Boundaries of the Firm:

A Theory of Informational Uncertainty and Learning

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Abstract

This paper examines the determinants of the boundaries of a firm. In contrast to much of the existing literature, we shy away from hold-up problems and instead focus on the firm's decision to redraw its boundaries in light of informational uncertainty. The firms faces uncertainty surrounding four key elements: (i) the profitability of a new market, (ii) whether the firm has the skills necessary for this new market, (iii) the compatibility of the new business with the firm's existing portfolio, and finally (iv) the competitive environment. We find that a firm's boundaries are determined dynamically as it tries to learn about these factors. The firm's optimal learning strategy will either be to redraw its boundaries right away, or to invest a smaller amount to learn and put itself into a position to redraw its boundaries later, or to postpone any investment until uncertainty is resolved. We also apply our model to business portfolio decisions and the incentives for firms with varying market power to pursue technological innovations. The interactions between the elements of uncertainty generate a number of empirically testable predictions, including a positive relationship between the level of competition and both the frequency of firm restructuring activity and the incentives to develop technological innovations.

1 Introduction

What determines the optimal boundaries of a firm? This question, which motivates this paper, has been one of central importance to the theory of the firm, as well as in finance and corporate strategy. It is also a question that has preoccupied corporate executives, as evidenced by waves of spectacular mergers and acquisitions deals that totalled over $1.6 trillion in 1997 alone, as well as divestitures worldwide. We have recently witnessed megamergers such as Aol.com and Time-Warner, Exxon and Mobil, and Daimler-Benz and Chrysler, as well as divestitures of various business units by companies like Kodak, Sara Lee, and Anheuser Busch.

The boundaries of firms are being constantly reconfigured through activities like acquisitions and divestitures. But our understanding of how these boundaries should be drawn remains incomplete. In the theory of the firm, these questions were first studied by Coase (1937). His insight was that the boundaries of firms are determined by the transaction costs of coordinating production under imperfect information; these costs may mean that it is less costly to include certain activities within the firm than to subject them to market exchange. This insight has been subsequently fleshed out and refined by Williamson (1975, 1985), Grossman and Hart (1986), and Hart and Moore (1990). What has emerged is an improved understanding of the role of firm boundaries in providing incentives. Much of this understanding has come from an examination of the "hold-up" problem (e.g., Klein, Crawford and Alchian (1978) and Grout (1984)). This analysis has shown that when transacting parties must make relationship-specific investments in an environment of incomplete contracting, it is sometimes better to integrate the transacting parties into a single firm.[1] The reason is that, as independent contractors, one of the parties may find itself being "held-up" by the other, thereby unable to get an adequate return on its relationship-specific investment after the investment is made. The resulting dilution of investment incentives may make market-mediated transactions prohibitively expensive.[2]

While these contributions have significantly enhanced our understanding about why firms exist and the benefits they offer relative to market-mediated transactions, they leave unattended some

[1] More recently, Rajan and Zingales (1998) provide a novel interpretation of why transactions take place within a firm, as opposed to the marketplace. They argue that by bringing these transactions within the firm, the firm has a greater ability to restrict employee access to key firm resources. The firm thereby empowers (i.e., provides access) only to those employees who make firm-specific investements.

[2] Countervailing forces are suggested in the important work of Berle and Means (1932). They focus on the agency problems associated with the separation of ownership and control, which are particularly common to large organizations. This literature has led to insightful work on security design (see Aghion and Bolton (1992)), as well as on internal organizational issues, such as internal capital markets. See Gertner, Scharfstein, and Stein (1994) for work on this issue, and Bolton and Scharfstein (1998) for an overview of these and other theory of the firm issues.

interesting features of firms. As Holmstrom and Roberts (1998) point out:

> "It seems to us that the theory of the firm, and especially work on what determines the boundaries of the firm, has become too narrowly focused on the hold-up problem and the role of asset-specificity...
>
> Information and knowledge are at the heart of organizational design... In light of this, it is surprising that the leading economic theories of firm boundaries have paid almost no attention to the role of organizational knowledge. The subject certainly deserves more scrutiny.

The challenge then is to begin to develop a theory of the firm based on information uncertainty and learning that can explain firm boundary choices in settings in which hold-up problems are small and relationship-specific investments may be high.[3] We develop such a theory in this paper. The theory helps us to answer the following questions:

- When is it optimal for a firm to expand scope?
- When should a firm incorporate new assets and replace old ones?
- What are the incentives of existing players in an industry to invest in innovations?
- How does the competitive environment affect the firm's boundary choice decision?

To address these questions in an environment of information uncertainty and learning, consider a firm that has an existing portfolio of assets. In redrawing its boundaries, the firm must decide whether to add a new asset to its portfolio and/or divest an existing asset. This decision must be made in light of three key information uncertainties: (i) will the new asset be profitable in a market demand sense?; (ii) will the firm have the necessary skill to manage the new asset?; and (iii) even if the asset is profitable and the firm possesses the skill to manage it, will this asset be compatible with the firm's existing portfolio? If these uncertainties are large enough, the firm may decide not to acquire the new asset. But then it loses the potential benefit of a profitable new asset. However, if it makes an irrevocable investment in the new asset, it may end up with an unprofitable investment because there is no market demand, or the firm discovers either that it lacks the skill to manage the asset, or that the new asset is incompatible with its existing portfolio.

The key for the firm in redrawing its boundaries is to figure out its optimal *learning strategy* that helps resolve these uncertainties. The easiest way to learn would simply be to wait until the

[3] We also abstain from the agency problems presented by Berle and Means (1932).

uncertainties are sufficiently resolved. But by then it may be too late because a competitor may have moved in and captured the necessary first-mover advantages.

This means that there are two ways in which a firm can learn about its skill in managing the new asset as well as the compatibility of this asset with its current portfolio. One is to make a small investment, establish a "toe-hold" and learn, after which it can decide whether to redraw its boundaries to fully include this new asset in its portfolio. In this case, the large investment would not be made until after demand uncertainty is resolved and the firm learns about skill and compatibility. The other is to go in with a large investment in the new asset right away – before any of the three uncertainties are resolved – and immediately redraw its boundaries.

Whenever a small investment is sufficient to learn about skills and compatibility, it is likely to be preferable to making a large investment right away. But there are instances in which the small investment produces insufficient information about skill and compatibility. In addition, it increases the likelihood of a competitor jumping in with a large investment and stealing the firm's first-mover advantage.

The pros and cons of the two learning strategies are now evident. The advantage of a small toe-hold investment to learn relative to plunging with a large investment right away is that it can resolve the necessary information uncertainty without risking a larger investment that may end up being wasted. The disadvantage is twofold: (i) the small investment may fail to produce the desired learning and, (ii) it also increases the odds of the firm losing its first-mover advantage to a competitor. Boundaries are thus determined by firms' attempts to *learn* about both the skills needed to operate effectively in a profitable new area and portfolio compatibility, as well as by the desire to exploit *first-mover advantages* in the new area.

While the general model considers *new* investment opportunities, we also apply it to business portfolio decisions and investments in new technologies. Focussing first on portfolio decisions, we show that as competition rises, it becomes more attractive for firms to enter new markets early and with large investments. Consequently, the frequency of mergers and acquisitions goes up with competition. However, these early-entry decisions represent full-scale investments made prior to learning, and are thus more error-prone. The prediction then is that divestitures – intended to correct previous asset-acquisition errors – also become more frequent in more competitive industries. The model thus highlights the importance of competition as a determinant of firm boundaries. We also show that the likelihood that a firm restructures early is increasing in the liquidation value of the firm's assets. The greater the value the firm receives if it chooses to divest assets, the more

likely it will be to make risky portfolio decisions.

Investments in new technologies also help to redraw boundaries. We find that if a technological innovation is unlikely to be introduced by other firms, then an established, larger firm has a weaker incentive to make these investments than newer, smaller firms. On the other hand, if the innovation is very likely to be introduced by others, the larger firm has a greater incentive to pursue it. In other words, how a firm redraws its boundaries depends both on its existing asset portfolio and its size, as well as the likelihood that an innovation is to come about. The intuition is that a large existing player already reaps most of the benefits of being a market leader, and therefore gains less from innovating. This is analogous to a firm's reluctance to introduce new products that simply cannibalize its existing ones. However, if the innovation represents a simple refinement to the existing technology, it will choose to pursue to prohibit the smaller firm from picking up market share. But if the potential innovation is unlikely to be available to others, the larger firm is more likely to forego the innovation.

This way of looking at firm boundaries allows us to develop a theory in which informational uncertainty and learning are at the heart. Moreover, this theory produces numerous testable empirical predictions that we discuss later in the paper.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 contains an equilibrium analysis of strategic investments in scope expansion. Sections 4 and 5 present applications of our model to business portfolio decisions and innovation incentives, respectively. Section 6 discusses the applicability of the model to various industries and formulates the empirically testable predictions of the model. Section 7 concludes. Much of the algebraic detail is relegated to the Appendix.

2 Model Setup

We develop a general model that is concerned with how a firm optimally chooses to redraw its boundaries with respect to new investment opportunities. There are three dates, $t \in \{0, 1, 2\}$, covering two periods. There is universal risk neutrality and the riskless rate is normalized to zero.

