 3 of this paper we discuss the effect of the secondary's initial metallicity (Z= $10^{-5} \ldots 2\,10^{-2}$) on long–term standard CV evolution. We neglect any chemical pollution of the outer convective envelope of the secondaries which might arise

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from accretion of highly metal–rich material from an expanding nova envelope after a nova outburst (see Stehle & Ritter 1997 for a discussion). In Sec. 4 we apply the CV population synthesis technique (Kolb 1993, Kolb & de Kool 1993) to compute the present intrinsic population of PopII CVs from evolutionary sequences for PopII CVs, weighted according to a time–dependent CV birthrate. By extracting a visual magnitude–limited sample from the intrinsic CV population we predict the observable distribution of both PopI and PopII CVs. We discuss our results in Sec. 5. Before we critically examine the relation of Howell & Szkody's sample of HGL CVs in Sec. 2 we note that throughout the remainder of the paper the superscripts $^{\rm I}$ and $^{\rm II}$ refer to values which belong to the population I or II, respectively.

## 2. Observations of PopII CVs

Howell & Szkody (1990) were the first to point out that the probability for a HGL CV to belong to the population II increases with increasing distance $z$ from the Galactic plane. They tried to select CVs with $z > 350$pc from the General Catalogue of Variable Stars (Kukarkin et al. 1969, Kholopov et at. 1985) by a distance estimate from the observed apparent magnitude and an assumed uniform absolute magnitude of $7.5^{\rm mag}$ in quiescence. The sample was limited to CVs with galactic latitude $b \gtrsim 40°$ and lightcurve variations with an amplitude $> 1.9^{\rm mag}$. As a main result the selected sample shows a slightly higher fraction of CVs with orbital periods below the period gap $(2/3)$ than the sample of all CVs with determined orbital periods $(2/5$, e.g. Ritter & Kolb 1995).

In principle Howell & Szkody's method to identify HGL CVs as PopII CVs is certainly justified, but it turns out that the selection criteria they applied were too weak to give a proper PopII CV sample (see also Augusteijn & Stehle 1995):

(i) Despite the fact that the vertical stratification of the Galaxy still is matter of debate (see e.g. Freeman 1987) a crude estimate for the intrinsic probability

$$p_{\rm int}^{\rm II}(z) = \frac{n^{\rm II}(z)}{n^{\rm I}(z) + n^{\rm II}(z)} \qquad (1)$$

that a CV at a given distance $z$ from the Galactic plane is a member of the population II can be obtained from spatial distributions $n(z)$ derived from star counts of single stars (Gilmore, Wyse & Kuijken 1989). By comparing the exponential number density decrease of single stars in the thin Galactic disk (scale height $H^{\rm I}$ = 300pc) with the thick component ($H^{\rm II}$ = 1500pc) we derive $p_{\rm int}^{\rm II}(z = 350{\rm pc}) = 5\%$, where we assumed that the observed single star value $n^{\rm II}(z = 0)/n^{\rm I}(z = 0) = 0.02$ (Gilmore & Reid 1983) is also valid for the ZAMS binary distribution. With these values we find $z_{0.5} \simeq 1500$pc for the distance $z_{0.5}$ where $p_{\rm int}^{\rm II}(z_{0.5}) = 50\%$. A somewhat more optimistic model is obtained with star count values of M stars. In that case Gould, Bahcall & Flynn (1996) find $H^{\rm I}$ = 213pc, $H^{\rm II}$ = 714pc, and $n^{\rm II}(z = 0)/n^{\rm I}(z = 0) = 0.085$. Then $p_{\rm int}^{\rm II}(z = 350{\rm pc}) = 23\%$ and $z_{0.5} \simeq 700$pc. We will use these values for further investigations in Sec. 4. We note that $p_{\rm int}^{\rm II}(z = 350{\rm pc})$ gets even smaller when the thin disk component is compared with halo stars instead of stars in the thick disk.

(ii) For the observable CV population the above estimate is optimistic as we shall show that PopII CVs are intrinsically fainter, hence selection effects (see Sect. 4) operate against PopII CVs, in favour of PopI CVs.

(iii) The absolute magnitude of magnetic CVs (i.e. AM Her, DQ Her systems) is less well constrained and therefore these systems should be excluded from the sample.

Additionally, in order to properly select PopII CV candidates a sufficiently accurate distance determination is essential. Recently, more refined distance estimates (e.g. with Bailey's method using infrared photometry data) by Szkody & Howell (1992), Howell, Szkody & Cannizzo (1995) and in particular Sproats, Howell & Mason (1996) have shown that the method applied by Howell & Szkody (1990) systematically overestimates the distance, i.e. most of the CVs in their sample are closer to the Galactic plane ($z < 350$pc) than originally thought.

Despite these points we do not question the existence of PopII CVs in general. The following systems at very high distance from the Galactic plane are excellent PopII candidates: Among the 7 faint CVs discovered by searching on overlapping sections of digitized SRC-J sky survey plates for variability (Drissen et al. 1994) one object is estimated to be at $z$ = 2900pc. Hawkins & Véron (1987) report another CV at a distance $z$ = 3000pc. Such observations indicate the existence of a large number of faint CVs which are theoretically expected (see below) but evidently difficult to detect. Some of these faint CVs must belong to the population II.

An independent method to find PopII CVs by anomalies in the systems' $\gamma$–velocities was applied by van Paradijs, Augusteijn & Stehle (1995), but apart from some PopII candidates no convincing sample of high velocity CVs was found.

Although a statistically significant observed sample of PopII CVs is still missing we nevertheless derive theoretically in the next sections in what respect collective properties of PopII and standard PopI CVs are expected to differ due to the different underlying formation and evolution.

## 3. The influence of the secondary's chemical composition on the secular evolution of CVs

### *3.1. Description of model parameters*

In practically all observed CVs the mass ratio $q = M_1/M_2$ ($M_1$ denotes the WD mass, $M_2$ the secondary mass) is such that the systems are dynamically and thermally stable against mass transfer. It is commonly accepted that mass transfer from the secondary to the WD is driven by loss of orbital angular momentum, e.g. due to gravitational radiation and magnetic braking (see e.g. King 1988, Ritter 1996). Within the framework of the disrupted magnetic braking model (Rappaport, Verbunt & Joss 1983, Spruit & Ritter 1983) the latter is assumed to operate only as long as the secondary is not fully convective, whereas gravitational wave emission is a natural general relativistic consequence of the binary's changing quadrupole

**Table 1.** A collection of numerical calculations of CV evolution with the stellar evolution code of Mazzitelli (1989). For all calculations the chemical composition is Y = 0.25 and Z = $10^{-4}$ but they differ in the WD mass $M_1$ and initial secondary mass $M_{2,\mathrm{i}}$. Listed is the mass $M_{\mathrm{conv}}$ where the secondaries become fully convective, the ratio $(\tau_{\mathrm{KH,e}}/\tau_{\mathrm{M}})_{\mathrm{u}}$ shortly before the CVs enter the period gap, the upper and lower period value of the period gap ($P_{\mathrm{u}}$ and $P_{\mathrm{l}}$ respectively), the minimum period $P_{\mathrm{min}}$ and the corresponding secondary mass $M_{\mathrm{min}}$. The first column indicates the prescription used to calculate the loss of orbital angular momentum. GR indicates gravitational radiation only, VZ magnetic braking according to Verbunt & Zwaan (1981) with the free parameter $f_{\mathrm{VZ}}$ and MS according to Mestel & Spruit (1987) with $n$ as a free parameter. Magnetic braking is assumed to operate only as long as the secondary is not fully convective.

