An S matrix analysis of the Z resonance

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The S matrix ansatz is a rigorously model independent approach to describe the cross-sections and asymmetries in e^+e^- annihilation. Using the cross-sections and asymmetries measured with the L3 detector during the 1990 and 1991 running period, we determine the mass and the width of the Z boson, the contributions of the Z exchange and of the γZ interference. Including the polarization of the τ lepton in the analysis, the leptonic helicity amplitudes of the scattering process are determined assuming lepton universality. The results are compared with other model independent ansatzes as realized in ZFITTER. A systematic bias of the Z mass due to the γZ interference term is detected, which leads to an underestimation of the error on m_Z for model independent determinations.

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1. Introduction

The successful operation of LEP has allowed a precise measurement of the e^+e^- annihilation cross-sections and asymmetries near the Z resonance. The mass, total and partial widths of the Z boson have been determined with an excellent accuracy. The experimental results confirm the Standard Model with percent precision [1–3].

In this paper we investigate to what extent a satisfactory description of the experimental data on the Z line shape can be reached with minimal assumptions. We base this study on an S matrix approach, the details of which are explained elsewhere [4]. The scattering process is described by the superposition of massless and massive boson exchange, without making detailed assumptions about the dynamics of the process.

For the total cross-section the S matrix approach is equivalent to the model independent approach derived earlier [5]. Other model independent approaches to the Z line shape have been described in the literature [6–8] and used by L3 in previous studies [1]. All of these studies have in common that the interference between the massless and the massive boson exchange for hadronic reactions is fixed to the value it assumes in the Standard Model. This treatment was shown to be sufficient at the previous level of accuracy, since the interference term is suppressed in the vicinity of the Z resonance. The present accuracy of lineshape measurements allows, however, to determine limits on the value of this interference term for total cross-sections as well as asymmetries, and for leptonic and hadronic final states separately. We also study the influence of this term on the value of the Z mass, and the potential bias caused by fixing it to its Standard Model value. A discussion of the theoretical predictions for the interference term in the Standard Model and its measurability can be found in ref. [9].

We use our experimental measurements of the total cross-sections, the forward-backward asymmetries for all leptonic and hadronic Z decay channels as well as the polarization of tau leptons from Z decay. The total luminosity used is 17.2 pb^{-1} (corresponding to about 40 000 leptonic and 423 000 hadronic events) collected with the L3 detector in 1990 and 1991.

2. The L3 detector

The L3 detector at LEP covers 99% of the full solid angle. It is designed to measure energy and position of leptons, photons and jets with high precision. A detailed description of the detector and its performance can be found elsewhere [10].

The detector consists of a time expansion chamber (TEC) for the tracking and vertex reconstruction of charged particles, a high resolution electromagnetic calorimeter made of about 11 000 bismuth germanium oxide (BGO) crystals, a hadron calorimeter (HCAL) with uranium absorber and brass proportional wire chambers and a high precision muon spectrometer, consisting of three layers of multi-wire drift chambers. A cylindrical array of 30 scintillation counters is installed in the barrel region between the BGO and the HCAL. The luminosity is measured by the luminosity monitors, two electromagnetic calorimeters, situated symmetrically on either side of the interaction point. Each calorimeter is a finely segmented and azimuthally symmetric array of 304 BGO crystals covering the polar range $24.93 < \theta < 69.94$ mrad. All detectors are inside a 12 m inner diameter solenoidal magnet which provides a uniform magnetic field of 0.5 T along the beam direction.

3. Z lineshape measurements

Operating the LEP storage ring in the vicinity of the Z mass with high luminosity permits a detailed study of the lineshape of the Z resonance. We have performed measurements of the reactions

(1) $e^+e^- \rightarrow$ hadrons,

(2) $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$,

(3) $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$,

(4) $e^+e^- \rightarrow e^+e^-(\gamma)$.

The analysis methods used for these reactions are described in detail elsewhere [1,11]. The cross-sections and asymmetries determined with the data taken in the 1990 and 1991 runs of LEP have been published [1]. Additionally, we include the average τ polariza-

tion [12] at $\sqrt{s} = 91.222$ GeV, $\mathcal{P}_\tau = -0.132 \pm 0.026 \pm 0.021$.