2.1 Players and Investment Opportunities

There are two firms at $t = 0$, denoted A and B. Firm A is the representative firm for which we characterize its optimal behavior, while firm B serves to provide competition. We take the

production decisions of firm B to be endogenous to the model, but we let its market *entry* decision be exogenous. These firms face a new investment opportunity that can be added to their existing business. The precise nature of these business activities is not important for the analysis. What is important is that the existing business is an activity quite familiar to the firms and thus there is no uncertainty about the skill needed to operate effectively. The same cannot be said for the new activity. It represents an activity that the firm has not participated in so far, and thus, there is uncertainty about the skill necessary for effective operation.

The question is when to invest in this new activity, if at all. We interpret an investment in the new activity as the firm redrawing its boundaries. Since the market for the new activity opens at $t = 1$, demand is realized at $t = 1$ and this is observed by all. The aggregate demand for the new activity at $t = 1$ is denoted by $\widetilde{\Omega}$, and it is random when viewed at $t = 0$. That is, $\widetilde{\Omega}$ takes a value $\Omega > 0$ with probability (w.p.) $\eta \in (0, 1)$, and a value 0 w.p. $1 - \eta$. Uncertainty about $\widetilde{\Omega}$ is completely resolved at $t = 1$ before firms commit to their production choices, but not necessarily before the firms commit to their investment choices. This implies that the investment I could be wasted if the investment is made before demand is realized and it is ultimately discovered at $t = 1$ that $\Omega = 0$. For simplicity, we assume that the salvage values of these assets are zero. We relax this assumption in Section 4.

2.2 Skills Uncertainty and Production Costs

Producing in the new activity requires specific skills, and higher skills translate into lower production costs. We assume that the cost of producing in the firm's existing activity is deterministically known at $t = 0$, and thus the decisionmaking about this activity is uninteresting for our purposes. By contrast, the per-unit production cost in the new activity is stochastic at $t = 0$. The idea is that the firm knows for sure the production cost in the activity it is familiar with, but is uncertain about the cost of participating in the new activity. We assume that the two firms have identical, yet uncorrelated skills. High skills mean a low per-unit product cost $c = \underline{c}$, and low skills mean high per-unit production cost $c = \overline{c}$, where $0 < \underline{c} < \overline{c} < \infty$. A firm has high skills w.p. δ and low skills w.p. $1 - \delta$. Hence, both firms' expected per-unit production costs are given by

$$E(c) = \delta\underline{c} + [1 - \delta]\overline{c}. \tag{1}$$

2.3 Investment Timing and Learning

At $t = 0$, the representative firm A faces two uncertainties regarding the new activity: skill and demand. For now, we suppress the possible uncertainty about whether the new asset is compatible with the firm's existing portfolio. This complication will be introduced in Section 4. The skill of firm B is not uncertain. As mentioned above, for either firm to participate in this market, it must make an investment. This investment is I. This irreversible investment could represent the costs of assembling human resources and/or tangible assets necessary to compete in the new activity. Whichever firm makes the investment I *first* gets to produce as the Stackleberg leader, as long as it invests just *before* market demand is realized. This first-mover benefit must be weighed against the risk that the investment is lost if demand fails to materialize. If both firms wait until demand is realized, neither firm gains a Stackleberg advantage and they compete as Cournot duopolists.

Now, although full production in the new market doesn't commence until the second period (beginning at $t = 1$), we characterize how the representative firm A prepares for this production period. While the firm can choose to wait until $t = 1$ before making any decision, the firm can prepare for this market early (at $t = 0$) in one of two different ways. The first early-entry strategy entails making a small investment $k > 0$ at $t = 0$. Investing $k > 0$ at $t = 0$ represents a small toe-hold investment in the new activity in the *first* period.[4] The second way to enter early is to fully commit to the new activity by investing I at $t = 0$. The first strategy can be thought of as the firm putting itself in the *position* to redraw its boundaries in the future. That is, it acquires the option to play in this new market. The second strategy represents the firm redrawing its boundaries immediately by fully committing I at $t = 0$. We assume that $k < I$.

One could interpret the small investment k as forming an alliance or joint venture with another company, or as buying a small company that already engages in the new activity. The idea is that upon investing k at $t = 0$ and trying the business on a smaller scale in the first period, the firm *may* learn its skill and hence c in the new activity before making the larger investment I. This learning will occur w.p. $\lambda \in (0, 1)$. Thus, at the end of the first (trial) period, w.p. λ the firm will know exactly whether $c = \underline{c}$ or $c = \overline{c}$, and w.p. $1 - \lambda$ it will learn nothing about c. Alternatively, the firm can invest the full amount I at $t = 0$. If the full investment is made at $t = 0$, the firm learns c *for sure* by the end of the first period.

Besides the possibility that the firm won't learn its skill, the other disadvantage of investing

[4] We do not allow for the investment k to be made at $t = 1$. This is a harmless assumption since the firm needs to invest the full amount I before it can begin production anyway. Thus, the investment in k would then be redundant.

only k at $t = 0$ is that the other firm may beat it to market by investing I before market demand is realized at $t = 1$. Recall that we assume that whichever firm first invests I becomes the Stackleberg leader. If firm B invests I at $t = 1$ before firm A and before demand is realized, it gains the Stackleberg leader position and firm A is forced to follow if it wants to participate. Firm B's early entry strategy is taken to be exogenous and we assume that it enters early w.p. $\gamma \in (0, 1)$. The probability γ could be viewed as a measure of the potential competition in this new activity. A high value of γ implies that many firms have identified this new market, whereas a low value implies that few have.

If firm A invests only k and firm B has not committed I already (occurring w.p. $1 - \gamma$), firm A can choose whether to invest I before or after demand is realized at $t = 1$, if at all. The value of being first to market will be determined by what firm A learns in the first period, if anything. By contrast, if firm A invests I early at $t = 0$, it *guarantees* itself the Stackleberg leader position. The disadvantage of entering early with the full investment I is that it is sunk prior to knowing whether there will be any demand. With probability $1 - \eta$, demand in the second period does not materialize and I is lost completely.

Instead of investing early in any way, the firm can also wait until $t = 1$ before making *any* investment at all. The benefit of such late entry is that demand is realized at $t = 1$ *before* the firm redraws its boundaries and has to commit I. The firm then abstains from investing I when demand turns out to be zero. However, the disadvantages of entering late are that the firm will not know its *actual* skill (either $\underline{c}$ or $\overline{c}$) and it cannot be the Stackleberg leader since the toe-hold investment of k is the minimum investment necessary to play as the leader.

2.4 Product Market Structure

We assume symmetric information throughout the paper. Conditional on a positive demand for the new activity at $t = 1$, each firm competes by choosing its production quantity, given by q_i, where $i \in \{A, B\}$. The per-unit price of the new activity, given by P, is determined by the inverse demand function

$$P(\Omega, Q) = \Omega - Q, \tag{2}$$

where $Q = q_A + q_B$ is the total quantity produced by both firms A and B. Production then starts, actual costs are realized, and revenues (or losses) are collected at $t = 2$ when the game ends.

Demand for the new activity is such that there is room for both firms to compete. That is, the demand structure allows for positive profits to both competitors, except when the firm has low

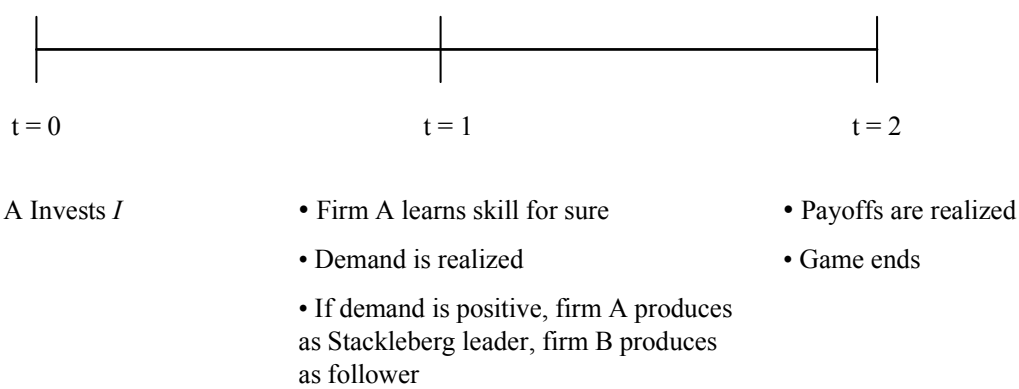
skill (i.e., $c = \overline{c}$). For simplicity, we specify demand such that the firms are indifferent between producing and not producing when $c = \overline{c}$, i.e.,

$$\Omega = \overline{c}. \tag{3}$$

Under this specification, Ω is large enough to make it strictly profitable for a firm with $\underline{c}$ or $E(c)$ to produce profitably, while insuring that at cost $\overline{c}$, the firm's profit is zero.[5] This low-skill node yields an indifference point at which we assume the firm would choose to exit the market. Thus, if the firm invested k at $t = 0$ and learned that $c = \overline{c}$, it would not invest I. For a firm that had invested I already at $t = 0$, a discovery that $c = \overline{c}$ also means market exit, in this case treating I as a sunk cost.

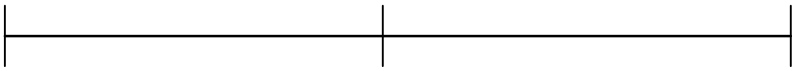
Figures 1, *2*, and *3* summarize the sequence of events in each of firm A's three possible investment strategies.