| | $M_1$ | $M_{2,\mathrm{i}}$ | $M_{\mathrm{conv}}$ | $\frac{\tau_{\mathrm{KH,e}}}{\tau_{\mathrm{M}}}\Big\vert_{\mathrm{u}}$ | $P_{\mathrm{u}}$ | $P_{\mathrm{l}}$ | $M_{\mathrm{min}}$ | $P_{\mathrm{min}}$ |
|---|---|---|---|---|---|---|---|---|
| $f_{\mathrm{VZ}}\Vert n$ | $M_\odot$ | $M_\odot$ | $M_\odot$ | | h | h | $M_\odot$ | h |
| VZ 1.0 | 0.7 | 0.6 | 0.2914 | 3.249 | 2.805 | 2.137 | | |
| GR | 1.0 | 0.41 | | | | 2.746 | | |
| VZ 1.0 | 1.0 | 0.42 | 0.3244 | 2.623 | 2.838 | 2.309 | | |
| VZ 1.0 | 1.0 | 0.47 | 0.3173 | 2.570 | 2.764 | 2.264 | | |
| VZ 1.0 | 1.0 | 0.6 | 0.3148 | 2.648 | 2.730 | 2.255 | 0.0752 | 0.916 |
| VZ 1.0 | 1.0 | 0.9 | 0.3134 | 2.422 | 2.724 | 2.245 | 0.0747 | 0.914 |
| VZ 1.0 | 1.3 | 0.6 | 0.3300 | 2.241 | 2.712 | 2.328 | | |
| MS 1.0 | 1.0 | 0.6 | 0.3560 | 1.497 | 2.680 | 2.438 | | |
| MS 1.2 | 1.0 | 0.6 | 0.3288 | 2.387 | 2.710 | 2.322 | | |

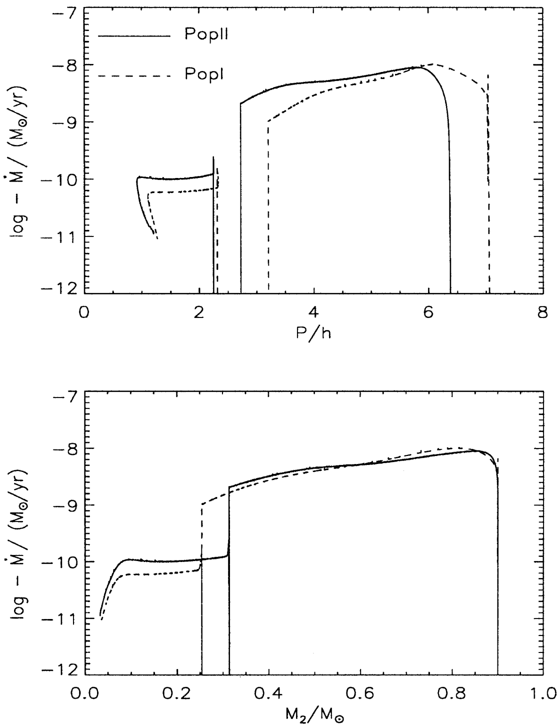


**Fig. 1a and b.** Mass transfer rate $\log|\dot{M}|$ versus binary period $P$ (**a**) and the secondary mass $M_2$ (**b**) for a PopI and PopII CV. The secondary's chemical composition is ($Z^{\mathrm{I}} = 0.02, Y^{\mathrm{I}} = 0.28$) and ($Z^{\mathrm{II}} = 10^{-4}, Y^{\mathrm{II}} = 0.25$) respectively. The WD mass $M_1 = 1M_\odot$ is constant along the sequences, the initial secondary mass is $M_{2,\mathrm{i}} = 0.9M_\odot$. See also Table 1 and 2.

moment and is unavoidable for all secondary masses (see e.g. Shapiro & Teukolsky 1983).

From the spin–down timescale of young main–sequence G stars with an age between several $10^6$ and $10^9$ yr Verbunt & Zwaan (1981) estimated that the timescale of magnetic braking is of order $\tau_{\mathrm{VZ}} \simeq 1\ldots3\ 10^9$yr and scales with the secondary's stellar radius $R_2$ and radius of gyration $r_{\mathrm{g},2}$ as $\tau_{\mathrm{VZ}} \sim R_2 r_{\mathrm{g},2}^{-2}$. Physical models for magnetic braking involve an ionized stellar wind launched along magnetic field lines (Weber & Davis 1967, Mestel & Spruit 1987) and predict that the braking rate is not sensitive to the low–mass star's chemical composition. Therefore we apply the Verbunt & Zwaan (1981) formulation of magnetic braking also for CVs with low metallicity secondaries. In this way, for a given WD mass, the evolution of a CV is completely determined by the stellar structure of the secondary which we compute numerically with the stellar evolution code of Mazzitelli (1989), adapted to treat mass transfer (for a more detailed description see Kolb & Ritter 1992 and references therein).

For the comparative discussion of CV evolution below it will be helpful to remember that PopII low–mass stars are smaller ($R^{\mathrm{II}} < R^{\mathrm{I}}$), hotter ($T_{\mathrm{eff}}^{\mathrm{II}} > T_{\mathrm{eff}}^{\mathrm{I}}$) and more luminous ($L^{\mathrm{II}} > L^{\mathrm{I}}$) than PopI low–mass stars (see e.g. D'Antona & Mazzitelli 1985, D'Antona 1987). Our model computations confirm the finding by D'Antona & Mazzitelli (1982) that the transition mass $M_{\mathrm{L}}$ from the main sequence to the degenerate phase changes only slightly with metal and helium content. The actual input physics used to calculate the internal structure of brown dwarfs, still a matter of debate (see, e.g., Burrows & Liebert 1993 and references therein, Burrows et al. 1993, Saumon et al. 1995), determines the value and precise dependencies of $M_{\mathrm{L}}$. Similarly, the limiting mass $M_{\mathrm{conv}}$ below which ZAMS stars are fully convective is larger for PopII stars ($0.41M_\odot$ for $Z = 10^{-4}$, $Y = 0.25$) than for PopI stars ($0.35M_\odot$ for $Z = 0.02$, $Y = 0.25$), but again the precise values depend on the stellar structure input physics. The stellar radius $R(M_{\mathrm{conv}})$ of stars with mass $M_{\mathrm{conv}}$ is, however, for both cases almost the same. The gyration radius $r_{\mathrm{g}}$ is a measure of how centrally condensed a star is, and increases with the mass fraction of the convective envelope (e.g. Ruciński 1988). Thus (for $M > M_{\mathrm{conv}}$) it increases with decreasing stellar mass along the low–mass ZAMS.

### *3.2. Secular evolution of PopI and PopII CVs*

In Fig. 1 we compare the secular evolution of a PopI CV ($Z_2^{\mathrm{I}} = 0.02, Y_2^{\mathrm{I}} = 0.28$) with that of a PopII CV ($Z_2^{\mathrm{II}} = 10^{-4}, Y_2^{\mathrm{II}} = 0.25$) in a $\log|\dot{M}|$–$P$ and $\log|\dot{M}|$–$M_2$ diagram, where $\dot{M}$ is the mass transfer rate and $P$ the orbital period. In both evolutionary sequences the WD mass $M_1 = 1M_\odot$ was kept constant and the secondary was assumed to evolve chemically only by nuclear burning. Further characteristic quantities of the sequences can be found in Tables 1 and 2.

