Chapter 2

Credit Frictions and the Comovement between Durable and Non-durable Consumption

Abstract

Frictions in lending between households have been proposed as a solution to the difficulties New-Keynesian models have in predicting a decline in both durable and non-durable consumption following a monetary tightening. By revisiting a standard New-Keynesian framework with collateral constraints, it is shown that the presence of such credit frictions in fact makes it more difficult to generate the joint decline. The intuitive reasons behind this result are provided, which should be helpful in developing models that are more successful in generating a positive comovement between durables and non-durables.

2.1 Introduction

An undesirable feature of standard New-Keynesian models is that they tend to generate counterfactual comovements between durable and non-durable consumption, as pointed out by Barsky, House, and Kimball (2003, 2007). For low levels of durable price stickiness, these models typically predict that during a monetary contraction, non-durable purchases will decrease, while durable purchases will, remarkably, *increase*. In the case of fully flexible durable prices, the predicted expansion in the durable goods producing sector is so large that the monetary tightening has almost no effect on total aggregate output.[1] These

[1] The literature typically focuses on completely flexible durable prices, for which case the comovement problem is most severe. According to Barsky, House, and Kimball (2007), prices of new homes are

predictions are in sharp contrast with the conventional wisdom and empirical evidence that especially durable consumption falls during a monetary tightening.[2]

Barsky, House, and Kimball (2003, BHK henceforth) suggest that one reason why standard models have difficulties matching the empirical evidence could be that they assume frictionless financial markets. After a monetary tightening credit constraints may become tighter, and a reduced ability to borrow could then force credit-constrained households to decrease durable purchases. Monacelli (2009) formalizes this argument by extending the standard model to feature credit-constrained households, which in equilibrium borrow from households that are relatively patient. He shows that *if* one also allows for a moderate degree of price-stickiness in the durable goods producing sector, his model is able to generate a positive comovement between durables and non-durables.

This chapter revisits the framework of Monacelli (2009) and disentangles the contribution of the credit frictions from the effects that arise from the assumption that durable prices are somewhat sticky. By comparing the results of his model to a stripped-down version without frictions in financial markets, it is shown that without credit frictions it is *easier* to generate a positive comovement, that is, less stickiness of durable prices is needed. In the model of Monacelli (2009), credit-constrained households do, in fact, reduce their durable purchases after a monetary tightening. But the lending households increase their durable purchases so much that the response of aggregate durable purchases is more positive than in the version of the model without credit frictions. Also, in the case of fully flexible durable prices, the presence of credit frictions leads to a *positive* response of total aggregate output to a monetary tightening, whereas the model without credit frictions predicts a flat response.

To understand why the frictions in the market for household loans are unhelpful in solving the comovement problem, it is important to keep in mind that standard credit

arguably flexible because they are usually the outcomes of negotiations. Bils and Klenow (2004) report a median price duration of only two months for new cars.

[2]For empirical evidence on the effects of monetary shocks on durable and non-durable consumption, see for example Bernanke and Gertler (1995), Barsky, House, and Kimball (2003), and Monacelli (2009).

frictions, including those considered by Monacelli (2009), do not eliminate equilibrium in the bond market. If borrowing by the credit-constrained households is reduced as a consequence of tighter credit constraints, then the bond market will only remain in equilibrium if lending by the other households decreases by the exact same amount. Since buying durables is an alternative way of saving, the lending households can avoid a large revision of their intertemporal plans by purchasing more durables instead of saving through bonds. Therefore, the forced reduction in borrowing can be expected to have a limited effect on aggregate consumption of durables and non-durables.[3] In the model of Monacelli (2009), additional undesirable effects are generated by the fact that the borrowers' incentives to buy durables depend positively on the tightness of the borrowing constraint, whereas the other households do not face a binding credit constraint. This chapter demonstrates analytically that it is precisely this feature, that causes Monacelli's model to have more difficulties in generating a negative response of durable purchases to a monetary tightening, than a version of the model without credit frictions. Models with additional forms of heterogeneity between borrowers and lenders could offer better hopes of solving the comovement puzzle.

2.2 Two sticky-price models with consumer durables

Two New-Keynesian models with durable and non-durable consumption are analyzed. The first one replicates the credit friction model of Monacelli (2009), which describes a standard sticky-price economy augmented with collateral constraints and heterogeneous households that borrow and lend.[4] The second one is the same model, but with household heterogeneity eliminated so that it reduces to a standard New-Keynesian model in which credit frictions are not relevant.

[3]The importance of general equilibrium effects has been demonstrated in a different context by Thomas (2002). In her model, the effects of lumpy firm investment on aggregate quantities vanish because of the behavior of agents on the other side of the *goods* market, that is, the behavior of consumers.

[4]For a more detailed description of this model, the reader is referred to Monacelli (2009).

2.2.1 The model with credit frictions

The credit friction model features two types of households with different rates of time preference, that is, one group of households is more patient than the other. Because in equilibrium the impatient households borrow from the patient households, they are referred to as *borrowers* and *savers*, respectively. Borrowing by the impatient households is restricted by a collateral constraint, which guarantees the existence of a well-defined steady state.[5] The size of the total population is normalized to one and the fraction of borrowers is set equal to ϖ.

Borrowers. Impatient households maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, D_t, N_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(\left[(1-\alpha)^{\frac{1}{\eta}} (C_t)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (D_t)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}) - \frac{\nu N_t^{1+\varphi}}{1+\varphi} \right\},$$

where C_t is non-durable consumption, D_t is the *stock* of durable goods, N_t is labor supply, and α, η, ν, and φ are preference parameters. In every period, the borrowers face the following constraints:

$$P_{c,t} C_t + P_{d,t} \left(D_t - (1-\delta) D_{t-1} \right) + R_{t-1} B_{t-1} = B_t + W_t N_t, \tag{2.1}$$

$$R_t B_t \leq (1-\chi)(1-\delta) E_t \left\{ D_t P_{d,t+1} \right\}, \tag{2.2}$$

where $P_{c,t}$ is the price of non-durables, $P_{d,t}$ is the price of durables, R_t is the gross nominal interest rate, B_t is the nominal amount of debt, and W_t is the nominal wage. Equation (2.1) is the budget constraint. Equation (2.2) is a collateral constraint and states that the level of debt must be such that debt servicing in the next period cannot exceed a fraction $(1-\chi)$ of the expected value of the depreciated current durable stock one period ahead. Therefore, χ can be interpreted as a downpayment requirement. It is assumed

[5] Similar constraints are considered in Kiyotaki and Moore (1997) and Iacoviello (2005).