Figure 1: Firm A Invests I Early



[5] These profits are calculated conditional on the firm having already invested I. Fixing demand this way is completely inconsequential to the analysis.

Figure 2: Firm A Invests k Early



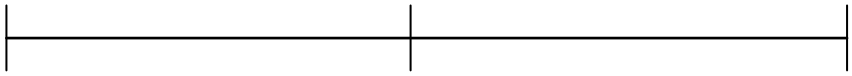
t = 0 t = 1 t = 2

- Firm A Invests *k*

- Firm A learns skill w.p. λ
- Firm B enters before demand is realized w.p. γ, producing as Stackleberg leader.
  - Demand is realized
  - If demand is positive, firm A produces as follower (either low or average cost depending on whether skill is learned)
- Or firm B doesn't enter early (w.p. 1- γ)
  - Firm A can enter before demand and produce as the Stackleberg leader. Demand is realized.
  - Or firm A can wait until demand is realized and produce as a Cournot duopolist.

- Payoffs are realized
- Game ends

Figure 3: Firm A Waits to Invest



t = 0 t = 1 t = 2

- Firm A does not invest

- Firm A does not learn skill
- Firm B enters before demand is realized w.p. γ, producing as Stackleberg leader.
  - Demand is realized
  - If demand is positive, firm A produces as follower (average cost)
- Or firm B doesn't enter early (w.p. 1- γ), demand is realized, and if positive, firms A and B compete as Cournot duopolists with average costs.

- Payoffs are realized
- Game ends

3 Equilibrium Analysis

We begin by first characterizing the second-period output decisions of the two firms. Next, we consider firm A's decision at $t = 0$, i.e., whether it should enter the new market early or late, and with what level of capital commitment if it enters early.

3.1 Second-Period Analysis

Production decisions are made at $t = 1$ once uncertainty about $\widetilde{\Omega} \in \{0, \Omega\}$ has been resolved. Hence, we focus on the case where $\Omega > 0$ is realized. If $\Omega = 0$, there is no demand and the game ends, with the investment made earlier being lost. The different cases corresponding to the firms' production costs and competitive positions are listed below and derived in the Appendix.

- Both firms wait to observe $\Omega > 0$ at $t = 1$, then they will both invest I and compete as Cournot duopolists facing per-unit production costs of $E(c)$.
- Firm A invests I at $t = 0$. It then learns its skill at $t = 1$ w.p. 1. If $\Omega > 0$ at $t = 1$, firm A produces if and only if it has $c = \underline{c}$. Investing I early also guarantees that the firm gains the Stackleberg leader advantage for the second period. If $\Omega = 0$, it exits the market without recovering I.
- Firm A invests $k < I$ at $t = 0$. It then learns its skill at $t = 1$ w.p. λ, and doesn't learn its skill w.p. $1 - \lambda$. If $\Omega = 0$, it does not invest any further, regardless of what it learns about its skill. If the firm learns it skill, then it produces if and only if $c = \underline{c}$, choosing to exit the market if$c = \overline{c}$. If the firm doesn't learn its skill, it produces at $c = E(c)$. Recall that investing k at $t = 0$ early allows for the possibility that the *competing* firm B will invest I *first*, thereby gaining the Stackleberg advantage. This occurs w.p. γ.
  - If firm B does not enter early (occurring w.p. $1-\gamma$), firm A can choose whether to invest I before or after demand is realized, if at all.
  - If firm B does invest I before demand is realized, firm A will wait until demand is realized before committing I. Given the uncertainty surrounding firm B's entry behavior, firm A will find that investing k early could result in it acting as either a Stackleberg leader or follower, or as a Cournot duopolist if neither firm invests before demand is realized.

In the Appendix, we define both firm's production decisions and ultimate profits for each of the possible skills and competitive positions. These results (contained in *Table 1* of the Appendix) emphasize the two primary benefits to entering the new market early with the full investment I at $t = 0$. First, the firm can make better production decisions because of skill discovery w.p. 1 rather than w.p. λ as with investing only k early. This allows it to compete as the bigger player if $c = \underline{c}$ and not at all if $c = \overline{c}$. Second, early entry with I makes firm A the Stackleberg leader, whereas it achieves this position only w.p. $1 - \gamma$ if it enters early with k. These benefits are balanced against the disadvantage that the investment I will be wasted with early entry if no demand materializes.

3.2 First-Period Analysis

We can now back up to time $t = 0$ to determine the expected value of each of the three investment strategies for firm A. These are then used to characterize the conditions under which the representative firm (A) finds one investment strategy preferable to another.

Let Ψ represent firm A's expected profits at time 0, inclusive of all possible investments. The three cases we need to examine are (i) firm A waits until to $t = 1$ to invest, (ii) firm A invests I at $t = 0$, and (iii) firm A invests k at $t = 0$.

Firm A Waits Until *t=1*

In this case, firm A waits until $t = 1$ to observe whether $\Omega > 0$. If demand is positive, firm A will invest I and produce. If firm B waits as well (occurring w.p. $[1 - \gamma]$), then they each compete as Cournot duopolists with $c = E(c)$. However, if firm B jumps in early with the investment I (occurring w.p. γ), it will produce as the Stackleberg leader. Firm A follows, and both produce at $c = E(c)$. Firm A's expected profit from waiting to invest is:

$$\begin{aligned} \Psi_A(\text{Wait}) &= \Pr(\Omega > 0) \times \left[\begin{array}{c} \Pr(\text{B enters early}) \times \Pi(\text{Follower, Avg. Cost}) \\ + \Pr(\text{B enters late}) \times \Pi(\text{Cournot, Avg. Cost}) \end{array} \right] \\ &= \eta \left[\begin{array}{c} \gamma \left[\frac{1}{16} \delta^2 \left[\overline{c} - \underline{c} \right]^2 - I \right] \\ + [1 - \gamma] \left[\frac{1}{9} \delta^2 \left[\overline{c} - \underline{c} \right]^2 - I \right] \end{array} \right], \end{aligned} \tag{4}$$

where $\Pi()$ represents firm A's second-period profits. As can be seen above, firm A only invests if demand is positive. Therefore, the investment I is never lost, even if demand fails to materialize. However, the expected value of waiting to invest is decreasing in γ. That is, the more likely it is

that a competitor will beat firm A to market at $t = 1$, the less desirable it is to wait to invest. We summarize this in the next result.

Lemma 1

The expected value of waiting to invest is decreasing in the probability γ that a competitor will enter the market early. Moreover, the expected value of waiting to invest is bounded below by zero.

Firm A Invests *I* At *t=0*

If firm A invests I at $t = 0$, it learns its skill for sure before making its decision to produce or exit. Importantly, firm A *guarantees* itself the Stackleberg leader position. However, the investment I is put at risk in that the firm may have to exit the market without recovering any portion of I. The expected profit from investing I early is

$$\begin{aligned} \Psi_A(\text{Invest } I) &= [\Pr(\Omega > 0) \times \Pr(\text{High Skill}) \times \Pi(\text{Leader, Low Cost})] - I \\ &= \eta\delta\frac{1}{8}\left[2-\delta\right]^2\left[\overline{c}-\underline{c}\right]^2 - I. \end{aligned} \tag{5}$$

Since firm A locks in the leader position early, the expected profit is independent of γ, the probability of early competitive entry. It is important to note that (5) can be negative as $\eta \to 0$, whereas the expected value of waiting to invest is bounded from below by zero.

Lemma 2

The expected value of investing I at $t = 0$ is increasing in the probability η that demand materializes. Moreover, the expected value of investing I at $t = 0$ is negative for sufficiently small η.

The lemma is straightforward. If there is a sufficiently low probability of positive demand, the first-mover benefit of being the Stackleberg leader is exceeded by the expected loss of I. However, if a positive demand is sufficiently likely to materialize, the first-mover advantage of producing as a Stackleberg leader dominates.

Firm A Invests *k* at *t=0*

If firm A invests k at $t = 0$, w.p. λ it learns its skill and makes an efficient decision to produce ($c = \underline{c}$) or exit ($c = \overline{c}$). However, w.p. $1 - \lambda$, it will learn nothing about its skill by the end of the first period and will produce at cost $c = E(\dot{c})$. Moreover, investing k doesn't guarantee that firm A will be first to market. In the case the competitor invests I first (occurring w.p. γ), firm A must play the role of the follower whenever it chooses to produce.