Due to the smaller PopII stellar radii the corresponding evolutionary track appears to be shifted to shorter periods in the

**Table 2.** The same as Table 1 but for different chemical compositions Y and Z (see column 1 and 2). All listed calculations are computed with a constant WD mass of 1 $M_\odot$ and an initial secondary mass $M_{2,\mathrm{i}} = 0.6 M_\odot$ with the Verbunt & Zwaan (1981) description of magnetic braking ($f_{\mathrm{VZ}} = 1$).

| Y | Z | $M_{\mathrm{conv}}$ | $\frac{\tau_{\mathrm{KH,e}}}{\tau_{\mathrm{M}}}\Big\vert_{\mathrm{u}}$ | $P_{\mathrm{u}}$ | $P_{\mathrm{l}}$ | $M_{\mathrm{min}}$ | $P_{\mathrm{min}}$ |
|---|---|---|---|---|---|---|---|
| | | $M_\odot$ | | h | h | $M_\odot$ | h |
| 0.23 | $10^{-5}$ | 0.3328 | 2.641 | 2.766 | 2.285 | 0.0792 | 0.853 |
| 0.25 | $10^{-5}$ | 0.3250 | 2.534 | 2.720 | 2.273 | 0.0769 | 0.844 |
| 0.27 | $10^{-5}$ | 0.3174 | 2.447 | 2.676 | 2.258 | 0.0753 | 0.835 |
| 0.23 | $10^{-4}$ | 0.3227 | 2.768 | 2.780 | 2.271 | | |
| 0.25 | $10^{-4}$ | 0.3148 | 2.648 | 2.730 | 2.255 | 0.0752 | 0.916 |
| 0.27 | $10^{-4}$ | 0.3070 | 2.553 | 2.688 | 2.245 | 0.0737 | 0.907 |
| 0.23 | $10^{-3}$ | 0.2762 | 3.254 | 2.869 | 2.212 | 0.0749 | 1.010 |
| 0.25 | $10^{-3}$ | 0.2702 | 3.147 | 2.817 | 2.195 | 0.0734 | 1.000 |
| 0.27 | $10^{-3}$ | 0.2640 | 3.081 | 2.765 | 2.182 | 0.0716 | 0.989 |
| 0.25 | 0.01 | 0.2680 | 3.476 | 3.189 | 2.319 | 0.0752 | 1.091 |
| 0.25 | 0.02 | 0.2657 | 3.598 | 3.327 | 2.362 | 0.0742 | 1.120 |
| 0.28 | 0.02 | 0.2545 | 3.571 | 3.212 | 2.314 | | |

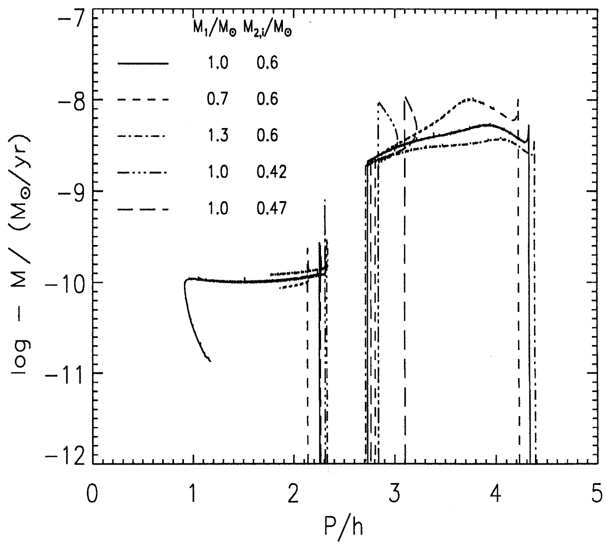


**Fig. 2.** Mass transfer rate $|\dot M|$ versus orbital period $P$ for PopII CVs. The sequences differ only in the WD mass $M_1$ and the initial secondary mass $M_{2,\mathrm{i}}$. See also Table 1.

log $|\dot M|$–$P$ plane. Thus the turn–on periods, i.e. the period where the secondary fills its critical Roche volume for the first time, differ by 54min, for example. For $M_2 < 0.57 M_\odot$, where the mass of the convective envelope is large, $r^{\mathrm{I}}_{\mathrm{g},2}$ and $r^{\mathrm{II}}_{\mathrm{g},2}$ are comparable, so that $|\dot M|$ is larger for PopII CVs because of the smaller secondary radius $R_2$. In contrast, the smaller convective envelope of PopII stars in the regime $M_2 > 0.57 M_\odot$ corresponds to a smaller gyration radius which overcompensates the dependence of $\tau_{\mathrm{VZ}}$ on $R_2$, leading to a somewhat smaller mass transfer rate.

When the secondary becomes fully convective and magnetic braking vanishes the system enters a detached phase and mass transfer stops (see e.g. Kolb 1996). The orbital period where $|\dot M|$ drops sharply is referred to as $P_{\mathrm{u}}$. Because of thermal relaxation the secondary's radius subsequently decreases to its equilibrium value. Mass transfer resumes at an orbital period $P_{\mathrm{l}}$ when the Roche lobe — still shrinking through gravitational radiation — catches up with the stellar radius. Recently Stehle, Ritter & Kolb (1996) showed explicitly that the period width $\Delta P_{\mathrm{gap}} = P_{\mathrm{u}} - P_{\mathrm{l}}$ of the detached phase increases with the ratio of Kelvin–Helmholtz time $\tau_{\mathrm{KH}} = G M_2^2 / R_2 L_2$ and mass transfer timescale $\tau_{\mathrm{M}} = |M_2/\dot M|$, taken at $P_{\mathrm{u}}$. Since $\tau^{\mathrm{II}}_{\mathrm{KH}}(P^{\mathrm{II}}_{\mathrm{u}}) < \tau^{\mathrm{I}}_{\mathrm{KH}}(P^{\mathrm{I}}_{\mathrm{u}})$ because the larger surface luminosities of PopII stars dominate over the effect of the corresponding smaller radii, and since $\tau^{\mathrm{II}}_{\mathrm{M}}(P^{\mathrm{II}}_{\mathrm{u}}) \simeq \tau^{\mathrm{I}}_{\mathrm{M}}(P^{\mathrm{I}}_{\mathrm{u}})$, the period width $\Delta P^{\mathrm{II}}_{\mathrm{gap}}$ is only half that of PopI CVs, $\Delta P^{\mathrm{I}}_{\mathrm{gap}}$ (namely $\Delta P^{\mathrm{II}}_{\mathrm{gap}}$ = 28.7 min and $\Delta P^{\mathrm{I}}_{\mathrm{gap}}$ = 53.3 min). Since $\tau_{\mathrm{KH}}$ is shorter a PopII secondary deviates less from thermal equilibrium than the PopI secondary, i.e. is less bloated when it enters the detached phase; hence $P^{\mathrm{II}}_{\mathrm{u}} < P^{\mathrm{I}}_{\mathrm{u}}$.

The higher central temperature of PopII stars (arising from the need to maintain a larger surface luminosity) results in a higher limiting mass $M^{\mathrm{II}}_{\mathrm{conv}} > M^{\mathrm{I}}_{\mathrm{conv}}$ ($M^{\mathrm{I}}_{\mathrm{conv}} = 0.25 M_\odot$ compared to $M^{\mathrm{II}}_{\mathrm{conv}} = 0.31 M_\odot$ for stars in thermal equilibrium). The effects of the generally smaller radius $R^{\mathrm{II}}_2$ and the larger mass $M^{\mathrm{II}}_{\mathrm{conv}}$ on the binary period nearly compensate, so that the PopI and PopII values of the turn–on period $P_{\mathrm{l}}$ after the detached phase differ by only 4 min.

The timescale of orbital angular momentum loss due to gravitational radiation scales as $\tau_{\mathrm{GR}} \propto R_2^4$, and thus $|\dot M^{\mathrm{II}}| \simeq 1.4 |\dot M^{\mathrm{I}}|$ after the detached phase. As a general rule, because of a generally higher mass transfer rate $|\dot M^{\mathrm{II}}|$, the secular evolution timescale of PopII CVs is smaller than for PopI CVs.