4. The S matrix formalism

The matrix element for the exchange of a photon and a Z boson in e^+e^- annihilation into massless fermions can be written as

$$\mathcal{M}^{fi}(s) = \frac{R_\gamma}{s} + \frac{R_Z^{fi}}{s - s_Z} + \sum_{n=1}^{\infty} F_n^{fi}(s_Z)(s - s_Z)^n,$$

$$i = 0, 3, \tag{1}$$

with

$$s_Z = \overline{m}_Z^2 - i\overline{m}_Z \overline{\Gamma}_Z.$$

The pole position for the Z-boson is given by s_Z. R_γ and R_Z^{fi} are the residuals for the photon and Z boson respectively. R_γ is defined by

$$R_\gamma = \frac{\alpha(s)}{\alpha} Q_e Q_f. \tag{2}$$

Q_f is the charge of the corresponding fermion and $\alpha(s)$ the running QED coupling constant.

The coefficients R_Z^{fi} describe the four helicity amplitudes for the Z exchange:

$$R_Z^{f0} = R_Z(e_L^- e_R^+ \longrightarrow f_L^- f_R^+),$$

$$R_Z^{f1} = R_Z(e_L^- e_R^+ \longrightarrow f_R^- f_L^+),$$

$$R_Z^{f2} = R_Z(e_R^- e_L^+ \longrightarrow f_R^- f_L^+),$$

$$R_Z^{f3} = R_Z(e_R^- e_L^+ \longrightarrow f_L^- f_R^+). \tag{3}$$

The coefficients $F_n^{fi}(s)$ of the power series in eq. (1) describe non-resonant contributions to the scattering process. As shown in refs. [4,5] these are numerically small around the Z resonance and are neglected for our analysis.

The cross-sections, σ_i, arising from the corresponding $\mathcal{M}^{fi}$ can be combined to four linearly independent cross-sections, observable at LEP:

$$\sigma_{tot}(s) = +\sigma_0 + \sigma_1 + \sigma_2 + \sigma_3,$$

$$\sigma_{fb}(s) = +\sigma_0 - \sigma_1 + \sigma_2 - \sigma_3, \tag{4}$$

$$\sigma_{pol}(s) = +\sigma_0 - \sigma_1 - \sigma_2 + \sigma_3,$$

$$\sigma_{pol}^{fb}(s) = +\sigma_0 + \sigma_1 - \sigma_2 - \sigma_3. \tag{4 cont'd}$$

The cross-sections can be parametrized as follows:

$$\sigma_A(s) = \tfrac{4}{3}\pi\alpha^2 \left(\frac{r_A^\gamma}{s} + \frac{s r_A^f + (s - \overline{m}_Z^2) j_A^f}{(s - \overline{m}_Z^2)^2 + \overline{m}_Z^2 \overline{\Gamma}_Z^2} \right),$$

$$A = tot, fb, pol, fbpol, \tag{5}$$

where r_A^γ is the photon exchange term:

$$r_A^\gamma = \begin{cases} |R_\gamma|^2 & \text{if } A = tot, \\ 0 & \text{if } A \neq tot. \end{cases} \tag{6}$$

The Z exchange term, r_A^f, and the γZ interference term, j_A^f, are given by

$$r_A^f = \tfrac{1}{4} \sum_{i=1}^{4} \{\pm 1\} |R_Z^{fi}|^2 + 2\frac{\overline{\Gamma}_Z}{\overline{m}_Z} \text{Im } C_A^f,$$

$$j_A^f = 2 \text{Re } C_A^f,$$

$$C_A^f = R_\gamma^* \left(\tfrac{1}{4} \sum_{i=1}^{4} \{\pm 1\} R_Z^{fi} \right), \tag{7}$$

where $\{\pm 1\}$ indicates that the sign of $|R_Z^{fi}|^2$ and of R_Z^{fi} corresponds to the sign of σ_i in eq. (4).

For the hadron channel one has to sum r_A^γ, r_A^f and j_A^f over all colours and open flavours. The asymmetries are defined by

$$\mathcal{A}_A(s) = \frac{\sigma_A(s)}{\sigma_{tot}(s)}, \quad A \neq tot. \tag{8}$$

Photonic corrections are included by convolution, for details see ref. [4].

Eqs. (1)–(8) completely define the framework of an S matrix analysis of the Z lineshape. However, when comparing Z parameters to other approaches, the following clarification is necessary:

First, it should be noted that in the S matrix approach the total width, $\overline{\Gamma}_Z$, is constant in contrast to the parametrization of the Breit–Wigner resonance of the Z lineshape, where Γ_Z is a result of quantum corrections, which are s-dependent. This leads to a transformation of the Z mass, m_Z, to $\overline{m}_Z$ and of the total Z width, Γ_Z, to $\overline{\Gamma}_Z$ [13]:

$$\overline{m}_Z = [1 + (\Gamma_Z/m_Z)^2]^{-1/2} m_Z \approx m_Z - 34 \text{ MeV},$$

$$\overline{\Gamma}_Z = [1 + (\Gamma_Z/m_Z)^2]^{-1/2} \Gamma_Z \approx \Gamma_Z - 1 \text{ MeV}. \tag{9}$$

Thus, the mass, $\overline{m}_Z$, obtained from S matrix fits should be shifted by -34 MeV and the total width, $\overline{\Gamma}_Z$, by -1 MeV, with respect to the results obtained from the standard procedure [1].