that the collateral constraint always binds.[6] Let ψ_t be defined as the ratio of the Lagrange multiplier of the borrowing constraint to the Lagrange multiplier of the budget constraint. The optimality conditions of the borrowers' maximization problem are:

$$\frac{-U_{n,t}}{U_{c,t}} = w_t, \tag{2.3}$$

$$q_t U_{c,t} = U_{d,t} + \beta (1-\delta) E_t \{U_{c,t+1} q_{t+1}\} + (1-\chi)(1-\delta) U_{c,t} q_t \psi_t E_t \{\pi_{d,t+1}\}, \tag{2.4}$$

$$R_t \psi_t = 1 - \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{R_t}{\pi_{c,t+1}} \right\}, \tag{2.5}$$

where $-U_{n,t}$, $U_{d,t}$ and $U_{c,t}$ are, respectively, the marginal utilities of leisure, durables and non-durables, $w_t \equiv W_t/P_{c,t}$ is the real wage in units of non-durables, $q_t \equiv P_{d,t}/P_{c,t}$ is the relative price of durables, and $\pi_{j,t}$ is gross inflation in sector j, with $j \in \{\text{non-durables, durables}\}$. Equation (2.3) is the standard optimality condition for labor, which equates the marginal utility of leisure to the product of the real wage and the marginal utility of non-durable consumption. The right-hand side of Equation (2.4) is the *shadow value of durables* to the borrowers, which is the sum of the immediate utility gain they derive from a marginal unit of durables, the discounted expected value of the undepreciated part of the durable next period, and a term reflecting their utility gain from the additional borrowing capacity. This last term is proportional to ψ_t, which measures the tightness of the borrowing constraint. At the optimum, the shadow value of durables must be equal to the marginal utility gain that is derived from buying q_t non-durable goods. Equation (2.5) is the first-order condition for debt.

Savers. The patient households or savers have a discount factor $\gamma > \beta$ and their variables are characterized by a tilde. Savers receive all firm profits, so their budget

[6] It can be shown that the borrowing constraint is binding in the deterministic steady state. This paper checks the validity of the assumption that it also binds outside the steady state by performing an accuracy test. The test is described in the Appendix 2.A. The results show that the assumption is not problematic for the calibration considered in this paper.

reads:

$$P_{c,t}\widetilde{C}_t + P_{d,t}\widetilde{I}_{d,t} + R_{t-1}\widetilde{B}_{t-1} = \widetilde{B}_t + W_t\widetilde{N}_t + \frac{\Pi_t}{(1-\varpi)}, \tag{2.6}$$

where Π_t is total nominal firm profits. The optimality conditions of savers and borrowers are very similar, with the important difference that to the savers the collateral constraint is not relevant, since they are net lenders. This implies that $\widetilde{\psi}_t = 0$ and consequently, the durable first-order condition for the savers can be written as:

$$q_t\widetilde{U}_{c,t} = \widetilde{U}_{d,t} + \gamma(1-\delta)E_t\left\{\widetilde{U}_{c,t+1}q_{t+1}\right\}. \tag{2.7}$$

Comparing the right hand sides of Equations (2.4) and (2.7) makes clear that the shadow value of durables is fundamentally different for borrowers and savers, as the latter are not restricted by a collateral constraint.

Firms. Final goods producers create bundles from intermediate goods according to the Dixit-Stiglitz aggregator. The final durable and non-durable goods are sold to the households. Intermediate goods firms face a quadratic cost of price adjustment, following Rotemberg (1982). The output of intermediate goods producer i in sector j is simply equal to labor input, that is, $Y_{j,t}(i) = N_{j,t}(i)$. For a symmetric equilibrium, the optimality conditions of the intermediate non-durable and durable producers can be written, respectively, as:

$$(1-\varepsilon_c) + \varepsilon_c w_t = \vartheta_c(\pi_{c,t}-1)\pi_c - \gamma\vartheta_c E_t\left[\frac{\widetilde{U}_{c,t+1}}{\widetilde{U}_{c,t}}\frac{Y_{c,t+1}}{Y_{c,t}}(\pi_{c,t+1}-1)\pi_{c,t+1}\right], \tag{2.8}$$

$$(1-\varepsilon_d) + \varepsilon_d\frac{w_t}{q_t} = \vartheta_d(\pi_{d,t}-1)\pi_{d,t} - \gamma\vartheta_d E_t\left[\frac{\widetilde{U}_{c,t+1}}{\widetilde{U}_{c,t}}\frac{q_{t+1}}{q_t}\frac{Y_{d,t+1}}{Y_{d,t}}(\pi_{d,t+1}-1)\pi_{d,t+1}\right] \tag{2.9}$$

where $Y_{j,t}$ is output in sector j, ε_j is the elasticity of substitution between intermediate goods and ϑ_j is the price adjustment cost parameter.[7] When price adjustment costs are

[7] Firms discount future profits by the stochastic discount factor of the savers, i.e., the savers are their owners.

zero, prices are set according to a constant markup over nominal marginal costs, which is the nominal wage in this model. Thus, when durable prices are fully flexible, i.e., when $\vartheta_d = 0$, the real wage in units of durables w_t/q_t is constant and from Equation (2.9) it can be seen to equal $(\varepsilon_d - 1)/\varepsilon_d$.

Market clearing conditions and monetary policy. Clearing of the markets for non-durables, durables, bonds and labor requires:

$$Y_{c,t} - \frac{\vartheta_c}{2}(\pi_{c,t} - 1)^2 Y_{c,t} = \varpi C_t + (1-\varpi)\widetilde{C}_t, \tag{2.10}$$

$$Y_{d,t} - \frac{\vartheta_d}{2}(\pi_{d,t} - 1)^2 Y_{d,t} = \varpi (D_t - (1-\delta) D_{t-1}) + (1-\varpi)\left(\widetilde{D}_t - (1-\delta)\widetilde{D}_{t-1}\right), \tag{2.11}$$

$$0 = \varpi B_t + (1-\varpi)\widetilde{B}_t, \tag{2.12}$$

$$Y_{c,t} + Y_{d,t} = \varpi N_t + (1-\varpi)\widetilde{N}_t. \tag{2.13}$$

The model is closed by the following monetary policy rule:

$$\frac{R_t}{R} = \left(\frac{\widetilde{\pi}_t}{\widetilde{\pi}}\right)^{\xi_\pi} \exp(\varepsilon_t), \tag{2.14}$$

where $\widetilde{\pi}_t = \pi_{c,t}^{1-\tau}\pi_{d,t}^{\tau}$ is a composite inflation index, R and $\widetilde{\pi}$ are the steady-state levels of the nominal interest rate and the inflation index, respectively, and ε_t is an exogenous shock.[8]

2.2.2 The model without credit frictions

The model with credit frictions can be modified to obtain a standard New-Keynesian model without credit frictions, by simply setting the fraction of borrowers, ϖ, equal to zero. So the model without credit frictions features a representative household, behaving like the savers in the model with credit frictions. With heterogeneity across households eliminated, debt equals zero in equilibrium and collateral constraints are irrelevant.