The secondary mass $M_{2,\mathrm{min}}$ at the minimum period (the orbital period where $P$ begins to increase again with further decreasing secondary mass) changes very little with the secondary's chemical composition. This is not surprising given the composition insensitivity of the transition mass $M_{\mathrm{L}}$ from the main–sequence to the degenerate phase mentioned above. The $\Delta P_{\mathrm{min}}$ = 11.5 min shorter PopII minimum period found in our calculations is mainly due to the generally smaller PopII stellar radius. We note that in our models for PopI CVs, we find a minimum period which is systematically shorter than the observed value $\sim$ 80 min, probably due to the neglect of rotational and tidal corrections in the stellar structure equations. Nevertheless we are convinced that the differential change we find is independent of the absolute calibration. We therefore expect that PopII CVs may have orbital periods as small as $\sim$70 min.

### *3.3. PopII CV evolution for different initial parameters*

In addition to the evolutionary sequences (hereafter: CV sequences) described in the previous paragraph we show in Fig. 2 PopII secular evolution tracks with different initial conditions, again with the Verbunt & Zwaan (1981) formulation for magnetic braking. The main parameters of the computations are summarized in Table 1.

The differential change of $|\dot M(P)|$ for different WD and initial secondary masses is the same as for PopI CVs and well known from previous investigations (e.g. Kolb 1993): above the period gap $|\dot{M}|$ increases with decreasing WD mass, the position and width of the detached phase in period space changes only little. Sequences with the same WD mass but different $M_{2,\mathrm{i}}$ converge to a single track (Stehle et al. 1996). In CVs with predominantly convective secondaries $P$ increases for a short phase after the onset of mass transfer since the adiabatic mass–radius exponent of the secondary determining the initial reaction on mass loss is less than the critical value $1/3$.

A sequence where the magnetic braking rate was calculated from Hameury et al. 's (1988) implementation of the Mestel & Spruit (1987) model is similar to the corresponding sequence obtained with magnetic braking according to Verbunt & Zwaan (1981), see Table 1 and Stehle (1993).

From those six sequences in Table 1 with $M_{2,\mathrm{i}} \geq 0.6 M_\odot$ where the time spent before the secondary becomes fully convective is long enough for the system to approach the system's uniform, initial condition independent evolution (cf. Stehle et al. 1996) we find a linear relation between $M_{\mathrm{conv}}$ and $\tau_{\mathrm{KH}}/\tau_{\mathrm{M}}$. A least square fit which includes the limiting mass of the unperturbed ZAMS (model $\tau_{\mathrm{KH}}/\tau_{\mathrm{M}} = 0$) gives

$$\frac{M_{\mathrm{conv}}}{M_\odot} = 0.41 - 0.03654 \left(\frac{\tau_{\mathrm{KH}}}{\tau_{\mathrm{M}}}\right) \quad (2)$$

with a rms deviation of 0.0046. This is due to the decreasing central temperature with increasing deviation from thermal equilibrium.

Finally we note that the turn–on period $P_{\mathrm{to}}(M_{2,\mathrm{i}} = 0.41 M_\odot)$ = 2.75h for the sequence with an initially fully convective secondary ($M_{2,\mathrm{i}} = 0.41 M_\odot$) is longer than the period $P_{\mathrm{u}}$ for most PopII evolutionary sequences we computed (see Table 1). Hence we expect that the period region of the detached phase does not appear as a completely empty "gap" in the period distribution of an ensemble of PopII CVs (see Sec. 4).

### 3.4. CV evolution as a function of metallicity

In this section we study CV evolution as a function of the secondary's metallicity ($Z = 10^{-5}, 10^{-4}, 10^{-3}, 0.01, 0.02$) and He–content ($Y = 0.23, 0.25, 0.27$).

For a better comparison we adopt $M_1 = 1.0\,M_\odot$ = const. and $M_{2,\mathrm{i}} = 0.6\,M_\odot$ for all these calculations. Some values of interest are listed in Table 2. In Fig. 3 we compare sequences with Y= 0.25 but different metallicity in a $\log|\dot{M}| - P$ diagram. Fig. 4 shows $P_{\mathrm{u}}$, $P_{\mathrm{l}}$ and $P_{\mathrm{min}}$ as a function of $\log Z$ for a He content of $Y = 0.25$. We summarize our results as follows:

(i) $P_{\mathrm{u}}$ and $P_{\mathrm{min}}$ increase with $\log Z$ whereas $P_{\mathrm{l}}$ changes only marginally, as explained in Sec. 3.2.
(ii) The period width of the detached phase increases mainly between $Z = 10^{-3}$ and $Z = 0.02$. This is due to the decrease of the stellar luminosity with changing surface opacity which increases most significantly in that regime.
(iii) $P_{\mathrm{min}}$ for $Y = 0.25$ can be approximated by

$$P_{\mathrm{min}}/\mathrm{h} = 1.258 + 0.084 \log(\mathrm{Z}) \quad (3)$$

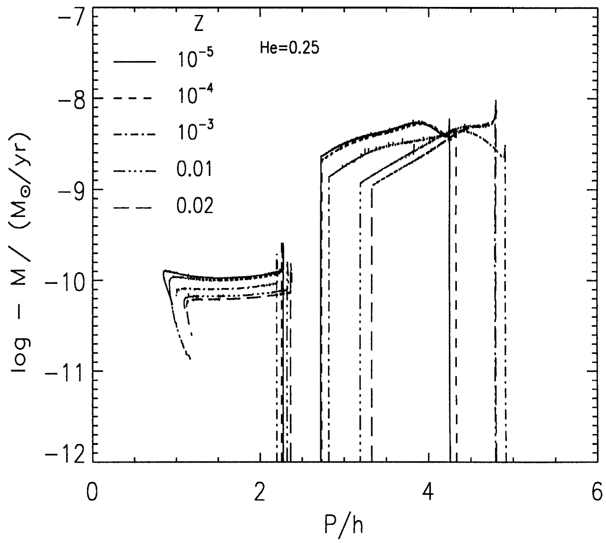


**Fig. 3.** CV evolution as a function of the secondary's metallicity (from $Z = 10^{-5}$ to $Z = 0.02$). For all sequences $Y = 0.25$. We uniquely chose $M_{2,\mathrm{i}} = 0.6\,M_\odot$ and $M_1 = 1\,M_\odot$ = const. Further parameters are summarized in Table 2.

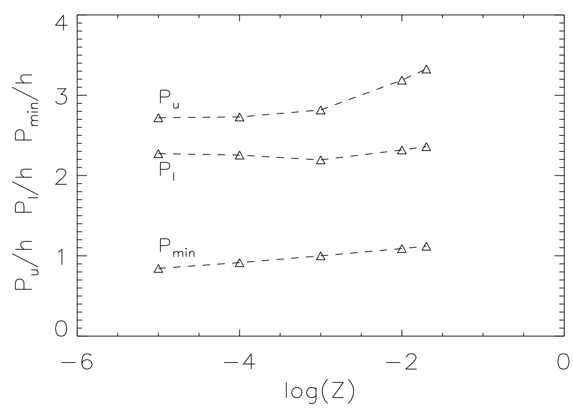


**Fig. 4.** Upper $P_{\mathrm{u}}$ and lower $P_{\mathrm{l}}$ edge of the detached phase and period minimum $P_{\mathrm{min}}$ as a function of metallicity $Z$ for a helium content $Y = 25$. The runs of $P(Z)$ are only marginally influenced by the choice of $Y$ (see, e.g., Table2).

with a rms deviation of $\sigma = 0.0066$. Although we expect that the precise value of $P_{\mathrm{min}}$ is sensitive to the input physics, in particular to surface opacities and the equation of state, the differential change as expressed by the slope $\partial P_{\mathrm{min}}/\partial \log(Z)$ in Eq. (3) should be independent of these factors. This should also be true if corrections to the stellar structure equations for rotational and tidal deformation (see Nelson, Chau & Rosenblum 1985), neglected in our computations, are properly taken into account.