Second, with vector and axial vector couplings of the Z to the fermions, eqs. (3) can be expressed as

$$R_Z^{f0} = \kappa \, (\widehat{g}_v^e + \widehat{g}_a^e)(\widehat{g}_v^f + \widehat{g}_a^f),$$

$$R_Z^{f1} = \kappa \, (\widehat{g}_v^e + \widehat{g}_a^e)(\widehat{g}_v^f - \widehat{g}_a^f),$$

$$R_Z^{f2} = \kappa \, (\widehat{g}_v^e - \widehat{g}_a^e)(\widehat{g}_v^f - \widehat{g}_a^f),$$

$$R_Z^{f3} = \kappa \, (\widehat{g}_v^e - \widehat{g}_a^e)(\widehat{g}_v^f + \widehat{g}_a^f), \qquad (10)$$

with

$$\kappa = \frac{G_\mu m_Z^2}{\sqrt{2}\, 2\pi\alpha}. \qquad (11)$$

Interpreting the couplings, $\widehat{g}_v$ and $\widehat{g}_a$, as effective parameters, the weak corrections are absorbed in the couplings.

For leptonic channels, the cross-sections (eqs. (4)), or respectively, the helicity amplitudes (eqs. (3)), can all be measured separately, assuming lepton universality. However, for hadronic reactions the contributing flavours cannot all be separated. Therefore, one can only measure the sum of all contributions according to eq. (7), i.e. in terms of r_A^{had} and j_A^{had}. In previous model independent studies [1], terms relating to j_{tot}^{had} were evaluated using the Standard Model relation

$$\widehat{g}_v^f = \widehat{g}_a^f \left(1 - 4|Q_f|\sin^2\widehat{\theta}_W\right), \qquad (12)$$

with $\sin^2\widehat{\theta}_W$ taken from the leptonic lineshape.

5. S matrix analysis

We used the program SMATASY [14] together with ZFITTER version 4.53 [7]. SMATASY relies on the S matrix ansatz for the total cross-section and for the three asymmetries. It is a generalization of the existing ZFITTER branch ZUSMAT, which considers only the total cross-section. Initial and final state QED corrections are taken into account by convolution in $\mathcal{O}(\alpha^2)$, higher order corrections for initial state radiation are considered with common

photon exponentiation. Interference between initial and final state is neglected for radiative corrections.

The data listed in ref. [1] have systematic uncertainties in addition to their statistical errors. These are caused by selection bias, theoretical uncertainties, limited Monte Carlo statistics etc. We consider a partial error correlation calculating a χ^2,

$$\chi^2 = \Delta^T V^{-1} \Delta, \qquad (13)$$

where Δ is a column vector with elements such as $(\sigma^{th} - \sigma^{exp})$ and $(A^{th} - A^{exp})$ and V is the $N \times N$ error correlation matrix between measurements. The diagonal elements of V are given by the quadratic sum of statistical and systematic errors, while the off-diagonal elements are given by the product of the common systematic errors. This can be generalized also to the common systematic error between different data sets. The procedure to implement the LEP energy uncertainty is described in detail elsewhere [15].

6. Results

To study the influence of the γZ interference on the final results of $\overline{m}_Z$ and $\overline{\Gamma}_Z$ the fits in the following sections are performed in two steps:

(a) All parameters except the photon exchange, r_A^γ, are left free.

(b) In addition to r_A^γ, the contributions to the γZ interference, j_{tot}^{had}, are fixed to the value expected by the Standard Model.

In order to reduce the number of free parameters, lepton universality is assumed. The photon exchange term, r_{tot}^γ (see eq. (2)), is fixed using the running coupling constant value at LEP energies, $|\alpha^{-1}(s)| = 128.8$. The quality of all fits is good. The χ^2 per degree of freedom varies between 0.75 and 0.78. The results of the S matrix approach are compared to those of the model independent ansatzes of ZFITTER.