[8] In order to enhance comparability with Monacelli (2009), I use the same monetary policy rule.

As explained by BHK (2007), the comovement problem is driven by a key property of standard representative household models, which is the quasi-constancy of the shadow value of durables. This means that the household cares little about the timing of durable purchases. Recall that the shadow value of durables is the right-hand side of the durable optimality condition (2.7) and note that this equation can be rewritten as follows:

$$q_t \widetilde{U}_{c,t} = E_t \sum_{k=0}^{\infty} \gamma^k (1-\delta)^k \widetilde{U}_{d,t+k} \approx const. \tag{2.15}$$

The reason the shadow value of durables for the representative agent is quasi-constant, is that the marginal utility of durables depends on the *stock* of durables, which is not much affected by variations in the *flow* of durables. Also, the shadow value of durables depends for an important part on the marginal utility of durables in the distant future, which is even less sensitive to temporary shocks. Because the shadow value of durables is near-constant, the relative price of durables q_t and the marginal utility of non-durable consumption $\widetilde{U}_{c,t}$ move in opposite directions. If prices of durables are flexible relative to prices of non-durables, the relative price of durables q_t falls during a monetary tightening, creating more incentives for households to purchase durables. At the same time, the decrease in q_t must be accompanied by an increase in the marginal utility of non-durables $\widetilde{U}_{c,t}$, which is associated with a lower level of non-durable consumption.

More insight in the comovement problem can be obtained by considering the special case of fully flexible durable prices. In the absence of durable price adjustment costs, the real wage in units of durables, w_t/q_t, is constant. As a consequence, monetary policy shocks are neutral with respect to total real activity. This follows from the labor optimality condition, which can be expressed as a condition equating the marginal utility of leisure to the product of the real wage in units of durables w_t/q_t and the shadow value of durables, which equals $q_t\widetilde{U}_{c,t}$:

$$-\widetilde{U}_{n,t} = \frac{w_t}{q_t} q_t \widetilde{U}_{c,t}.$$

Given that the real wage in units of durables is constant and the shadow value of durables is quasi-constant, the same holds for total employment (and total output). Also, the relative durable price q_t decreases after a monetary tightening, and the quasi-constancy of $q_t\widetilde{U}_{c,t}$ implies that non-durable consumption also decreases. Since total employment remains roughly constant, the production of durables must expand. Hence, durable purchases comove negatively with non-durable purchases.[9]

2.3 Comparing the two models

A solution to the comovement problem requires a model that predicts a fall in both non-durable and durable purchases following a monetary tightening as well as a rise of the nominal interest rate.[10] Monacelli (2009) shows that his credit friction model with moderate price-stickiness in the durable goods sector is able to generate these predictions. In this section, the results of Monacelli's model are compared to those of a version of the model *without* credit frictions, under the same calibration and normalization of the steady state.[11] The model is solved using a first-order perturbation method in logarithms, which allows one to exploit the observational equivalence between the log-linearized versions of the Rotemberg model with quadratic price adjustment costs and the Calvo-Yun model. The parameter values are displayed in Table 2.1.

Figure 2·1 plots, for both models, the Impulse Response Functions (IRFs) of the nominal interest rate, of aggregate durable and non-durable purchases, and of aggregate

[9] This argument abstracts from resources lost because of price changes. In a log-linearized version of the model, these losses are equal to zero.

[10] It is important to consider the nominal interest rate, because increasing price stickiness in the durable goods sector is helpful in solving the comovement problem, but it actually makes it more difficult to generate a realistic response of the nominal interest rate, as shown by BHK (2007) and Monacelli (2009).

[11] To facilitate comparison to the numerical results in Monacelli (2009), the parameter τ reflecting the weight of durables in the composite inflation index in the monetary policy rule is set to zero. This means that monetary policy only responds to inflation in the non-durable goods sector. Appendix 2.B investigates the consequences of adopting the more realistic assumption that monetary policy also responds to prices of durables. This is shown to make it more difficult to obtain the desired comovements in the model without credit frictions, and even impossible in the model with credit frictions. Appendix 2.B also discusses results under a monetary policy rule that responds to both output and inflation.

Table 2.1: Parameter settings

parameter	description	Model with credit frictions	Standard model
β	discount factor borrowers	0.98	-
γ	discount factor savers	0.99	0.99
δ	depreciation rate durables	0.01	0.01
ε_c	el. of subst. between nondurable varieties	6	6
ε_d	el. of subst. between durable varieties	6	6
η	el. of subst. between durables and nondurables	1	1
ξ_π	coefficient on inflation in monetary policy rule	1.5	1.5
ϖ	share of borrowers	0.5	0
ρ	persistence parameter monetary policy shocks	0.5	0.5
χ	parameter in borrowing constraint	0.25	-
φ	inverse elasticity of labor supply	1	1