## 4. A model for a PopII CV population

To predict the intrinsic and observable distribution of CV system parameters (e.g. the orbital period distribution $n(P)$) we have to synthesize a complete CV population from individual evolutionary sequences. As already described elsewhere in detail (e.g. de Kool 1992, Kolb 1993) this population synthesis procedure consists of three steps: determining the CV birthrate, calculating the secular evolution and correcting for selection effects.

### 4.1. Time–dependent CV birthrate

The time–dependent formation rate $\widehat{b}(M_{1,\mathrm{i}}, M_{2,\mathrm{i}}, t_{\mathrm{ZZ}})$ of newborn CVs, i.e. of systems appearing as CVs for the first time, was computed by de Kool (1992). $\widehat{b}$ is given as an explicit function of the initial WD and secondary mass $M_{1,\mathrm{i}}$, $M_{2,\mathrm{i}}$, and the time $t_{\mathrm{ZZ}}$ elapsed since formation of the ZAMS binary. Below we use Model 3 of de Kool (1992) which is based on the following assumptions: both ZAMS binary component masses form independently from the same Miller & Scalo (1979) initial mass function, the distribution of the initial binary separation $a$ is flat in $\log a$, and the common envelope (CE) ejection efficiency $\alpha_{\mathrm{CE}}$ is 1 (see de Kool (1992) for a definition of $\alpha_{\mathrm{CE}}$). We restrict our study to CVs with unevolved main–sequence secondaries (channel L in de Kool 1992) and ignore systems containing evolved secondaries (channel I). These contribute $\lesssim 10\%$ to the total intrinsic population (Kolb & de Kool 1993). Since no sufficiently dense grid of PopII single star evolutionary sequences is currently available, we assume that the evolution up to the CV state is the same for PopI and PopII stars. In particular, we use the same underlying mass–radius relations (Webbink 1988, Politano 1996) needed to determine the pre–CE evolution. Fortunately it turns out that the dominant influence on a PopII CV population comes from the time–dependence of the *star formation rate* (see below), not so much from the detailed shape of $\widehat{b}$.

A convolution with the star formation history $SF(t)$, the formation rate of ZAMS binaries, converts $\widehat{b}$ to the CV birthrate $b(M_{1,\mathrm{i}}, M_{2,\mathrm{i}}, t_{\mathrm{ZACV}})$, as a function of "galactic time" $t_{\mathrm{ZACV}}$, measured on a time axis with origin at the onset of our Galaxy's formation:

$$b(M_{1,\mathrm{i}}, M_{2,\mathrm{i}}, t_{\mathrm{ZACV}}) = \int_0^{t_{\mathrm{ZACV}}} \widehat{b}(M_{1,\mathrm{i}}, M_{2,\mathrm{i}}, t_{\mathrm{ZACV}} - t_{\mathrm{SF}})\, SF(t_{\mathrm{SF}}) \mathrm{d}t_{\mathrm{SF}}. \quad (4)$$

For $SF(t_{\mathrm{SF}})$ we arbitrarily adopt a step function

$$SF(t_{\mathrm{SF}}) = \begin{cases} s & \text{for} \quad t_{\mathrm{SF}} \in \{0 \ldots t_{\max}\} \\ 0 & \text{for} \quad t_{\mathrm{SF}} \in \{t_{\max} \ldots t_{\mathrm{Gal}}\} \end{cases} \quad (5)$$

where $t = t_{\mathrm{Gal}}$ denotes the present time, and $s$ = const. We assume $t_{\mathrm{Gal}} = 10^{10}$yr for the age of our Galaxy and $t_{\max} = 10^9$yr for the duration of the initial PopII star forming period. Varying these parameters has little influence on our results (see also Kolb & Stehle 1996).

### 4.2. The secular evolution

As described by Kolb (1993) the secular evolution defines a mapping from the initial CV configuration space with coordinates ($M_{1,\mathrm{i}}$, $M_{2,\mathrm{i}}$, $t_{\mathrm{ZACV}}$) to the present ($t = t_{\mathrm{Gal}}$) configuration space consisting of configuration vectors $\mathbf{A} = (M_1, M_2, |\dot{M}|, P)$. Similarly, the distribution function $b$ in the initial space is mapped to the distribution function $n(\mathbf{A}) = n(M_1, M_2, |\dot{M}|, P)$ in the final space. $n(\mathbf{A})$ represents the intrinsic distribution of the present CV population and is described in more detail in the next section.

Numerically the mapping is accomplished by covering the initial configuration space with a sufficiently dense grid of CV sequences, computed with a generalized bipolytrope code (Kolb & Ritter 1992). The code was supplemented by a new calibration designed to reproduce the PopII CV sequences described in Sec. 3.2 and 3.3. This calibration allows proper treatment of CVs with an initial secondary mass in the range $M_{2,\mathrm{i}} \in [0.11, 0.9] M_{\odot}$, i.e. secondaries with a convective envelope. With this restriction we miss only about 8 % of the total initial configuration space of CVs with unevolved secondaries.

### 4.3. The intrinsic distribution

We compare the intrinsic distribution of PopI and PopII CVs differentially in the following way. First (case 1) we discuss the effect the different secular evolutions cause by assuming a constant star formation rate with $t_{\max} = t_{\mathrm{Gal}}$ and $s^{\mathrm{I}} = s^{\mathrm{II}} = 1$ for both populations. In a second step (case 2) we study the influence of a reduced duration of the star formation period by assuming $t_{\max} = 10^9$yr for the population II; here the choice $s^{\mathrm{II}} = 10$ ensures that the total number of ZAMS binaries is the same in all three population models. For both cases we show in Figs. 5 and 6 the relative number density $n(P)$ and $n(|\dot{M}|)$ on a logarithmic scale, obtained by integrating the intrinsic distribution over the respective other parameters, e.g.:

$$n(P) = \int\int\int n(\mathbf{A})\ \mathrm{dM_1\, dM_2\, d|\dot{M}|}. \quad (6)$$

The differences in the secular evolution of PopI and PopII CVs noted above clearly reappear as differences of the intrinsic distributions in case 1 (Fig. 5). The most striking feature of the PopII distribution is the lack of a well–defined period gap. The detached phase of individual sequences is too small in period space to reappear as an empty period region in the total population; rather this region is effectively smeared out mainly by systems born with a fully convective secondary. As a consequence, $n(\log P)$ drops in the "gap" by less than a factor $\sim 10$. Furthermore, the PopII distribution is shifted towards somewhat shorter binary periods and slightly higher mass transfer rates, mainly because of the smaller stellar radii of PopII secondaries. The shorter PopII evolution timescale leads to a smaller fraction of systems driven by magnetic braking ( 0.15% versus 0.29%), and to a larger fraction of systems which have already passed the period minimum (75% instead of $\simeq$ 67%). The latter are those systems in the $n(\log |\dot{M}|)$ distribution with extremely small mass transfer rate ($\log |\dot{M}| \lesssim$ few $10^{-11} M_{\odot}/\mathrm{yr}$).
The two density spikes in $n(|\dot{M}|)$ at $\sim 10^{-10} M_{\odot}/\mathrm{yr}$ represent gravitational radiation driven systems below the period gap with