If the competitor does not enter early (occurring w.p. $[1 - \gamma]$), firm A can choose to invest I just before demand is realized to become the Stackleberg leader, or wait until demand is realized and compete as a Cournot duopolist. Either strategy may be optimal, and would depend on market conditions (η) and what it learned in the first period. Thus, there are two relevant decision nodes for firm A when firm B does *not* enter the market early. These are when firm A learns its skill and when firm A learns nothing about its skill. For simplicity, we assume that market conditions are such that it is always optimal for firm A to invest I before demand is realized to gain the Stackleberg leader position whenever possible.[6]

Since firm B enters early w.p. γ, firm A's expected profit from waiting to invest is:

$$\Psi_A(\text{Invest } k) = \left[\begin{array}{l} \Pr(\text{B enters early}) \times \Pr(\Omega > 0) \times \left\{ \begin{array}{c} \left[\begin{array}{c} \Pr(\text{Learn}) \times \Pr(\text{High Skill}) \times \\ [\Pi(\text{Follower, Low Cost}) - I] \end{array}\right] \\ + \left[\begin{array}{c} \Pr(\text{Doesn't Learn}) \times \\ [\Pi(\text{Follower, Avg. Cost}) - I] \end{array}\right] \end{array} \right\} \\ + \Pr(\text{B enters late}) \times \left\{ \begin{array}{c} \left[\begin{array}{c} \Pr(\text{Learn}) \times \Pr(\text{High Skill}) \times \\ [\Pr(\Omega > 0) \times \Pi(\text{Leader, Low Cost})] - I \end{array}\right] \\ + \left[\begin{array}{c} \Pr(\text{Doesn't Learn}) \times \\ [\Pr(\Omega > 0) \times \Pi(\text{Leader, Avg. Cost})] - I \end{array}\right] \end{array} \right\} \\ -k \end{array}\right]$$

$$= \left[\begin{array}{c} \gamma\eta \left[\begin{array}{c} \lambda\delta \left[\frac{1}{16}[3 - 2\delta]^2 [\overline{c} - \underline{c}]^2 - I\right] \\ + [1 - \lambda] \left[\frac{1}{16}\delta^2 [\overline{c} - \underline{c}]^2 - I\right] \end{array}\right] \\ + [1 - \gamma] \left[\begin{array}{c} \lambda\delta \left[\eta\frac{1}{8}[2 - \delta]^2 [\overline{c} - \underline{c}]^2 - I\right] \\ + [1 - \lambda] \left[\eta\frac{1}{8}\delta^2 [\overline{c} - \underline{c}]^2 - I\right] \end{array}\right] \end{array}\right] - k. \tag{6}$$

[6] This could easily be relaxed and simply added as another payoff node to consider. All of the qualitative results would be unaffected. In the Appendix, we derive the parametric conditions under which this is true.

Similar to the expected value of investing I, the expected value of investing k is increasing in η. In fact, as $\eta \to 0$, the expected value of investing k is negative as well. Contrary to investing I, however, this expected value *does* depend on the potential threat of competitors, as well as on the likelihood that firm A learns its skill. This is presented as the following result.

Lemma 3

The expected value of investing k at $t = 0$ is strictly increasing in the probability λ that firm learns its skill and the probability η of a positive market demand, and it is strictly decreasing in the probability γ of early competitive entry. For sufficiently high uncertainty about market demand (low η), the expected value of investing k is negative.

As the probability that the firm learns its skill by making only the limited investment increases (high λ), the greater is the likelihood that the firm will be in a position at $t = 1$ to make an efficient production decision. This increases the desirability of investing k. However, if the probability that a competitor enters to take the Stackleberg leader position increases, the expected value of the first-mover advantage diminishes. That is, the firm is forced to become a follower, which is less profitable than being a leader.

3.3 Optimal Early Investment Strategies

We now turn our attention to the comparison of each of these strategies. The question is: when a firm is considering redrawing its boundaries (i.e., investing I), should it wait until demand uncertainty is resolved, or make a small initial investment k to "try" out the activity, or jump in with the large investment I right away? This decision will depend on demand uncertainty η, the probability λ that the firm can learn its skill by trying the business in only a limited way in the first period, the probability γ that a competitor will beat the firm to market, as well as the potential profitability of the market.

To answer the question, we need to compare $\Psi_A(\text{Wait})$, $\Psi_A(\text{Invest } I)$ and $\Psi_A(\text{Invest } k)$ from (4), (5) and (6). As noted in Lemmas 1, 2 and 3, $\Psi_A(\text{Wait}) \geq 0$, but both $\Psi_A(\text{Invest } I) \lesseqgtr 0$ and $\Psi_A(\text{Invest } k) \lesseqgtr 0$. Furthermore, for $\eta = 0$, we see that

$$\Psi_A(\text{Invest } I) < \Psi_A(\text{Invest } k) < \Psi_A(\text{Wait}) = 0.$$

For $\eta = 1$, we see that

$$\Psi_A(\text{Invest } I) > \Psi_A(\text{Invest } k) > \Psi_A(\text{Wait}) > 0.$$

This leads to the following result.

Theorem 1

There exits a critical probability of positive demand η, denoted $\widehat{\eta}_k$, such that the firm prefers to invest k early instead of waiting to invest for all $\eta > \widehat{\eta}_k$. This threshold is decreasing in the probability λ that firm learns its skill. There also exists a critical η, denoted $\widehat{\eta}_I$, such that the firm prefers to invest I early instead of waiting to invest for all $\eta > \widehat{\eta}_I$. This threshold is decreasing in the probability γ that a competitor enters the market early.

The theorem is intuitive. What it says is that when there is high demand uncertainty (low η), waiting to invest dominates both early investment strategies. However, as λ increases, it becomes more likely that the small investment k will enable the firm to learn its skill. Hence, investing k instead of waiting becomes optimal for relatively small values of η. When comparing investing I early to waiting to invest, the firm will prefer to invest I early for smaller values of η as the probability that a competitor beats it to market (γ) increases. However, even when early investment is optimal, the question remains whether this early investment should be I or k. We consider this next.

We see that firm's A early investment will be to make the full investment I rather than the toe-hold investment k if and only if

$$\eta\delta\frac{1}{8}\left[2-\delta\right]^2\left[\overline{c}-\underline{c}\right]^2 - I > \left[\begin{array}{c} \gamma\eta\left[\begin{array}{c}\lambda\delta\left[\frac{1}{16}\left[3-2\delta\right]^2\left[\overline{c}-\underline{c}\right]^2 - I\right] \\ +\left[1-\lambda\right]\left[\frac{1}{16}\delta^2\left[\overline{c}-\underline{c}\right]^2 - I\right]\end{array}\right] \\ +\left[1-\gamma\right]\left[\begin{array}{c}\lambda\delta\left[\eta\frac{1}{8}\left[2-\delta\right]^2\left[\overline{c}-\underline{c}\right]^2 - I\right] \\ +\left[1-\lambda\right]\left[\eta\frac{1}{8}\delta^2\left[\overline{c}-\underline{c}\right]^2 - I\right]\end{array}\right]\end{array}\right] - k. \tag{7}$$

We define $\widehat{\eta}_{I-k}$ as the η that sets (7) to equality. This leads to our next result.

Theorem 2

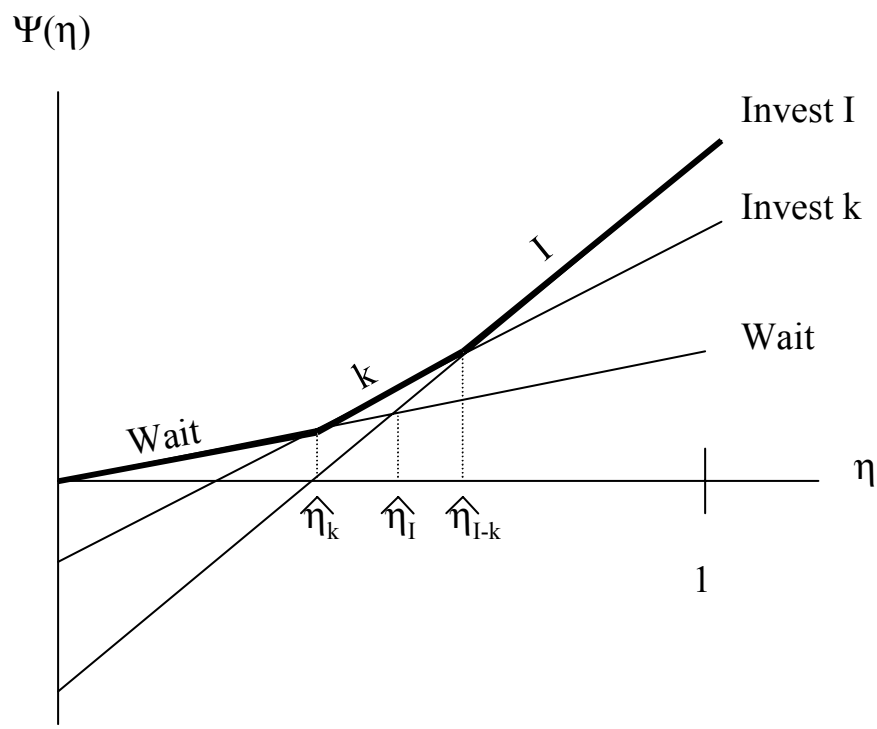
There exits a critical probability (η) of positive demand, denoted $\widehat{\eta}_{I-k}$, such that the firm prefers to invest I early instead of investing k early for all $\eta > \widehat{\eta}_{I-k}$. This threshold is increasing

in the probability (λ) that firm learns it skill upon making the small investment and decreasing in the probability (γ) that a competitor enters early.

Theorem 2 clarifies the relationship between investing I and k. If market demand is very likely to be positive, investing I has three advantages. First, it allows the firm to learn its skill for sure, rather than w.p. $\lambda < 1$. Second, the firm is never preempted by a competitor. Lastly, it can avoid making a redundant investment k. However, investing k allows the firm to possibly learn about its skill (w.p. λ) at a relatively low cost and can help avoid the inefficiency of investing I in states of the world in which its skill is inadequate (which happens w.p. $1-\delta$). This means that we need to compare k with the expected cost saving $\lambda\,[1-\delta]\,I$. The nature of competition plays a role in this early entry decision as well. As the probability that a competitor enters early (γ) increases, it becomes more likely that firm A will have to sacrifice the first-mover advantage if it only invests k.