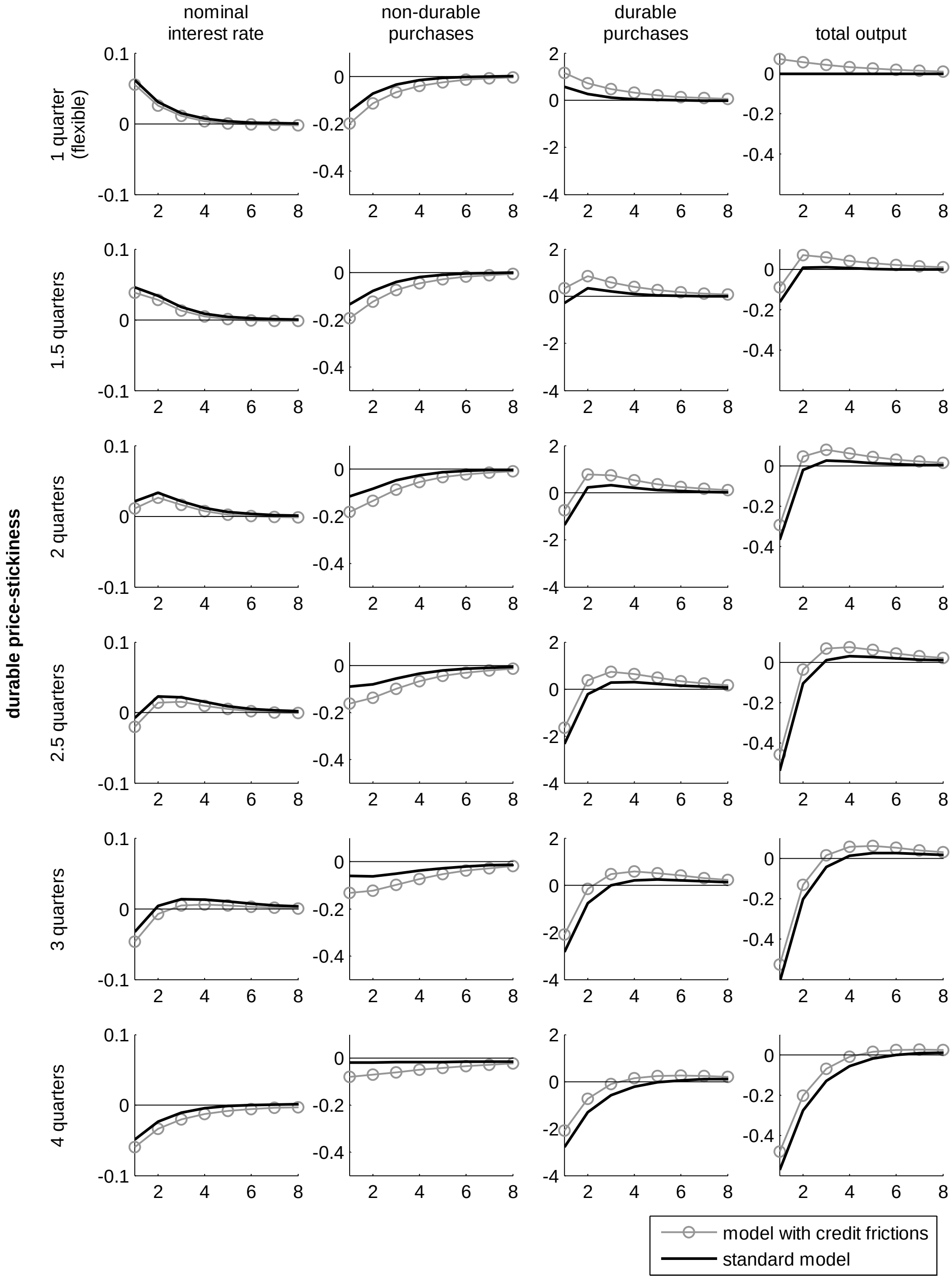
Notes: “Model with credit frictions” refers to the model of Monacelli (2009) that features patient households, who lend to impatient households who are at a credit-constraint. “Standard model” refers to the representative household model without credit frictions and with zero debt in equilibrium.

total output in reaction to a monetary tightening. Each row corresponds to a different level of durable price-stickiness.[12]

First consider the model with credit frictions. The top row shows the results for the case of fully flexible durable prices. The responses of durable and non-durable purchases display the comovement problem as reported in the literature. But whereas BHK (2007) found that in their model with frictionless financial markets total output remains almost constant after a monetary tightening, the model with credit frictions even predicts an *increase* in total output, which is at odds with the typical decrease found in empirical studies. The IRFs in the bottom row correspond to four-quarter durable price-stickiness, in which case prices of durables and non-durables are equally sticky. For this calibration, both durable and non-durable purchases decrease, but the nominal interest rate moves in the wrong direction. The figure also replicates the finding of Monacelli (2009) that there exists a small range of intermediate levels of durable price-stickiness for which the model

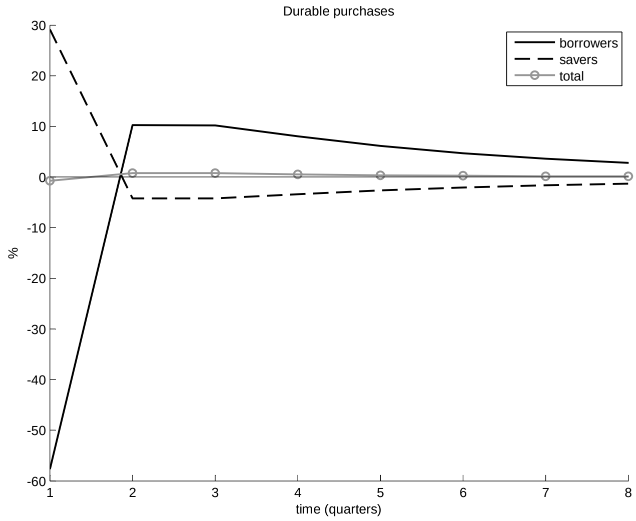
[12] In the context of the model, a monetary tightening is defined as an increase in the exogenous shock variable ε_t, even though the nominal interest rate actually goes down for some calibrations.

Figure 2·1: Responses to a monetary tightening in the model with and without credit frictions for the first eight quarters. The rows correspond to different degrees of durable price stickiness. Responses are plotted as percentage deviations from the steady state.



Notes: In the literature on models with durables, it is common to sum output components by their steady state prices, see for example BHK (2007) and Iacoviello and Neri (2009). Following this tradition, the output measure is constructed as $Y_{c,t} + qY_{d,t}$, where q is the relative price of durables in the steady state, which equals one.

Figure 2·2: Responses to a monetary tightening of total durable purchases, as well as durable purchases by borrowers and by savers in the model with credit frictions.



Notes: Durable price stickiness is set to two quarters. Responses are plotted as percentage deviations from the steady state.

is successful in generating a positive comovement between durables and non-durables, as well as an increase in the nominal interest rate.

Now consider the model without credit frictions. The IRFs in the top row show that the version of the model without credit frictions also predicts a negative comovement between durable and non-durable purchases in the flexible durable price case. Without credit frictions, however, total output remains constant instead of displaying the counterfactual increase after a monetary tightening. The IRFs for the case where durable prices are as sticky as non-durable prices (the bottom row), show that the nominal interest rate also moves in the wrong direction in the model without credit frictions. But most importantly, the figure shows that, for any level of durable price-stickiness, removing the credit frictions leads to a less negative response of non-durable purchases and a less positive response of durable purchases. It is, thus, more difficult to generate the positive comovement in the model with credit frictions, as one needs to rely on a larger degree of durable price-stickiness.