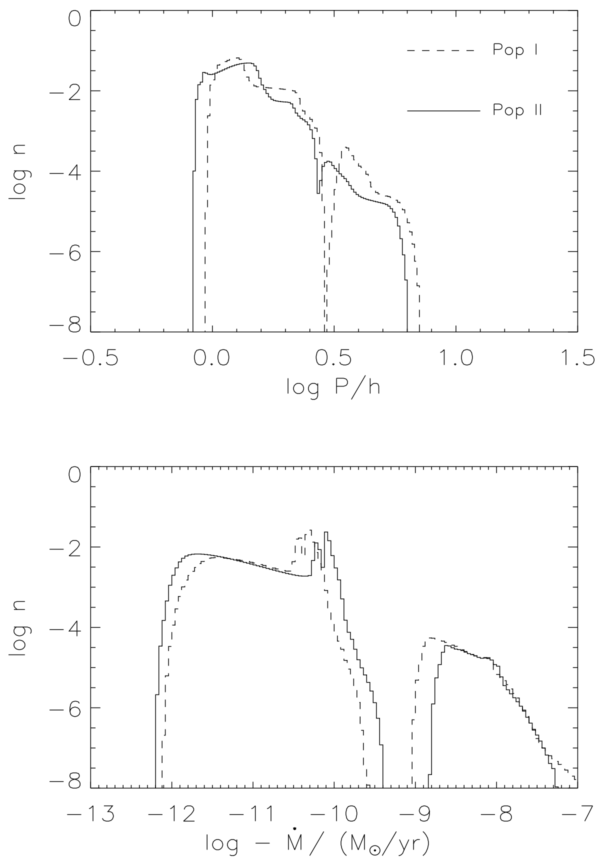


**Fig. 5.** The intrinsic distribution $n(\log P)$ and $n(\log|\dot{M}|)$ for PopI (dashed) and PopII CVs (full line), both with $t_{\rm max} = 10^{10}$yr and constant star formation rate $s^{\rm I} = s^{\rm II} = 1$. Note that both axes are plotted on a logarithmic scale.

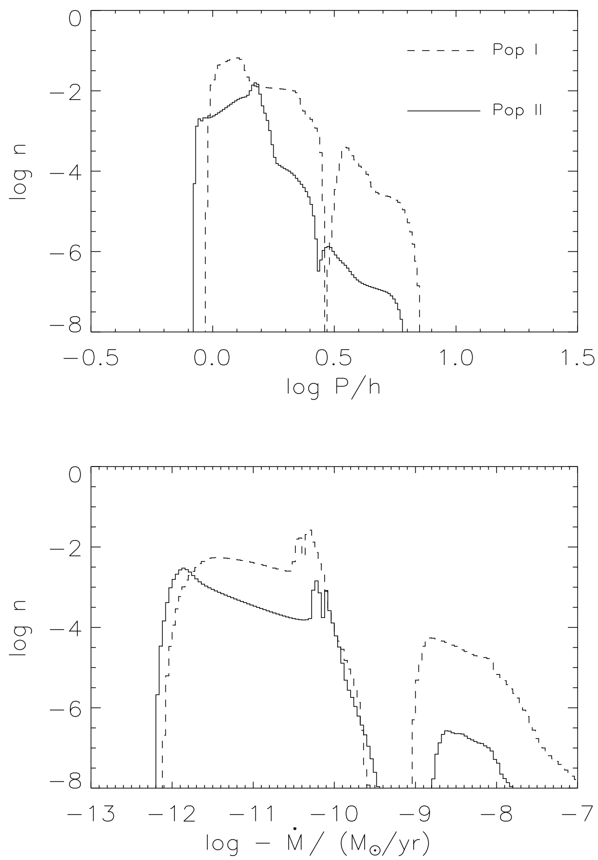


**Fig. 6.** As Fig.5, but with $t^{\rm II}_{\rm max} = 10^9$yr and $s^{\rm II} = 10$.

a (low–mass) He–WD or a (high–mass) CO–WD and a non degenerate secondary. See Kolb (1993) for a further discussion on the shape of the intrinsic distribution.

In case 2 PopII CVs are reduced to an old population (Fig.6). As a result, the fraction of systems above the period gap decreases to 0.007%, and at the period gap the density drops now by only a factor $\sim 5$. The fraction of CVs which have evolved past the minimum period, however, increases to 94%. The discovery of such systems clearly represents an observational challenge as they are characterized by a very low mass transfer rate, a correspondingly small intrinsic luminosity, and presumably a very long dwarf nova recurrence time.

These results are remarkable as they show that only about 1 in $10^4$ PopII CVs is above the period gap, about a factor of 30 . . . 50 less than for the case of PopI CVs. Hence a smaller fraction of the total PopII CV population is intrinsically bright enough to be observable; see Fig. 7, where we plot the distribution of CVs over absolute visual magnitude $M_{\rm v}$.

We derive the luminosity in visual light of a CV with a set of intrinsic system parameters from the bolometric luminosity

$$L_{\rm bol} = \frac{G\,M_1\,\dot{M}_{\rm acc}}{R_1}, \tag{7}$$

neglecting any source of luminosity other than accretion onto the WD, combined with bolometric corrections, taken from Dünhuber (1993). These were obtained by assuming a stationary optically thick accretion disk with a boundary layer of elliptical shape and black–body radiation from any surface element according to its local effective temperature. To determine the WD radius $R_1$ we apply the relation given by Nauenberg (1972). As dwarf novae are mainly detected during outburst we adopt for systems with an unstable disk somewhat arbitrarily a mass accretion rate $\dot{M}_{\rm acc}$ during outburst which is 10 times the secular mean mass transfer rate.

### 4.4. The observable distribution

To predict the observable fraction of CVs we have to correct the intrinsic distribution for detectability and selection effects. Introducing the detection probability $p_{\rm s}(\mathbf{A}, d)$ of a CV with intrinsic parameters $\mathbf{A} = (P, M_1, M_2, |\dot{M}|)$ at a distance $d$ from Earth, the relative number density of the observable distribution reads

$$N(\mathbf{A}) = \int n(\mathbf{A})\,p_{\rm s}(\mathbf{A}, d)\,p_{\rm Gal}(\mathbf{r})\;{\rm d}^3{\rm r}, \tag{8}$$

where the integration is over the volume of the Galaxy (for practical reasons we integrate over $d \le 5$kpc). The period dis-

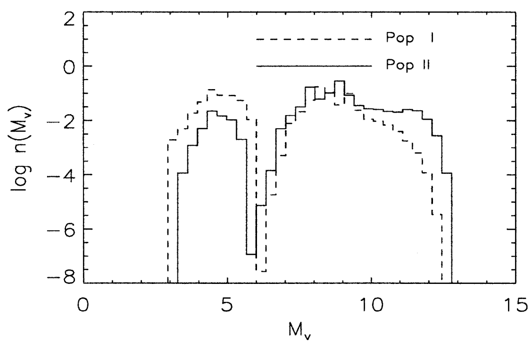


**Fig. 7.** The intrinsic distribution $n(M_\mathrm{v})$ as a function of the visual absolute magnitude. $M_\mathrm{v} = M_\mathrm{v}(P, M_1, M_2, |\dot{M}_\mathrm{acc}|)$ is taken from Dünhuber (1993). We assume that dwarf novae are detected in outburst where we arbitrarily set $\dot{M}_\mathrm{acc} = 10|\dot{M}|$.

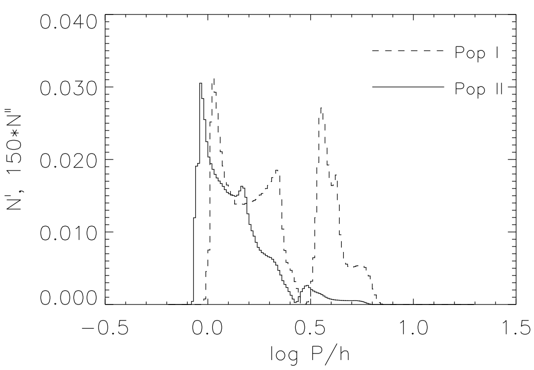


**Fig. 8.** The observable distribution $N^\mathrm{I}(\log P)$ and 150 × $N^\mathrm{II}(\log P)$, in arbitrary units. $N^\mathrm{II}$ is multiplied by 150 to yield comparable curves. The galactic model is that of Gould et al. (1995), selection effects were treated according to Eq. (9).

tribution $N(P)$ is derived from $N(\mathbf{A})$ in the same way as $n(P)$ from $n(\mathbf{A})$, Eq. (6). $p_\mathrm{Gal}(\mathbf{r})$ describes the normalized spatial distribution of the CV subpopulation in the Galaxy, taken from Galactic models. As CVs are mainly found by chance and no complete, systematic survey with well–defined selection criteria and a sufficient number of systems exists, the specification of $p_\mathrm{s}$ is somewhat arbitrary. A commonly used assumption is that the detectability of an object and its identification as a CV is highly correlated with the visual apparent brightness $l_\mathrm{v}$.