In *Figure 4* we have graphically summarized the firm's optimal strategies. When η is very low ($\eta < \widehat{\eta}_k$), demand uncertainty is very high, the firm avoids investing *anything* (k or I) at $t = 0$ and prefers to invest at I at $t = 1$ because the *expected* cost of wasting the (early) investment more than offsets both the learning and the first-mover advantages. For slightly higher values of η but below a threshold ($\eta \in (\widehat{\eta}_k, \widehat{\eta}_{I-k})$), corresponding to moderately high demand uncertainty, it will still not be optimal to invest I at $t = 0$, but it will pay to invest the smaller amount k at $t = 0$ as long as skill can thereby be learned with a sufficiently high probability (λ high). For lower values of λ and $\eta \in (\widehat{\eta}_k, \widehat{\eta}_{I-k})$, the firm will invest neither k nor I at $t = 0$. Finally, for the highest values of η ($\eta > \widehat{\eta}_{I-k}$), corresponding to the lowest values of demand uncertainty, the firm will invest I at $t = 0$ because the learning and first-mover advantages more than offset the expected cost of wasting the investment. The cutoff $\widehat{\eta}_{I-k}$ declines as the probability that a competing firm will be first-to-market (γ) increases.

Figure 4: The Expected Value of Various Entry Strategies



4 Business Portfolio Decisions: Uncertainty About Portfolio Compatibility

In this section, we provide an extension of our general model that is applicable to business portfolio decisions. While our earlier model provides a general framework to analyze a firm's strategy for entering a new activity, we now examine a firm's decision to reconfigure its *existing* asset portfolio. We consider a single firm that currently consists of two assets, denoted A and B. The value of the assets is determined solely by the synergy between them, by a matching parameter that determines the joint value of these assets. We assume that the matching parameter between assets A and B is given by $\rho_{AB} \in (0, 1)$.

The scope of the firm is limited to two assets. This implies that it is possible to add another asset only if an existing asset is discarded. This scope limitation could be justified by limitations on human capital and decision-support systems within the firm. Suppose the firm is considering adding another asset called C, which can only be added if the firm reconfigures its portfolio by discarding either A or B. The decision is driven by a comparison of the matching parameters

between the different assets. While the matching parameter between the existing assets (ρ_{AB}) is known, the matching parameter between C and either A or B is only stochastically known. This uncertainty makes investment timing important. A firm may prefer to wait until it learns more about these matching parameters and allow for the uncertainty about the value of C to diminish. However, waiting might not be optimal if competitors may go after C as well. We assume away the uninteresting case that the asset C fits perfectly with both assets A and B (i.e., if matching parameters were 1 for both assets), as well as the case that the new asset C does not fit at all with either asset A or B (i.e., if matching parameters were 0 for both assets).

We formalize the uncertainty as follows. The random matching parameters are given by $\{\rho_{AC}, \rho_{BC}\} \in \langle\{-1,1\},\{1,-1\}\rangle$; where $\{\rho_{AC}, \rho_{BC}\} = \{-1,1\}$ w.p. $q \in (0,1)$, and $\{\rho_{AC}, \rho_{BC}\} = \{1,-1\}$ w.p. $1-q$. That is, the new asset C may fit perfectly with asset B (because $\rho_{BC} = 1$), but does not fit at all with asset A (because $\rho_{AC} = -1$). Alternatively, the opposite could be the case. The assumption that ρ_{AB} lies in the interior of $(0,1)$ means potential benefits to the firm from acquiring C.

Because of the uncertainty about the compatibility of C with its existing portfolio and the need to divest an existing asset if C is added, buying C could lead the firm to make the wrong acquisition/divestiture decision. The likelihood of error is highest for intermediate values of q, the probability that $\{\rho_{AC}, \rho_{BC}\} = \{-1,1\}$. Since $q = 0$ means that $\Pr(\{\rho_{AC}, \rho_{BC}\} = \{1,-1\}) = 1$ and there is no uncertainty that $\{A, C\}$ is optimal, the firm will acquire asset C and divest B for relatively low values of q. If q is large, the firm will acquire asset C and divest A. For intermediate values of q, the firm will retain its current portfolio and not acquire C.

As in the general model, we will now examine whether the firm – if it chooses to acquire C – will do so early or late. In contrast to investing in C early, investing late resolves uncertainty and helps to avoid suboptimal asset portfolio decisions. That is, if the firm acquires C early, it may erroneously determine which existing asset is compatible with and hence divest the wrong asset. However, going late is risky as well since a competitor may steal C. We can thus think of C as a single asset that is for sale, and only one firm can buy it. For completeness, we assume that if the firm has made a mistake and retained the wrong set of assets, it can recover only a portion of the assets' value.

4.1 Investment Strategies

We consider two investment strategies that are similar to our general model. We assume that the firm can make a small investment k that will reveal for sure which matching parameter set is relevant (i.e., if $\{\rho_{AC}, \rho_{BC}\} = \{-1, 1\}$ or $\{\rho_{AC}, \rho_{BC}\} = \{1, -1\}$). However, investing k at $t = 0$ delays the restructuring decision for one period (i.e., it takes one period to learn). In that time, a competitor may come in and steal asset C; this happens w.p. γ.

Alternatively, the firm can choose to immediately reconfigure its boundaries by acquiring C right away and divesting A or B. This is analogous to the firm making the full investment I in the previous model. We assume that the net cash flow effects of this transaction are zero.[7] With this strategy, the firm does not know which asset fits best with the newly acquired one. Hence, the value loss could be twofold. First, the firm might divest the wrong asset. Second, the firm would have given up the existing value of assets A and B given by $\rho_{AB} > 0$. However, with this strategy, the firm never loses C to a competitor. Another consideration is that if the firm makes the wrong asset choice, it can *divest* both of the poorly performing assets in the next period and recover an amount $[1 - \alpha]$. We interpret α as the liquidation cost of the asset portfolio.[8]

Invest in Learning and Delaying Asset Reconfiguration

The expected incremental value of investing k and waiting to reconfigure the assets, *relative* to retaining the current asset mix, is given by

$$\begin{aligned} E(\text{Invest } k) &= \gamma \times [0 - k] + [1 - \gamma] \times [[1 - \rho_{AB}] - k] \\ &= [1 - \gamma] \times [1 - \rho_{AB}] - k. \end{aligned} \tag{8}$$

This value can be decomposed as follows. With probability γ, the competitor steals asset C and hence there is no extra value to be had, even though k is lost. The loss in this case is the lost investment k. If the competitor doesn't jump in (occurring w.p. $1 - \gamma$), the firm learns for sure which asset fits best with C (gaining a value of 1) and divests the other asset (giving up ρ_{AB}). The benefit in this case, net of the learning cost k, is $1 - \rho_{AB} - k$.

[7] We could easily introduce individual asset prices without any changes to the qualitative results.

[8] There are a number of ways to interpret the recovery amount of $1 - \alpha$. For instance, this could represent the net cash flow effects of retaining asset C, divesting the asset that was originally retained from $\{A, B\}$, and re-acquiring the asset that was orignally discarded. Alternatively, it could represent the liquidation value of both assets. Either interpretation is consistent with the results.

Reconfigure Assets Immediately

If the firm chooses to reconfigure its portfolio immediately at time $t = 0$, it restructures with only imperfect knowledge about the matching parameters between the various combinations. Therefore, although it may make mistakes in its restructuring decisions, it will never lose C to a competitor or incur the cost k.

For low values of q, it is more likely that $\{\rho_{AC}, \rho_{BC}\} = \{1, -1\}$. Hence, the firm would like to acquire C and divest B. If it turns out that the firm has restructured incorrectly (i.e., it turns out that $\{\rho_{AC}, \rho_{BC}\} = \{-1, 1\}$), it sells assets A and C and recovers $1 - \alpha$. The expected incremental value of this strategy, relative to retaining A and B, would be

$$\begin{aligned} E(\text{Acquire } C, \text{ Divest } B) &= [1-q]\,[1-\rho_{AB}] + q\,[-1-\rho_{AB}+1-\alpha] \\ &= 1 - [1+\alpha]\,q - \rho_{AB}. \end{aligned} \tag{9}$$

Alternatively, for high values of q, it is more likely that $\{\rho_{AC}, \rho_{BC}\} = \{-1, 1\}$. The firm would then like to acquire C and divest A. Again, if it is wrong, it can recover $1-\alpha$ by selling the assets. The expected incremental value of this strategy is

$$\begin{aligned} E(\text{Acquire } C, \text{ Divest } A) &= [1-q]\,[-1-\rho_{AB}+1-\alpha] + q\,[1-\rho_{AB}] \\ &= -\alpha + [1+\alpha]\,q - \rho_{AB}. \end{aligned} \tag{10}$$

Upon examining (9) and (10), we see that the firm with portfolio $\{A, C\}$ has positive value if q is low ($q < \frac{1-\rho_{AB}}{1+\alpha}$), and the firm with portfolio $\{B, C\}$ has positive value for high values of q ($q > \frac{\alpha+\rho_{AB}}{1+\alpha}$). For intermediate values of $q \in \left[\underline{q}, \overline{q}\right]$, where $\underline{q} = \frac{1-\rho_{AB}}{1+\alpha}$ and $\overline{q} = \frac{\alpha+\rho_{AB}}{1+\alpha}$, the firm will not restructure and simply retain portfolio $\{A, B\}$.