To obtain more insight in this result, consider the responses for the borrowers and savers separately, as displayed in Figure 2·2 for two-quarter durable price-stickiness. The figure confirms the intuition that during a monetary contraction, constrained households reduce their amount of durable purchases. However, the figure also shows that this decline is accompanied by an increase in durable purchases by the savers. The results presented in Figure 2·1 make clear that this increase is so large that the response of *aggregate* durable purchases is more positive than in the model without credit frictions.[13]

2.4 Why do the credit frictions make the comovement problem more severe?

In Section 2.3 the model and calibration proposed by Monacelli (2009) were revisited, and it was shown that removing the credit frictions from the model makes it easier to generate a positive comovement between durables and non-durables. The purpose of the current section is to show that there is a simple reason for this undesirable result, and that one can expect this mechanism to be at play more generally in models with collateral constraints. At the center of the analysis is the fact that collateral constraints fundamentally affect the shadow value of durables for borrowers, but not for the other households. It is shown analytically that as a consequence the credit friction model generates (i) nearly the same responses of prices and wages as the model without credit frictions, (ii) a more positive response of aggregate employment, (iii) a more negative response of aggregate non-durable consumption. These results imply a more positive response of aggregate durable purchases to a monetary tightening.

For the *savers*, the shadow value of durables remains *quasi-constant* following a monetary shock. Because the savers are not credit-constrained they face the same optimization

[13] Figure 2·2 reveils another problem of the model with credit frictions, namely that it predicts extremely large volatilities for durable purchases by the borrowers and savers. While aggregate durable purchases falls by less than 0.75%, the savers increase their durable purchase by more than 29%, while durable purchases by the borrowers decrease by more than 57%.

problem as the representative household in the model without credit-frictions. Hence, the reasoning in Section 2.2.2 explaining the quasi-constancy of the shadow value of durables in the model without credit frictions also applies to the savers in the credit friction model. As a consequence, the introduction of credit frictions leaves the responses of prices, wages and the nominal interest rate nearly unchanged. To see this, define $\widetilde{V}_t$ as the shadow value of durables to the households that do not face a binding collateral constraint. Since their optimality condition for durables states that $\widetilde{V}_t = q_t \widetilde{U}_{c,t}$, their Euler equation can be written as:

$$1 = \gamma E_t \left\{ \frac{\widetilde{V}_{t+1}}{\widetilde{V}_t} \frac{R_t}{\pi_{d,t+1}} \right\}. \tag{2.16}$$

Note, also, that when the shadow value of durables is constant, the same holds for the real interest rate in units of durables. Also, the definition of the relative price of durables implies that $q_t = \frac{\pi_{d,t}}{\pi_{c,t}} q_{t-1}$. Log-linearizing these two equations, as well both pricing equations for the intermediate goods firms and the monetary policy rule, gives

$$\begin{aligned}
\widehat{R}_t &= E_t \widehat{\pi}_{d,t+1} + \widehat{\widetilde{V}}_t - \widehat{\widetilde{V}}_{t+1}, && (2.17) \\
\widehat{q}_t &= \widehat{\pi}_{d,t} - \widehat{\pi}_{c,t} + \widehat{q}_{t-1}, && (2.18) \\
\widehat{\pi}_{c,t} &= \frac{\varepsilon_c - 1}{\vartheta_c} \widehat{w}_t + \gamma E_t \widehat{\pi}_{c,t+1}, && (2.19) \\
\widehat{\pi}_{d,t} &= \frac{\varepsilon_d - 1}{\vartheta_d} \left(\widehat{w}_t - \widehat{q}_t \right) + \gamma E_t \widehat{\pi}_{d,t+1}, && (2.20) \\
\widehat{R}_t &= \xi_\pi \widehat{\pi}_{c,t} + \varepsilon_t, && (2.21)
\end{aligned}$$

where hatted variables denote log deviations from the steady state. The fact that the shadow value of durables for households who are not at a credit constraint is quasi-constant, implies that $\widehat{\widetilde{V}}_t - \widehat{\widetilde{V}}_{t+1} \approx 0$. Ignoring this term in Equation (2.17) leaves one with a subsystem that is the same for the model with and without credit frictions, and that consists of five equations in five unknowns ($\widehat{\pi}_{c,t}$, $\widehat{\pi}_{d,t}$, $\widehat{q}_t$, $\widehat{R}_t$, $\widehat{w}_t$). It follows that the IRFs of the endogenous variables contained in this subsystem are nearly the same in the model with and without collateral constraints. The two variables that will play a role in the analysis below are the relative price of durables q_t and the real wage in units of durables, w_t/q_t. Figure 2·3 confirms that the IRFs for these two variables are indeed nearly equal for both models.

An important feature of the credit friction model is that for *borrowers*, the shadow value of durables *rises* during a monetary tightening. The reason is that the collateral constraint tightens and the possession of additional durables permits more borrowing, since durables serve as collateral for loans. To see this formally, rewrite the borrowers' optimality condition for durables (2.4) as follows:

$$V_t = q_t U_{c,t} = \frac{U_{d,t} + \beta (1-\delta) E_t \{V_{t+1}\}}{1 - (1-\chi)(1-\delta)\psi_t \{\pi_{d,t+1}\}}. \tag{2.22}$$

A tightening of the collateral constraint, reflected by an increase in ψ_t, will, *ceteris paribus*, lead to an increase in the shadow value of durables for the borrowers V_t.[14]