6.1. A fit to the total cross-section and forward–backward asymmetry

We perform a fit to the leptonic and hadronic cross-section data and the leptonic forward–backward asymmetries according to eq. (5). Assuming lepton universality, one gets for case (a) 8 and for (b) 6 free parameters: $\overline{m}_Z$, $\overline{\Gamma}_Z$, and r_{tot}^{had}, j_{tot}^{had} for hadrons and

Table 1
Results of the S matrix fit to total cross-sections and forward–backward asymmetries: (a) all parameters except the photon exchange are left free; (b) in addition the γZ interference terms are fixed to the Standard Model expectation.

Parameter	Case (a)	Case (b)
$\overline{m}_Z$ (GeV)	91.152 ± 0.015	91.160 ± 0.010
$\overline{\Gamma}_Z$ (GeV)	2.494 ± 0.012	2.492 ± 0.012
r_{tot}^{lep}	0.141 ± 0.002	0.140 ± 0.002
j_{tot}^{lep}	0.032 ± 0.064	fixed to 0.0058
r_{fb}^{lep}	0.004 ± 0.001	0.004 ± 0.001
j_{fb}^{lep}	0.674 ± 0.087	0.675 ± 0.087
r_{tot}^{had}	2.859 ± 0.030	2.855 ± 0.029
j_{tot}^{had}	0.720 ± 0.700	fixed to 0.219

Table 2
Results of the S matrix fit to total cross-sections, forward–backward asymmetries and τ polarization: (a) all parameters except the photon exchange are left free; (b) in addition the hadronic γZ interference terms for the total cross-section are fixed to the Standard Model expectation.

Parameter	Case (a)	Case (b)
$\overline{m}_Z$ (GeV)	91.155 ± 0.013	91.160 ± 0.010
$\overline{\Gamma}_Z$ (GeV)	2.494 ± 0.012	2.492 ± 0.012
R_Z^{lep0}	0.429 ± 0.012	0.429 ± 0.012
R_Z^{lep1}	-0.370 ± 0.003	-0.370 ± 0.003
R_Z^{lep2}	0.323 ± 0.016	0.323 ± 0.016
r_{tot}^{had}	2.860 ± 0.030	2.856 ± 0.029
j_{tot}^{had}	0.620 ± 0.620	fixed to 0.219
m_Z (GeV)	91.189 ± 0.013	91.194 ± 0.010
Γ_Z (GeV)	2.495 ± 0.012	2.493 ± 0.012
$\widehat{g}_v^{lep}$	-0.037 ± 0.010	-0.037 ± 0.010
$\widehat{g}_a^{lep}$	-0.4991 ± 0.0019	-0.4988 ± 0.0019
$\sin^2 \widehat{\theta}_W$	0.2317 ± 0.0037	0.2316 ± 0.0037

r_{tot}^{lep}, j_{tot}^{lep}, r_{fb}^{lep}, j_{fb}^{lep} for leptons. The results are shown in table 1. Comparing both fits, one notices that $\overline{m}_Z$ increases by 8 MeV, when the γZ interference terms are fixed in fit (b), whereas the error on $\overline{m}_Z$ decreases by 5 MeV. That means that fixing the γZ interference introduces a systematic bias in the determination of the Z mass. For all the other parameters the mean values and errors remain unchanged. If one compares the values for $\overline{m}_Z$ with the results for m_Z, determined with ZFITTER (see table 3), one finds the expected offset between $\overline{m}_Z$ and m_Z of -34 MeV, only when the γZ interference term for the hadron channel j_{tot}^{had} is treated in the same way.

6.2. Determining the helicity amplitudes with the S matrix approach

In addition to the data set used in section 6.1, the τ polarization measured in 1991 is also taken into account. Assuming lepton universality and CP conservation three independent helicity amplitudes R_Z^{lep0}, R_Z^{lep1} and R_Z^{lep2} can be determined corresponding to eqs. (3). It should be noted here, that in the τ channel one has the possibility to determine all four amplitudes, because one can measure the total cross-section and all three asymmetries. The amplitudes R_Z^{lep1} and R_Z^{lep3} are equal by time reversal symmetry. The available information is not sufficient to express r_{tot}^{had} and j_{tot}^{had} by helicity amplitudes. Therefore, they still remain independent parameters. The number of free parameters is 7 in case (a) and 6 in case (b). For the

fit we assume the helicity amplitudes to be real.

The first part of table 2 shows the results. The second part of table 2 shows quantities which are derived from the parameters in the upper part of the table. m_Z, Γ_Z and $\overline{m}_Z$, $\overline{\Gamma}_Z$ are related by eq. (9). Refering to eqs. (10) one can write the amplitudes $R_Z^{lep i}$ as function of the couplings $\widehat{g}_v^{lep}$ and $\widehat{g}_a^{lep}$. Here we use R_Z^{lep0} and R_Z^{lep1} to determine the couplings and $\sin^2 \widehat{\theta}_W$, defined by eq. (12).