The above result characterizes the value of restructuring if it is done immediately. However, we need to compare the value of this strategy to the value of delaying restructuring by investing k instead. The expected incremental value of delaying (i.e., investing k to learn), relative to doing nothing, is given by (8). Assume that this value is positive. We can then calculate the incremental value of investing and restructuring today, relative to delaying investment. Using (9), (10), and (8), we have

$$\begin{aligned} \Psi_{Now-Delay}(\text{Acquire } C \text{ and Divest } B) &= E(\text{Acquire } C, \text{ Divest } B) - E(\text{Invest } k) \\ &= \left[\begin{array}{c} [1-q]\,[1-\rho_{AB}] + q\,[-1-\rho_{AB}+1-\alpha] \\ -\,[[1-\gamma]\times[1-\rho_{AB}] - k] \end{array}\right] \\ &= [1+\alpha]\,q + \gamma\,[1-\rho_{AB}] + k \end{aligned}$$

and

$$\begin{aligned}
\Psi_{Now-Delay}(\text{Acquire } C, \text{ Divest } A) &= E(\text{Acquire } C, \text{ Divest } A) - E(\text{Invest } k) \\
&= \left[\begin{array}{c} [1-q]\,[-1-\rho_{AB}+1-\alpha] + q\,[1-\rho_{AB}] \\ -\,[[1-\gamma]\times[1-\rho_{AB}]-k] \end{array} \right] \\
&= [1+\alpha]\,q - [1+\alpha] + \gamma\,[1-\rho_{AB}] + k.
\end{aligned}$$

This leads to the following result.

Theorem 3

If investing in k alone is valuable (i.e., $[1-\gamma]\times[1-\rho_{AB}]-k>0$), then the firm acquires C immediately, divesting B, when the probability that C fits well with B is low (i.e., $q<\underline{q}'$); invests k and waits until $t=1$ for intermediate values of q ($q'\in\left[\underline{q}',\overline{q}'\right]$); and acquires C, divesting A, when the probability that C fits well with B is high (i.e., $q>\overline{q}'$); where $\underline{q}'=\frac{\gamma[1-\rho_{AB}]+k}{1+\alpha}$ and $\overline{q}'=\frac{[1+\alpha]-\gamma[1-\rho_{AB}]-k}{1+\alpha}$.

If investing in k alone is not valuable (i.e., $[1-\gamma]\times[1-\rho_{AB}]-k\leq 0$), then the firm acquires C, divesting B, for low values of q ($q<\underline{q}$); doesn't restructure for intermediate values of q ($q\in\left[\underline{q},\overline{q}\right]$); and acquires C, divesting A, for high values of q ($q>\overline{q}$); where $\underline{q}=\frac{1-\rho_{AB}}{1+\alpha}$ and $\overline{q}=\frac{\alpha+\rho_{AB}}{1+\alpha}$.

The range over which a firm will reconfigure its portfolio early is decreasing in the cost of liquidating poorly performing assets (α). That is, the range over which the firm will wait to reconfigure its assets is increasing in α.

Observe that the range over which the firm will optimally restructure early, instead of restructuring late by first investing k, is increasing in the probability, γ, that a competitor could grab C. That is, $\underline{q}'$ is increasing, and $\overline{q}'$ is decreasing, in γ. The interesting implication of this result is that in a more competitive market (higher γ), there will be a greater number of acquisitions *and* divestitures because the firm is redrawing its boundaries more frequently. The divestiture result follows because acquisitions are being made in the presence of high informational uncertainty. Thus, mistakes are made more often, and these are later rectified through divestitures. This implies that the intertemporal volatility in the composition of real asset (portfolios) is increasing in the degree of competition.

The implication that the firm is forced to redraw its boundaries more quickly and more often in the face of competition provides a clear testable hypothesis. There are many industries in which either deregulation and/or improved access to information and capital have led to an observable increase in competition. Financial services and telecommunications are two examples. The model predicts an elevated level of mergers and acquisitions, to be followed by divestitures. Casual empiricism seems to bear this out for acquisitions in financial services and telecommunications, although some time will probably have to elapse before the divestiture prediction can be tested.

The parameter α is related to Shleifer and Vishny's (1992) liquidity costs. In some industries, reconfiguring assets might be very expensive (high α), and firms in these industries might be very reluctant to enter early and redraw their boundaries right away. We can also link our theory to Tobin's q. For high values of Tobin's q, the presence of growth opportunities increases the likelihood that these assets are highly firm-specific. The greater the specificity of these assets, the higher the liquidation cost (α) and the lower the incentive to restructure early.

5 Technological Innovations

Our theory also applies to new technology introduction decisions. Should firms introduce these technologies early or wait? In this second application, we consider two firms competing in the same industry, but with different technologies. One player, denoted firm E, currently has the superior technology and consequently, greater market share. The other firm, denoted firm N, has an inferior technology, giving it a smaller share of the market. We wish to compare the incentives of these two firms to invest in research to invent a new technology superior to the existing technology of the leading firm.

We model this problem in the context of Cournot competition across two dates. There is a deterministically-known market demand of Ω that is realized at both $t = 0$ and $t = 1$. The player with the (currently) superior technology has high skill, i.e., per-unit production cost of $\underline{c}$. The other player (N) has low skill, i.e., per-unit production cost of $\overline{c}$. We assume that $\underline{c} < \overline{c} \ll \Omega$. At $t = 0$, both firms compete as Cournot duopolists, earning profits of

$$\Pi_E(\text{status quo}) = \frac{1}{9}[\Omega - 2\underline{c} + \overline{c}]^2$$

and

$$\Pi_N(\text{status quo}) = \frac{1}{9}[\Omega - 2\overline{c} + \underline{c}]^2,$$

respectively. Naturally, $\Pi_N < \Pi_E$, since firm E has the cost (technology) advantage. We assume that Ω is always large enough to insure positive profits for even the weakest player.[9]

If nothing changes, these two firms will earn the same profits at time $t = 2$. However, we consider an investment of I at $t = 0$ that *may* produce a superior technology. This technology would allow for production at a per-unit cost of $\underline{\underline{c}} < \underline{c}$. Conditional on investing I, the firm succeeds w.p. δ in developing the technology. For now, we assume that only one firm can invest I.

If firm E develops the superior technology successfully, and firm N does not, the following profits for firms E and N obtain:

$$\begin{aligned} \Pi_E(\text{E develops successfully}) &= \frac{1}{9}[\Omega - 2\underline{\underline{c}} + \overline{c}]^2 \\ &> \Pi_E(\text{status quo}) \end{aligned}$$

and

$$\begin{aligned} \Pi_N(\text{E develops successfully}) &= \frac{1}{9}[\Omega - 2\overline{c} + \underline{\underline{c}}]^2 \\ &< \Pi_N(\text{status quo}). \end{aligned}$$

If the smaller player develops the superior technology, it will now have the lowest cost. This would result in the following

$$\begin{aligned} \Pi_E(\text{N develops successfully}) &= \frac{1}{9}[\Omega - 2\underline{c} + \underline{\underline{c}}]^2 \\ &< \Pi_E(\text{status quo}) \end{aligned}$$

and

$$\begin{aligned} \Pi_N(\text{N develops successfully}) &= \frac{1}{9}[\Omega - 2\underline{\underline{c}} + \underline{c}]^2 \\ &> \Pi_N(\text{status quo}). \end{aligned}$$

With the above profit expressions, we can write down the expected value to each firm from individually investing I today. For the bigger firm, it is given by

$$\begin{aligned} \Psi_E(\text{E invests I}) &= \delta\left[\Pi_E(\text{E develops successfully})\right] \\ &\quad + [1-\delta]\left[\Pi_E(\text{status quo})\right] \\ &= \frac{\delta}{9}[\Omega - 2\underline{c} + \underline{\underline{c}}]^2 + \frac{[1-\delta]}{9}[\Omega - 2\underline{c} + \overline{c}]^2. \end{aligned}$$

[9] A sufficient, but not necessary, condition is $\Omega > 2\overline{c}$.