Dünhuber & Ritter (1993; see also Dünhuber 1993) improved the study by Ritter (1986) and Ritter & Burkert (1986) and reinvestigated magnitude–limited PopI CV samples constructed under the assumption that *all* CVs brighter than a given brightness–limit $l_\mathrm{v,limit}$ are observed ($p_\mathrm{s}(l_\mathrm{v} > l_\mathrm{v,limit}) = 1$) and all other sources remain undetected ($p_\mathrm{s}(l_\mathrm{v} < l_\mathrm{v,limit}) = 0$). By comparing such models with the actually observed CV sample they found that the probability of detecting a CV decreases rapidly with decreasing apparent brightness. We use this result to specify the detection probability of a CV:

$$p_\mathrm{s}(m_\mathrm{v}) = \begin{cases} 1 & \text{for} \quad m_\mathrm{v} \le 8 \\ 10^{-2/3\,(m_\mathrm{v}-8)} & \text{for} \quad 8 \le m_\mathrm{v} \le 20 \\ 0 & \text{for} \quad m_\mathrm{v} \ge 20 \end{cases} \tag{9}$$

where $m_\mathrm{v} = m_\mathrm{v}(\mathbf{A}, d)$ is the apparent magnitude of the CV with distance $d$ to Earth. In Fig. 8 we show $N^\mathrm{I}(P)$ and $N^\mathrm{II}(P)$. These represent the model prediction for how an observed period distribution of a PopI or PopII CV sample should look.

Unless the limiting magnitude which determines where the sample begins to become incomplete is not much fainter than $8^\mathrm{mag}$, the resulting distribution is insensitive to this parameter. Similarly, the actual cut–off magnitude, if reasonably faint (here $20^\mathrm{mag}$), is not important. What matters most is the slope determining the increase of incompleteness for fainter systems. The slope adopted in (9) was calibrated such that the number of systems above and below the period gap in the predicted PopI CV period distribution is about the same. The resulting rather steep slope effectively means that the observed CVs represent a sample of quite nearby systems, as it is indeed confirmed by distance estimates (Warner 1995).

In fact, this property justifies the ansatz made in some previous studies (Kolb 1993, 1995, 1996) to estimate the observable CV period distribution by neglecting the $z$–dependence of the spatial CV distribution ($p_\mathrm{Gal}$ = const. in (8)) and assuming a step function for $p_\mathrm{s}(m_\mathrm{V})$ (completeness down to a limiting brightness, non–detection below). In this case the relative shape of $N(\mathbf{A})$ is independent of the limiting magnitude and can be written simply as

$$N(\mathbf{A}) = n(\mathbf{A}) V(\mathbf{A}) \, , \tag{10}$$

where $V(\mathbf{A}) \propto L_\mathrm{V}^{3/2}$ is the observable volume for systems with configuration $\mathbf{A}$ and visual luminosity $L_\mathrm{V}$. Following Dünhuber (1993) the latter can be approximated by

$$L_\mathrm{V} \propto M_1 |\dot{M}|^{3/4} \, , \tag{11}$$

so that transforming $n$ into $N$ simplifies to a multiplication with a selection factor $V$ which depends only on $\mathbf{A}$. Eqs. (9) and (11) do not of course apply to magnetic CVs where the accretion disk is either truncated at its inner rim or altogether absent.

As Fig. 8 indicates, we still expect most PopII CVs to be found below the period gap, whereas for Pop I CVs the period distribution is more balanced, consistent with observations (e.g. Ritter & Kolb 1995). We find that 33% and 5% of the CVs in the observable PopI and PopII sample are above the gap, respectively. The spike at the short–period cut–off (close to $\log P/h \sim 0.0$) in both distributions is caused by systems at the period minimum where $\dot{P}$ = 0 and therefore formally $n(P) \to \infty$. The fact that a similar feature is not present in the observed PopI CV period distribution (cf. Kolb 1996) could be due to a very long outburst recurrence time $t_\mathrm{rec}$, a selection

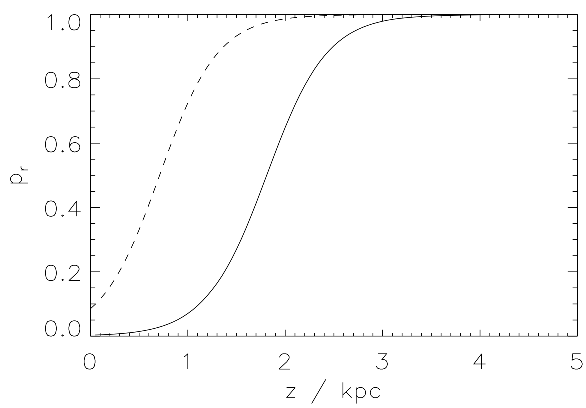


**Fig. 9.** Probability $p_{\mathbf{r}}$ that a CV at distance $z$ to the Galactic mid–plane belongs to the population II (full line). At $\sim$ 1.8kpc $p_{\mathbf{r}} \simeq 50\%$, essentially independent of the actual distance $r$ to the CV measured in the Galactic plane. Also shown (dashed line) is the corresponding intrinsic probability $p_{\rm int}^{\rm II}$, cf Eq. 1, that a given CV is PopII. Very close to the Galactic plane ($z < 50$ pc) $p_{\mathbf{r}} \to p_{\rm int}^{\rm II}$, i.e. the full line bends upwards and meets the dashed curve (this feature is not resolved in the figure).

effect not accounted for in the models and operating *against* the detection of these systems. As an example, $t_{\rm rec} \simeq 30$ yr for WZ Sge (e.g. Smak 1993), comparable to the recent time period over which CVs have been studied in detail. Similar systems or systems with even longer recurrence times could easily be missed. On the other hand, the larger discovery probability of dwarf novae with more frequent outbursts, arising purely from the fact that their brightness is *variable*, is also difficult to quantify and is not taken into account. A further reason for the apparent mismatch between model and observation at the minimum period might be that the simple accretion disk model used to compute the bolometric corrections fails for those low mass transfer rate systems ($|\dot{M}| < 10^{-11} M_\odot$/yr). Such disks are, at least in quiescence, likely to be optically thin. Due to the lack of detailed low $\dot{M}$ disk models we have no choice other than simply extrapolating (11) downwards to very small transfer rates.

These uncertainties and the fact that our current models seem to overestimate the number of observable low $\dot{M}$ CVs clearly call for more theoretical work to understand accretion disks and degenerate secondaries in CVs which have evolved past the minimum period, as well as for more observational effort to find intrinsically faint CVs. Some authors have suggested that TOADs represent good candidates for such post minimum period systems (e.g. Howell, Rappaport & Politano 1996).

We do not attempt to predict the total number of observable CVs, a number which depends strongly on the assumed selection function. Rather, we rely on a differential comparison of PopI and PopII CV distributions which is unaffected by this uncertainty in the absolute calibration. From such a comparison we predict that the detection probability of PopII CVs in a cylindrical box with 10kpc edge size and centered on the Sun is by a factor $\sim$ 230 smaller than for PopI CVs. We note that this value of course does not take into account the problem that we cannot distinguish the population class of a CV by its intrinsic parameters (see discussion).