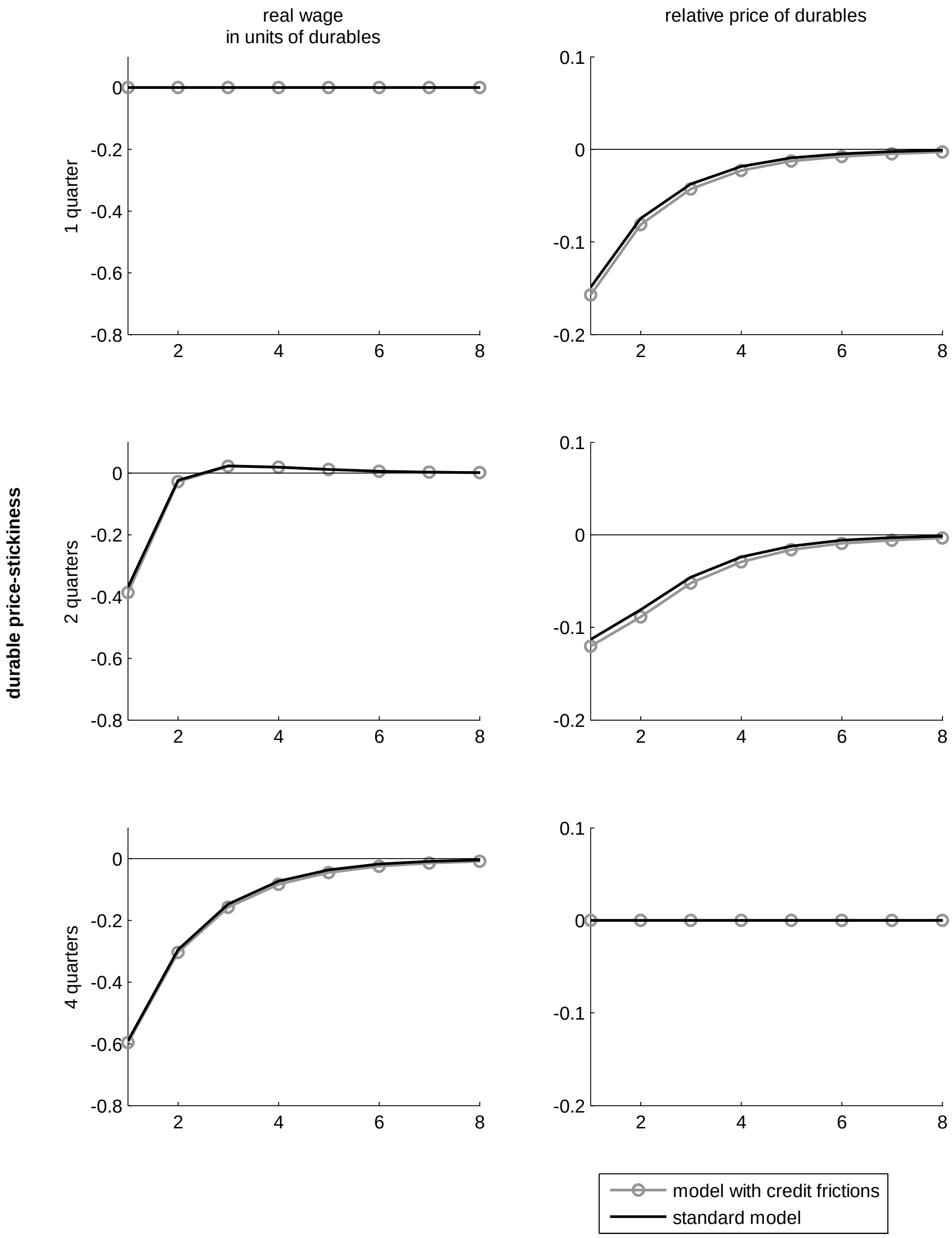
The rise in V_t explains why in the case of completely flexible durable prices, aggregate output increases after a monetary tightening in the model with credit frictions. To understand this, it is useful to combine the labor optimality conditions for the savers and the borrowers with the labor market clearing condition (2.13), in order to express total aggregate employment N_t^{agg} as:

$$N_t^{agg} \equiv \varpi N_t + (1-\varpi)\widetilde{N}_t = \frac{w_t}{q_t}\left[\frac{\varpi}{\nu} V_t + \frac{1-\varpi}{\widetilde{\nu}}\widetilde{V}_t\right]. \tag{2.23}$$

When durable prices are fully flexible, the real wage in units of durables w_t/q_t is constant, because durable prices are set according to a constant markup over the nominal wage. Given that $\widetilde{V}_t$ remains roughly constant, aggregate employment almost perfectly follows

[14] A monetary tightening also increases $U_{d,t}$, which pushes up V_t as well. However, this effect is relatively small, because variations in the stock of durables are small. Also, for sufficiently large levels of durable price stickiness, $E_t \{\pi_{d,t+1}\}$ decreases, which could offset the increase in ψ_t. However, given that the real interest rate in units of durables is quasi-constant, calibrations for which $E_t \{\pi_{d,t+1}\}$ increases are those for which the nominal interest rate decreases. In the numerical results this effect never dominates as V_t always increases.

Figure 2·3: Responses to a monetary tightening of the real wage in units of durables w_t/q_t and the relative price of durables q_t in the model with and without credit frictions for the first eight quarters.



Notes: The rows correspond to different degrees of durable price stickiness. Responses are plotted as percentage deviations from the steady state.

the rise in the shadow value of durables for borrowers, V_t, when a sudden monetary contraction takes place. Because labor is the only production input, the response of aggregate total output also increases.

In the more general case with possibly sticky durable prices, Equation (2.23) still explains why aggregate employment responds more positively in the model with credit frictions than in the version without. Recall that in both models $\widetilde{V}_t$ is quasi-constant and the response of w_t/q_t is nearly the same, whereas the shadow value for the borrowers, V_t, is only present in the model with credit frictions.[15] During a monetary tightening, V_t increases, shifting up the borrowers' labor supply curve. It follows from Equation (2.23) that this upward shift is behind the more positive response of aggregate employment (and output) in the credit friction model. The intuition is that during a monetary contraction the collateral constraint is particularly tight, which gives borrowers incentives to work more and use the additional wage income to dampen the decrease in durable purchases.

Similar logic can be applied to explain why the introduction of credit frictions lowers the response of aggregate non-durable consumption. Combine the durable optimality conditions for borrowers and savers with the non-durable market clearing condition (2.10) to express aggregate non-durable consumption, C_t^{agg}, as a function of the relative price q_t and the two shadow values of durables:

$$C_t^{agg} \equiv \varpi C_t + (1-\varpi)\,\widetilde{C}_t = q_t\,(1-\alpha)\left[\frac{\varpi}{V_t} + \frac{1-\varpi}{\widetilde{V}_t}\right]. \tag{2.24}$$

Given that in both models, with and without credit frictions, $\widetilde{V}_t$ is roughly constant and the response of q_t is the nearly the same, it is again the rise in V_t that drives the more pronounced decline of non-durable purchases in the credit friction model. For households that are not at a credit constraint, the quasi-constancy of the shadow value of durables implies that they decrease their non-durable purchases in proportion to the fall in the

[15] Recall that the model without credit frictions is obtained by removing the borrowers from the model, that is, by setting $\varpi = 0$.

relative price of durables. For credit-constrained households, however, there are additional reasons to substitute away from non-durables, because dampening the decline in durable purchases alleviates the tightening of the collateral constraint. Equation (2.24) makes clear that this is the reason for the stronger decline of aggregate non-durable purchases in the model with credit frictions.

It now follows that the introduction of collateral constraints leads to a more positive response of aggregate durable purchases. If the response of aggregate employment is more positive after adding credit frictions, and aggregate non-durable consumption falls more, then the aggregate resource constraint can only remain satisfied if production in the durable goods producing sector responds more positively. Consequently, the collateral constraints considered by Monacelli (2009) push the responses of durables and non-durables further in opposite directions, i.e., they make the comovement problem more severe.