For the smaller firm, we have

$$\begin{aligned}\Psi_N(\text{N invests I}) &= \delta\left[\Pi_N(\text{N develops successfully})\right] \\ &\quad [1-\delta]\left[\Pi_N(\text{status quo})\right] \\ &= \frac{\delta}{9}[\Omega - 2\underline{\underline{c}} + \underline{c}]^2 + \frac{[1-\delta]}{9}[\Omega - 2\overline{c} + \underline{c}]^2.\end{aligned}$$

While the expected benefits to each firm from developing the new technology are important, more critical is the expected losses each firm faces when they fail to develop the new technology. For the bigger firm, its expected value when the other firm invests I is

$$\begin{aligned}\Psi_E(\text{N invests I}) &= \delta\left[\Pi_E(\text{N develops successfully})\right] \\ &\quad + [1-\delta]\left[\Pi_E(\text{status quo})\right] \\ &= \frac{\delta}{9}[\Omega - 2\underline{c} + \underline{\underline{c}}]^2 + \frac{[1-\delta]}{9}[\Omega - 2\underline{c} + \overline{c}]^2.\end{aligned}$$

From the smaller firm's perspective, if the larger firm invests I, its expected value is

$$\begin{aligned}\Psi_N(\text{E invests I}) &= \delta\left[\Pi_N(\text{E develops successfully})\right] \\ &\quad [1-\delta]\left[\Pi_N(\text{status quo})\right] \\ &= \delta\frac{1}{9}[\Omega - 2\overline{c} + \underline{\underline{c}}]^2 + [1-\delta]\,\frac{1}{9}[\Omega - 2\overline{c} + \underline{c}]^2.\end{aligned}$$

Importantly, what this allows us to do is to then calculate the expected loss to each player if the other player successfully develop the new technology. For the big firm this is

$$\begin{aligned}E_E(\text{Loss if N develops}) &= \Pi_E(\text{N develops successfully}) - \Pi_E(\text{status quo}) \\ &= \frac{1}{9}[\Omega - 2\underline{c} + \underline{\underline{c}}]^2 - \frac{1}{9}[\Omega - 2\underline{c} + \overline{c}]^2 \\ &= \frac{1}{9}\left[\left[\underline{\underline{c}}^2 - \overline{c}^2\right] + \Omega\left[\underline{\underline{c}} - \overline{c}\right] - 2\underline{c}\left[\underline{\underline{c}} - \overline{c}\right]\right] < 0\end{aligned}$$

The expected loss to the smaller firm if the bigger firm successfully develops the new technology is

$$\begin{aligned}E_N(\text{Loss if E develops}) &= \Pi_N(\text{E develops successfully}) - \Pi_N(\text{status quo}) \\ &= \frac{1}{9}[\Omega - 2\overline{c} + \underline{\underline{c}}]^2 - \frac{1}{9}[\Omega - 2\overline{c} + \underline{c}]^2 \\ &= \frac{1}{9}\left[\left[\underline{\underline{c}}^2 - \underline{c}^2\right] + \Omega\left[\underline{\underline{c}} - \underline{c}\right] - 2\overline{c}\left[\underline{\underline{c}} - \underline{c}\right]\right] < 0\end{aligned}$$

Since $\underline{\underline{c}} < \underline{c} < \overline{c}$, the larger firm's expected loss exceeds the commensurate loss for the smaller firm. We now have the following result.

Theorem 4

The expected profit to the smaller firm N investing I today exceeds that to the bigger firm E from investing I today. Conversely, the potential cost to firm E of not investing while N does invest exceeds the cost to firm N of not investing when E does. This implies that the larger firm E is less likely to introduce an innovation than the smaller player N if it anticipates that others (such as firm N) have no access to it. However, the larger firm E will be more likely to innovate than the smaller firm N if it anticipates others will be able to successfully develop it.

This intuition is based on the impact of existing market share on the firm's incentive to innovate. A firm currently with a large market share has less to gain from investing in a new technology that will produce a market share increase than one with a smaller market share. However, an existing firm has *more* to lose if the smaller firm successfully develops the new technology. Therefore, if the larger firm assigns a high probability to the innovation being successfully developed, it will optimally choose to develop it itself. Smaller innovations, or moderate refinements to existing technology, have a higher success probability, and therefore the larger firm is more likely to take these on. However, for higher-potential innovations, the larger firm would assign a smaller probability (δ) of this innovation being successfully developed, and thereby foregoes these innovations. Smaller players have exactly the opposite tendencies, and will bypass small innovations (as there is little to lose from missing these), but always invest in the riskier, higher-potential innovations as there is much to gain. This explains why older players in the industry are less likely to make major innovations, without relying on the argument that inefficiencies and complacencies are more likely to be found in more dominant market players.

6 Empirical Implications and Industry Applications

The major empirical implications of our analysis are as follows:

1. The amount of investment a firm makes in a new market is decreasing in the uncertainty about the future payoffs from the investment. When this uncertainty is sufficiently low, the firm makes a large investment right away. When uncertainty is sufficiently high, the firm prefers to enter early with a smaller toe-hold investment. For even higher levels of uncertainty, the firm prefers to wait to invest until payoff uncertainty is resolved.

2. The amount of investment a firm makes in a new market is increasing in the degree of competition in that market. As competition increases, the strategy of entering early and with a large investment becomes more attractive.

3. An increase in competition leads to an increase in the intertemporal volatility of the composition of firms' real asset portfolios. Thus, firms acquire other firms and divest assets with greater frequency in more competitive industries. This restructuring tendency will be further increasing in the liquidation value of the firms' assets.

4. From an established (larger) firm's perspective, if it is unlikely that other (less established) firms have access to new technologies, the larger firm will be more reluctant to innovate than the smaller firm. However, if other firms are very likely to have access to a new technology, the larger firm will be more inclined than the smaller firms in introducing this new technology.

The first two predictions are readily testable and illuminate some recent strategic initiatives by companies. For example, consider the global appliance industry. For the leading companies in this industry, China was an attractive new market in the early 1990s. However, the major players also understood that this market was likely to become extremely competitive due to the anticipated entry of numerous large American and European companies. Consequently, many companies, including global appliance giant Whirlpool Corporation, entered with large investments in property, manufacturing plants, equipment and distribution systems. However, future appliance demand in China was highly uncertain. General Electric's appliance division was one of the companies whose estimate of this uncertainty was so high that it eschewed a large investment. Instead, it made a small investment by striking a joint venture deal with a national sales distributor in China and outsourcing all manufacturing. The intent was to "wait and see" whether large manufacturing investments would be justified in the future. To date, the more reserved strategy of GE has apparently been the wiser move.

As for the third prediction, there seems to be quite a bit of anecdotal evidence that is consistent with it. Because of deregulation and improvements in information technology, the financial services industry has become much more competitive. Not coincidentally, we are also witnessing massive consolidation through mergers and acquisitions in this industry. In recent years, Chemical Bank acquired Chase Manhattan, and Nations Bank acquired Boatmen's Bancshares and Barnett, and then merged with Bank of America. All of these acquisitions were dwarfed in significance by the marriage of Citibank and Travellers. In Europe, Union Bank of Switzerland and Swiss Bank

Corporation have combined. In Japan, the merged Tokyo-Mitsubishi bank has assets of over $700 billion.

Similar acquisitions are sweeping across other industries with heightened competition. For example, in pharmaceuticals, Sandoz and Ciba Gergy have merged to form Novartis. In telecommunications, America-On-Line first joined forces with Netscape and then with Time-Warner, and AT&T has acquired Tele-Communications Incorporated, the largest television firm in the US. While these developments are consistent with our theory, there is also the prediction that these industries will also be characterized by numerous divestitures in the future. In particular, this prediction would primarily apply to the examples where non-core businesses are brought into the corporate fold.

Consider now the fourth prediction. It has often been observed that large incumbents in a given industry are slow to introduce innovations, relative to smaller newcomers. In the telecommunications industry, for instance, analog companies dominated the market, and exhibited little interest in introducing digital (GSM) technology. It took newcomers like Rhythms, Covad and others to introduce powerful new digital applications. Similarly, it took a new company, Starlight Telecommunications, to begin offering basic, locally-operated telephone services in Africa (Somalia and Uganda); the plan was proposed first within GTE and rejected. In the context of our model, the established (analog) players might have anticipated a relatively small probability that other players would be able to introduce the digital technology quickly. Further, given the network benefits of the existing analog players, they might have thought that they could afford to delay digital because their perception was that it could not be quickly rolled out as a serious threat to main players' analog business. Thus, our model predicts that major innovations (such as the development of the GSM technology) are more likely to come from new players, whereas simpler refinements to existing technologies are more likely to come from the established players.

7 Conclusion

In this paper we have developed a theory of organizational boundaries based on informational uncertainty and learning. The basic idea is that the determinants of firm boundaries is a dynamic process, involving experimentation by firms attempting to learn about the payoff potential of new activities, as well as their own capabilities to realize this potential. There are four key elements that drive the model. First, there is uncertainty about future demand in the new market the

firm is considering for entry. Second, there is uncertainty about the firm's skill in operating in that market even if high demand materializes. Third, whenever a firm is thinking of redrawing its boundaries by adding an asset, it must worry about portfolio compatibility, having to do with how well that asset "fits" with the rest of its portfolio. And fourth, there is uncertainty about whether a competitor will come in and exploit the new opportunity first, thereby gaining a first-mover advantage.

We show that whether a firm redraws its boundaries by entering early with a large investment or by making a smaller toe-hold investment, or simply decides to wait and see, depends on the interaction of these four elements. This interaction produces numerous testable predictions as well as an economic description of how firms should evolve. In terms of the model's application to business portfolio decisions, we show that the frequency of corporate restructuring via acquisitions and divestitures is increasing in the competitiveness of the marketplace. And lastly, we show how the incentive to innovate depends on the size and market position of the firm, and particularly on the likelihood that other firms have access to the innovation. These applications illuminate how industry structure and competitiveness affect restructuring and innnovation decisions.