It is commonly thought that the affiliation of a CV to a population class can be determined by its Galactic position $\mathbf{r}$. Let $\mathcal{N}_{\mathbf{r}}$ denote the relative observability of a CV subsample located at $\mathbf{r}$, i.e.

$$\mathcal{N}_{\mathbf{r}} = n_0(\mathbf{r}) \int\int\int\int n(\mathbf{A})\, p_{\rm s}(m_{\rm v})\, {\rm dM_1\, dM_2\, d|\dot{M}|dP}, \qquad (12)$$

where $n_0(\mathbf{r})$ is the total number of ZAMS binaries of the corresponding population formed at location $\mathbf{r}$, $n(\mathbf{A})$ is the intrinsic distribution per unit number of ZAMS binaries, and $p_{\rm s}(m_{\rm v})$ is the detection probability defined in Eq. (9). Note that here $p_{\rm s}(m_{\rm v})$ is the same for all systems with the same absolute visual magnitude, $M_{\rm v}$, since $d$ is the same for all systems. Then the probability $p_{\mathbf{r}}$ that a CV at location $\mathbf{r}$ is member of the population II is

$$p_{\mathbf{r}} = \frac{\mathcal{N}_{\mathbf{r}}^{\rm II}}{\mathcal{N}_{\mathbf{r}}^{\rm I} + \mathcal{N}_{\mathbf{r}}^{\rm II}}. \qquad (13)$$

$p_{\mathbf{r}}$ is a function of both $z$ and $r$, but changes by less than 6% when $z \gtrsim 50$ pc and $r$ adopts any value within the Galactic disk. Hence, we show in Fig. 9 $p_{\mathbf{r}}$ as a function of $z$ only. Recall at this point our assumption from Sec. 2 that the intrinsic CV population is distributed in space similar to single star M–dwarfs. The dashed curve in Fig. 9 represents the corresponding intrinsic probability $p_{\rm int}^{\rm II}$ — as defined in Eq. (1) — that a CV at $\mathbf{r}$ is member of the population II. We determine $z_{0.5}$ for which $p_{\rm r}(z_{0.5}) = 50\%$ as $z_{0.5} = 1.8$kpc. This is much larger than the intrinsic value of $\sim$ 700 pc (see dashed curve in Fig. 9) since PopII CVs are on average much fainter. Hence our investigation suggests that a reliable PopII CV sample is only found at very high galactic latitudes.

Nearer to the Galactic midplane $p_{\mathbf{r}}$ is indeed a function of both $z$ and $r$, and for $d = \sqrt{r^2 + z^2} \to 0$, $p_{\mathbf{r}}$ reproduces the intrinsic distribution $p_{\rm int}^{\rm II}$, i.e. the solid curve eventually bends upwards and meets the dashed curve (not resolved in Fig. 9). For very small $z$ the precise shape of $p_{\mathbf{r}}$ strongly depends on the assumed detection probability and is therefore less well described by our model.

## 5. Discussion

The effects of the secondary star's metallicity on the long–term CV evolution have been examined in the context of the disrupted magnetic braking model. We have shown that CVs with a low metallicity secondary are characterized by a smaller period width of the detached phase, a somewhat shorter minimum period and a slightly higher mass transfer rate, causing a shorter CV evolution timescale.

By integrating over a grid of CV evolutionary sequences with different initial WD and secondary mass to a CV population model we predict a PopII CV period distribution where the "period gap", the well–known feature of the PopI CV period distribution, is tiny, essentially absent. The most significant differences of the PopII and PopI CV intrinsic distribution are caused by the restriction of the star formation period to the first $10^9$yr after the onset of Galaxy formation, i.e. by the fact that our model PopII CVs represent an old population. Accordingly, most CVs have evolved to short orbital period and only a marginal fraction still exists above the period gap. The average mass transfer rate of PopII CVs thus is much smaller and therefore they are intrinsically much fainter than PopI CVs.

Extracting a visual magnitude limited sample from the computed intrinsic distribution and taking into account incompleteness towards fainter magnitudes (as expressed by the detection probability Eq. (9)) demonstrates the difficulties in detecting PopII CVs. In such a computed sample only 1 out of 230 CVs belongs to population II. Since the actually observed sample represents a rather inhomogeneous ensemble of systems detected with various observational techniques or by chance, our computed incomplete magnitude limited sample can at best be a first approximation. When future observations with increased sensitivity lead to a more complete CV sample at fainter magnitudes we expect that the fraction of PopII CVs in this sample will increase.

From our discussion above it is clear that a direct, quantitative comparison of model populations like those derived in this paper with an actually observed sample of CVs is not very meaningful before observational surveys with well–defined selection criteria are performed.

This leads finally to the question whether our theoretical investigation can provide us with a strategy to identify PopII CVs positively, a problem even more severe than the difficulty of detecting them at all because of their intrinsic faintness. One might think of the following identifying criteria:

(i) metallicity of the secondary:

Stehle & Ritter (1997) showed that the accretion of metal–rich material from an expanding nova envelope onto and subsequent mixture into the outer convective layers of the secondary leads to a rapid increase in metallicity from initially $10^{-4}$ to $\sim 0.005$. Therefore PopII CVs will not manifest themselves as systems with a low metallicity secondary, but possibly as systems with an unusual metal composition more appropriate for nova–shell material than for solar–type stars. Depending on the type of nova (see Livio & Truran 1994) we expect neon respectively nitrogen to be overabundant. The latter should be prominent in large ratios $N/C$ and $N/O$ of up to 5...50 times the solar value.

(ii) high $\gamma$–velocities:

As we expect the space velocities of CVs to be similar to those of single stars and independent of the intrinsic parameters, a sample of high–$\gamma$ velocity CVs in the tail of the $\gamma$–velocity distribution might represent an old population. In the compilation of published $\gamma$–velocities done by van Paradijs, Augusteijn & Stehle (1996) some candidates are found. Most of them are magnetic (AM Her, DQ Her) systems, however, where the Doppler–shifts in the spectral lines originate mainly from the accretion stream. This is consistent with what we found in our study, namely that only a small fraction of the whole observable CV sample belongs to population II (see Sec. 4.4).

(iii) CVs with orbital periods 70min $\lesssim P \lesssim$ 80min:

The minimum period of PopII CVs turns out to be $\simeq$ 10min shorter than the PopI CV minimum period, hence the interval 70min $\lesssim P \lesssim$ 80min, likely to be inaccessible for PopI CVs with initially unevolved secondaries, may appear as a prime period range to look for PopII CVs. However, at the time PopII CVs reach the minimum period, the secondaries are likely to have accreted a non–negligible amount of metals from nova ejecta, see (i) above. This would raise the value of the minimum period for the corresponding system significantly. Hence we expect that only PopII CVs which form with a very small secondary mass will evolve to a minimum period which is significantly shorter than 80min. This is also true for PopII CVs with a He–WD primary as in these cases the metallicity of the nova ejecta is small (see Livio & Truran 1994) and as pollution of the secondary with He has no effect on the value of $P_{\rm min}$ (see Table 2).

(iv) high galactic latitude CVs:

Our investigation of the observable CV distribution shows that, although it is possible to obtain a sample of PopII CVs just by selecting CVs at sufficiently large distances $z$ from the Galactic mid–plane (as suggested by Howell & Szkody 1990), the lower limit $z_{\rm limit}$ for $z$ in order to achieve this must be much larger than the value of 350 pc adopted in previous work, typically $z_{\rm limit} \gtrsim$ 2000 pc. This is even true if any selection effects are neglected altogether and only the intrinsic distribution, deduced from recent Galactic models, is considered.

We finally conclude that PopII CVs exist, but they are difficult to detect and it is even more difficult to identify them positively as population II objects.

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