2.5 Concluding comments

In the model of Monacelli (2009), borrowers are different from lenders, because they face a binding collateral constraint, and precisely this feature reduces the model's ability to solve the comovement puzzle. Introducing other forms of heterogeneity between borrowers and lenders could be a promising way forward in solving the comovement puzzle.[16] For example, Carlstrom and Fuerst (2006) show that adding frictions in lending by *firms* to households helps to generate a positive comovement between durables and non-durables.[17]

[16] This may even be true in a model with collateral constraints, but the effects introduced by the additional form of heterogeneity would have to overturn the effects introduced by the collateral constraints.

[17] They also show that the introduction of sticky wages helps to solve the comovement puzzle.

Appendix 2.A Accuracy test

Monacelli (2009) shows that the borrowing constraint binds in the steady state. Following standard practice in the literature, he *assumes* that the borrowing constraint *always* binds, so that the model can be solved using perturbation methods. This chapter also follows this tradition. However, the question arises whether the assumption of an always-binding constraint is correct for a realistic calibration of the model, including the standard deviation of the shocks. To the best of my knowledge, this issue has not been addressed in the literature, except in Iacoviello (2005).[18] This is surprising because the properties of the model are potentially very different if the constraint does not always bind.

The accuracy test is closely related to the standard test of Judd (1992) that checks Euler equation errors and it does not require the use of a global solution technique. The basic idea is to calculate how much debt the borrowers would choose under the assumption that the constraint does not bind and see how often the chosen amount is less than what is allowed by the constraint. That is, the accuracy test checks how often the constraint is *not* binding.[19] The test is carried out by implementing the following steps:

1. Solve the model using a perturbation method (e.g. log-linearization).
2. Using the solution found in step 1, run a simulation of the state variables, and index simulated variables by $t = 1, ..., T$.
3. At each point in the simulation, shut off the borrowing constraint by setting ψ_t equal to zero. In that case, the first order conditions of the borrowers can be rewritten to

[18] Iacoviello (2005) investigates the non-linear solution of a simplified partial equilibrium version of his model.

[19] The amount of debt chosen by the borrowers in the absence of a borrowing constraint can be calculated easily from the non-linear equilibrium conditions under the assumption that prices, wages, the nominal interest rate, the state variables and the conditional expectations are those predicted by the model with an always-binding constraint. That is, the assumption is that these variables are consistent with a binding constraint and the procedure checks whether the chosen debt level is consistent with a binding constraint as well.

find expressions for non-durables, durables and labor:

$$\text{from Equation (5)} \quad : \qquad C_t^* = \frac{1}{\beta R_t} E_t \left\{ \pi_{c,t+1} C_{t+1} \right\}, \tag{2.25}$$

$$\text{from Equation (4)} \quad : \qquad D_t^* = \frac{\alpha}{1-\alpha} \left(\frac{q_t}{C_t^*} - \beta \left(1-\delta\right) E_t \left\{ \frac{q_{t+1}}{C_{t+1}} \right\} \right)^{-1}, \tag{2.26}$$

$$\text{from Equation (3)} \quad : \qquad N_t^* = \left(\frac{w_t \left(1-\alpha\right)}{\nu C_t^*} \right)^{1/\varphi}, \tag{2.27}$$

where the stars indicate that variables are calculated under the assumption that the constraint does not bind. The values of R_t, q_t, and w_t are calculated using the policy functions found in step 1. The policy functions from step 1 are also used to approximate the conditional expectations by Gauss-Hermite quadrature.

4. Calculate rel debt from the budget constraint, with real debt defined as $b_t \equiv B_t / P_{c,t}$:

$$b_t^* = C_t^* + q_t \left(D_t^* - \left(1-\delta\right) D_{t-1} \right) + R_{t-1} \frac{b_{t-1}}{\pi_t} - w_t N_t^*,$$

and again use the policy functions found in step 1 to calculate q_t, D_{t-1}, R_{t-1}, b_{t-1}, π_t, and w_t.

5. Compare the level of debt in the absence of a binding borrowing constraint b_t^* to b_t, where b_t is the amount of debt chosen when the borrowing constraint always binds, which is calculated using the policy function found in step 1. If b_t^* is lower than b_t at some points in the simulation, then it can be concluded that the borrowing constraint is not always binding. I also investigate the consumption errors that arise from falsely assuming that the constraint binds, by comparing C_t^* to C_t and $I_{d,t}^*$ to $I_{d,t}$ at the points in the simulation where the constraint is found to be non-binding.

Running a simulation of the model requires further assumptions about the distribution of the innovations to the shocks. The assumption here is that they are normally distributed with mean zero and standard deviation σ_u, which is to be calibrated. With larger shocks, a non-binding constraint is a more likely outcome. The strategy followed here is to relate output volatility predicted by the model to output volatility in the data. Output volatility is estimated to be 0.0087 over the sample period 1988q1-2007q4.[20] To remain agnostic about the importance of monetary shocks, several values for the standard deviation of the monetary shocks are considered. These values are chosen such that output volatility predicted by the model is a certain percentage of output volatility in the data, ranging from 10% to 100%.

Another factor that affects the likelihood of the borrowing constraint to be non-binding, is the difference between the discount factor of the savers γ and the discount factor of the borrowers β. In the extreme case where the two types of households are equally patient, that is, when $\beta = \gamma$, the borrowing constraint will never be binding as debt equals zero in equilibrium. Thus, the closer the two discount factors are, the less likely it is that the borrowing constraint always binds. To investigate this issue quantitatively, I not only run the test with β equal to its benchmark value 0.98, but I also repeat the test with β equal to 0.985 and 0.989. These values are very close to the discount factor of the savers γ, which is equal to 0.99 in all calibrations.

To evaluate accuracy, criteria are needed. First define the variable I_t, indicating whether the constraint is non-binding:

$$
\begin{aligned}
I_t &= 1 \text{ if } b_t - b_t^* > 0, \\
I_t &= 0 \text{ otherwise,}
\end{aligned}
$$

[20] The data series used for output is real GDP at a quarterly frequency, taken from the website of the Bureau of Economic Analysis. The log of this series is detrended using the HP-filter with $\lambda = 1600$.

and then define the following criteria:

$$\begin{aligned}
\text{criterion 1} &\equiv 100 \times \frac{\sum_{t=1}^{T} I_t}{T}, \\
\text{criterion 2} &\equiv 100 \times \frac{\sum_{t=1}^{T} \frac{|b_t^* - b_t|}{b_t} I_t}{\sum_{t=1}^{T} I_t}, \\
\text{criterion 3} &\equiv 100 \times \frac{\sum_{t=1}^{T} \frac{|C_t^* - C_t|}{C_t} I_t}{\sum_{t=1}^{T} I_t}, \\
\text{criterion 4} &\equiv 100 \times \frac{\sum_{t=1}^{T} \frac{|D_t^* - D_t|}{D_t - (1-\delta) D_{t-1}} I_t}{\sum_{t=1}^{T} I_t}.
\end{aligned}$$

The first criterion is the percentage of the cases where the constraint is found to be non-binding. The second, third, and fourth criteria are the average relative errors in the amount of debt, non-durable purchases, and durable purchases by the borrowers, respectively, conditional on the event of a non-binding borrowing constraint.