For the helicity fit one finds the same behaviour for $\overline{m}_Z$ as in the previous section. The error for $\overline{m}_Z$ increases when j_{tot}^{had} is left free. The mean values and errors for all other parameters are almost unchanged.

As a cross check we compare the results of the S matrix approach with the ZFITTER results using the same measurements as for the helicity fit. In ZFITTER two alternative model independent ansatzes are applied: the first is based on the assumption of real vector and axial vector couplings of the Z boson to fermions; the second relies on the assumption that scattering through the Z boson may be considered as subsequent formation and decay of a resonance described by the widths into the initial and final state fermions. The complications due to the handling of the γZ interference contribution for the hadronic final state, j_{tot}^{had}, are solved by fixing it to the value expected in the Standard Model. In order to check this proce-

Table 3
Results of a model independent fit to total cross-sections, forward–backward asymmetries and τ polarization using ZFITTER: (a) all parameters except the photon exchange are left free; (b) in addition the hadronic γZ interference terms for the total cross-section are fixed to the Standard Model expectation.

Parameter	Case (a)	Case (b)
m_Z (GeV)	91.187 ± 0.013	91.194 ± 0.009
Γ_Z (GeV)	2.492 ± 0.012	2.490 ± 0.012
$\widehat{g}_v^{\text{lep}}$	-0.040 ± 0.006	-0.040 ± 0.006
$\widehat{g}_a^{\text{lep}}$	-0.4989 ± 0.0016	-0.4986 ± 0.0016
Γ_{had}	1.751 ± 0.011	1.750 ± 0.011
$j_{\text{tot}}^{\text{had}}$	1.00 ± 0.86	fixed to 0.219
$\sin^2 \widehat{\theta}_W$	0.2300 ± 0.0030	0.2300 ± 0.0029

dure a five parameter fit to the leptonic and hadronic cross-section data and the leptonic forward–backward asymmetry and the τ polarization is performed.

In case (a) the term $j_{\text{tot}}^{\text{had}}$ is left free, and in case (b) the standard ZFITTER code is used with the fixed $j_{\text{tot}}^{\text{had}}$. The results are shown in table 3. The second part of table 3 shows $\sin^2 \widehat{\theta}_W$ derived from $\widehat{g}_v^{\text{lep}}$ and $\widehat{g}_a^{\text{lep}}$.

If one modifies the standard ZFITTER program to allow for a fit to the hadronic interference term $j_{\text{tot}}^{\text{had}}$, as it is done for the results (a) in table 3, one also finds that the mean value for m_Z decreases and that the error increases with respect to the standard ZFITTER results (b), by the same amount observed for the fits using the S matrix formalism.

A comparison of the second part of table 2 with table 3 shows that one gets the same results for the S matrix approach as with ZFITTER. The S matrix approach can, however, reproduce the mean value and error for m_Z only if the treatment of $j_{\text{tot}}^{\text{had}}$ is identical to the standard ZFITTER.

7. Conclusions

– The S matrix approach allows a general model independent investigation of the cross-sections and asymmetries measured in the vicinity of the Z resonance and a determination of the mass and the width of the Z boson.
– The S matrix approach can reproduce, with additional assumptions, the results of other model inde-

pendent ansatzes realized in ZFITTER.
– For model independent determinations, the fixing of the hadronic interference term for the total cross-section to the Standard Model expectation value leads to a systematic bias in the value of the Z mass and underestimates the systematical error on m_Z. This effect is also observed for ZFITTER. All other parameter are independent of $j_{\text{tot}}^{\text{had}}$. Although the γZ interference contribution to the hadronic final state, $j_{\text{tot}}^{\text{had}}$, is suppressed and its measurement is very poor, the influence of $j_{\text{tot}}^{\text{had}}$ on the value m_Z is not negligible. The procedure of fixing $j_{\text{tot}}^{\text{had}}$ by expressions predicted by the Standard Model should be checked when performing model independent fits to avoid misinterpretation of the results. An improved measurement of $j_{\text{tot}}^{\text{had}}$ is expected by running with higher luminosity at energies off the resonance (see [9]).

The results of the S matrix approach confirm the present values for m_Z and Γ_Z within their errors. The good agreement of the S matrix approach with the Standard Model fit values means that there is no evidence for new physics in the data.

Acknowledgement

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