8 Appendix

8.1 Derivation of Second-Period Profits

The second-period profits earned in all of these cases are solved in the same way. The key differences among them lies in the production costs faced by the firms and the extent to which one of the firms (if any) has a competitive (Stackleberg) advantage. We can solve the representative game by assigning a production cost c_A to firm A and c_B to firm B, where $c_A, c_B \in \{\underline{c}, E(c), \overline{c}\}$. We will first derive general expressions for the output quantities of the two firms and their expected profits under both Cournot and Stackleberg competition. We can then compute the outcome in any particular state by substituting in the production cost and competitive positions related to that state of nature and equilibrium choice.

We begin with Cournot competition with A and B choosing production levels simultaneously. Let $E(\Pi_A)$ and $E(\Pi_B)$ denote the expected profits of firms A and B, respectively, computed at $t = 1$. We do not yet concern ourselves with the decision problems at $t = 0$ (i.e., enter early versus late or invest I versus k), but only consider the profits in the relevant states at $t = 1$. Firms A and B will individually choose and commit to produce q_A and q_B units of output of the new activity to maximize their expected profits. The equilibrium outputs and expected profits in this Cournot case are then:

$$q_A = \frac{1}{3}[\Omega - 2c_A + c_B] \tag{11}$$

$$q_B = \frac{1}{3}[\Omega - 2c_B + c_A] \tag{12}$$

$$E(\Pi_A) = \frac{1}{9}[\Omega - 2c_A + c_B]^2 \tag{13}$$

$$E(\Pi_B) = \frac{1}{9}[\Omega - 2c_B + c_A]^2. \tag{14}$$

An immediate implication of equations (11) through (14) is that if firm A has the cost/skill advantage ($c_A < c_B$), then it has a greater market share and expected profits ($q_A > q_B$ and $E(\Pi_A) > E(\Pi_B)$). These inequalities are reversed if firm B has the cost advantage ($c_A > c_B$).

With Stackleberg competition, the firm investing I first (and before demand is realized at $t = 1$) becomes the Stackleberg leader. The other firm then reacts to the leader's production choice. Suppose that firm A is the Stackleberg leader, then firm B reacts to a production choice of q_A by firm A by producing

$$q_B = \frac{1}{2}\left[\Omega - q_A - c_B\right]. \tag{15}$$

Observe that firm A chooses its own production first, given firm B's reaction function in (15). The equilibrium outputs and expected profits for firm A as the Stackleberg leader are

$$q_A = \frac{1}{2}\left[\Omega - 2c_A + c_B\right] \tag{16}$$

$$q_B = \frac{1}{4}\left[\Omega - 3c_B + 2c_A\right] \tag{17}$$

$$E(\Pi_A) = \frac{1}{8}\left[\Omega - 2c_A + c_B\right]^2 \tag{18}$$

$$E(\Pi_B) = \frac{1}{16}\left[\Omega - 3c_B + 2c_A\right]^2. \tag{19}$$

Observe that if B is the leader and A is the follower, the expressions are just reversed.

Upon substituting $\Omega = \overline{c}$ from (3) into the above expressions, we arrive at both of the firm's output and expected profits for the different states and competitive positions.[10] This analysis is summarized in *Table 1*. In this table, the first column denotes firm A's entry decisions. Conditional on this decision, column two describes the states of the world that could occur. Columns 3 and 4 then contain the per-unit production costs and expected second-period profits for both firms A and B in that particular state.

[10] To insure that firms that have already committed I always find it profitable to produce whenever their per-unit production cost is less than the worst-case $\overline{c}$, we assume that $\delta > \frac{1}{2}$. This is sufficient for a follower to produce profitably when the leader has the low cost $\underline{c}$.

Table 1: Cost and Profit Outcomes

Entry Decision	**State of World/ Competition**	**Per-Unit Production Cost**	**Profit Outcomes**
Both Firms Wait Until $t = 1$	Cournot duopolists	$c_A = E(c)$ $c_B = E(c)$	$\Pi_A = \frac{1}{9}\delta^2 [\overline{c} - \underline{c}]^2$ $\Pi_B = \frac{1}{9}\delta^2 [\overline{c} - \underline{c}]^2$
Firm A Invests I at $t = 1$ first	A is Stackleberg leader	$c_A = E(c)$ $c_B = E(c)$	$\Pi_A = \frac{1}{8}\delta^2 [\overline{c} - \underline{c}]^2$ $\Pi_B = \frac{1}{16}\delta^2 [\overline{c} - \underline{c}]^2$
Firm A Invests k at $t = 0$	Learns skill w.p. λ; skill high w.p. δ (Cournot)	$c_A = \underline{c}$ $c_B = E(c)$	$\Pi_A = \frac{1}{9}[2 - \delta]^2 [\overline{c} - \underline{c}]^2$ $\Pi_B = \frac{1}{9}[2\delta - 1]^2 [\overline{c} - \underline{c}]^2$
	Learns skill w.p. λ; skill low w.p. $1 - \delta$ (Cournot)	$c_A = \overline{c}$ $c_B = E(c)$	$\Pi_A = 0$ $\Pi_B = \frac{1}{4}\delta^2 [\overline{c} - \underline{c}]^2$
	Doesn't learn skill w.p. $1 - \lambda$ (Cournot)	$c_A = E(c)$ $c_B = E(c)$	$\Pi_A = \frac{1}{9}\delta^2 [\overline{c} - \underline{c}]^2$ $\Pi_B = \frac{1}{9}\delta^2 [\overline{c} - \underline{c}]^2$
	Learns skill w.p. λ; skill high w.p. δ (Leader)	$c_A = \underline{c}$ $c_B = E(c)$	$\Pi_A = \frac{1}{8}[2 - \delta]^2 [\overline{c} - \underline{c}]^2$ $\Pi_B = \frac{1}{16}[3\delta - 2]^2 [\overline{c} - \underline{c}]^2$
	Learns skill w.p. λ; skill low w.p. $1 - \delta$ (Leader)	$c_A = \overline{c}$ $c_B = E(c)$	$\Pi_A = 0$ $\Pi_B = \frac{1}{4}\delta^2 [\overline{c} - \underline{c}]^2$
	Doesn't learn skill w.p. $1 - \lambda$ (Leader)	$c_A = E(c)$ $c_B = E(c)$	$\Pi_A = \frac{1}{8}\delta^2 [\overline{c} - \underline{c}]^2$ $\Pi_B = \frac{1}{16}\delta^2 [\overline{c} - \underline{c}]^2$
	Learns skill w.p. λ; skill high w.p. δ (Follower)	$c_A = \underline{c}$ $c_B = E(c)$	$\Pi_A = \frac{1}{16}[3 - 2\delta]^2 [\overline{c} - \underline{c}]^2$ $\Pi_B = \frac{1}{8}[2\delta - 1]^2 [\overline{c} - \underline{c}]^2$
	Learns skill w.p. λ; skill low w.p. $1 - \delta$ (Follower)	$c_A = \overline{c}$ $c_B = E(c)$	$\Pi_A = 0$ $\Pi_B = \frac{1}{4}\delta^2 [\overline{c} - \underline{c}]^2$
	Doesn't learn skill w.p. $1 - \lambda$ (Follower)	$c_A = E(c)$ $c_B = E(c)$	$\Pi_A = \frac{1}{16}\delta^2 [\overline{c} - \underline{c}]^2$ $\Pi_B = \frac{1}{8}\delta^2 [\overline{c} - \underline{c}]^2$
Firm A Invests I at $t = 0$	Learns skill; high w.p. δ (Leader)	$c_A = \underline{c}$ $c_B = E(c)$	$\Pi_A = \frac{1}{8}[2 - \delta]^2 [\overline{c} - \underline{c}]^2$ $\Pi_B = \frac{1}{16}[3\delta - 2]^2 [\overline{c} - \underline{c}]^2$
	Learns skill; low w.p. $1 - \delta$ (Leader)	$c_A = \overline{c}$ $c_B = E(c)$	$\Pi_A = 0$ $\Pi_B = \frac{1}{4}\delta^2 [\overline{c} - \underline{c}]^2$

8.2 Characterization of Firm A's Early Entry Strategy for k

If firm A learns its skill is high, it must compare the expected profits from investing early and producing as the leader with low cost, to the expected profits of investing only when demand is positive and producing with low cost as a Cournot duopolist. If firm B waits to invest and firm A learns its skill is high, firm A will invest I before demand is realized if and only if

$$\frac{1}{72}\eta\left[2-\delta\right]^2\left[\overline{c}-\underline{c}\right]^2-\left[1-\eta\right]I>0. \tag{20}$$

If it learns nothing about its skill, it must compare the expected profits from investing early, and producing as the leader with average cost, to the expected profits of investing only when demand is positive and producing with average cost as a Cournot duopolist. If firm B doesn't jump in and firm A learns nothing about its skill, firm A will invest I before demand is realized if and only if

$$\frac{1}{72}\eta\delta^2\left[\overline{c}-\underline{c}\right]^2-\left[1-\eta\right]I>0. \tag{21}$$

Observe that (21) implies (20) since $\delta^2 \leq \left[2-\delta\right]^2$. One can interpret these two conditions as measures of the value of the first-mover advantage. It is easy to see from (21) and (20) that as the viability of the market becomes more uncertain ($\eta \to 0$), the value of being first to market declines as both of these incremental profitability conditions become more difficult to satisfy. In fact, for $\eta = 0$, neither condition is satisfied. The reason is that being the first to enter a market with no demand offers no advantage. We assume that both (21) and (20) are true, but could readily relax this.

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