Table 2.2 reports the results. For the benchmark calibration with $\beta = 0.98$, the solution under the assumption of an always-binding constraint turns out to be accurate in the sense that in none of the points in the simulation the constraint becomes non-binding, even if the standard deviation of monetary policy shocks is calibrated to be so large that monetary policy shocks explain all output volatility present in the data.[21] Not surprisingly, the table also shows that as β approaches γ, the constraint becomes binding more often, resulting in serious inaccuracies, especially regarding durable purchases by the borrowers, $I_{d,t}$.

The procedure above is based on a standard accuracy test, in which the conditional expectations are calculated using a very accurate numerical integration procedure. The accuracy test executed here focuses on only one particular feature of the model, namely whether the constraint is binding or not. In this case, it is possible to use a much simpler procedure that avoids using numerical integration. In particular, instead of calculating

[21]This possibly changes if one calibrates the model to feature other types of shocks as well. Because results would depend on the particular choice of shocks and their relative volatilities, I have limited the analysis to monetary policy shocks only.

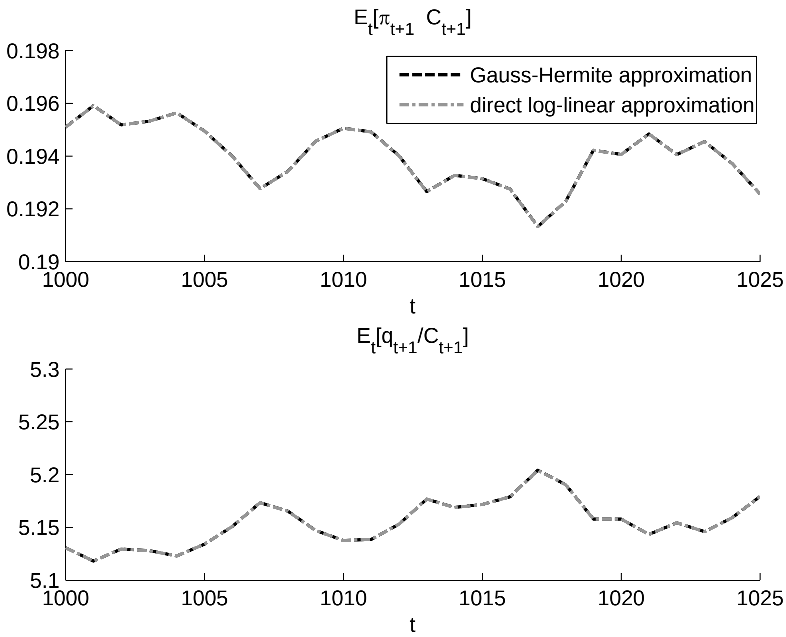
Table 2.2: Accuracy test for the model with credit frictions.

β	$\frac{\text{output volatility model}}{\text{output volatility data}}$	criterion1 (% non-binding)	criterion 2 (% error b_t)	criterion 3 (% error C_t)	criterion 4 (% error $I_{d,t}$)
0.98	10%	0.0	0.0	0.0	0.0
(benchmark)	25%	0.0	0.0	0.0	0.0
	50%	0.0	0.0	0.0	0.0
	100%	0.0	0.0	0.0	0.0
0.985	10%	0.0	0.0	0.0	0.0
	25%	0.0	0.0	0.0	0.0
	50%	0.0	0.0	0.0	0.0
	100%	1.1	0.0	0.0	1.2
0.989	10%	0.0	0.0	0.0	0
	25%	4.0	0.0	0.0	1.1
	50%	18.2	0.2	0.0	33.9
	100%	28.7	0.7	0.1	436.6

Note: The test is based on a simulation of length 51000, starting from the steady state and with the first 1000 observations discarded. Price stickiness of durables is set to 2 quarters. The percentages in the second column denote how much output volatility is predicted by the model with only monetary shocks, relative to output volatility observed in the data. Criterion 1 is the percentage of the cases in which the constraint is non-binding. Criterion 2, 3, and 4 are, respectively, the average of the absolute percentage errors in the amount of real debt, non-durable purchases and durable purchases by the borrowers, conditional on the event of a non-binding borrowing constraint. Conditional expectations are approximated using Gauss-Hermite quadrature with 10 nodes.

the conditional expectations in Equations (2.25) and (2.26) explicitly, one can use log-linear approximations. These can be obtained by simply adding the equations $x_{1,t} = E_t\left\{\pi_{c,t+1}C_{t+1}\right\}$ and $x_{2,t} = E_t\left\{\frac{q_{t+1}}{C_{t+1}}\right\}$ to the system solved in step 1.[22] Figure 2·4 plots the two conditional expectations calculated under both methods and suggests that differences are small. Table 2.3 compares criterion 1, calculated with both procedures, and shows that with the direct log-linear approximation of the conditional expectations, the constraint is non-binding somewhat more often, but the conclusions about accuracy would be the same in all cases.

Figure 2·4: Conditional expectations: Gauss-Hermite approximation versus direct log-linear approximation.



Notes: Model simulation with $\beta = 0.989$ and the standard deviation of the shocks calibrated such that output volatility in the model is 50% of output volatility in the data.

[22] I would like to thank Matteo Iacoviello for this idea.

Table 2.3: Accuracy test: comparing the Gauss-Hermite approximation of the conditional expectations to a direct log-linear approximation.

β	$\frac{\text{output volatility model}}{\text{output volatility data}}$	criterion 1 (% non-binding) Gauss-Hermite approximation	log-linear approximation
0.98	10%	0.0	0.0
(benchmark)	25%	0.0	0.0
	50%	0.0	0.0
	100%	0.0	0.0
0.985	10%	0.0	0.0
	25%	0.0	0.0
	50%	0.0	0.0
	100%	1.1	1.5
0.989	10%	0.0	0.0
	25%	4.0	4.3
	50%	18.2	20.1
	100%	28.7	33.3

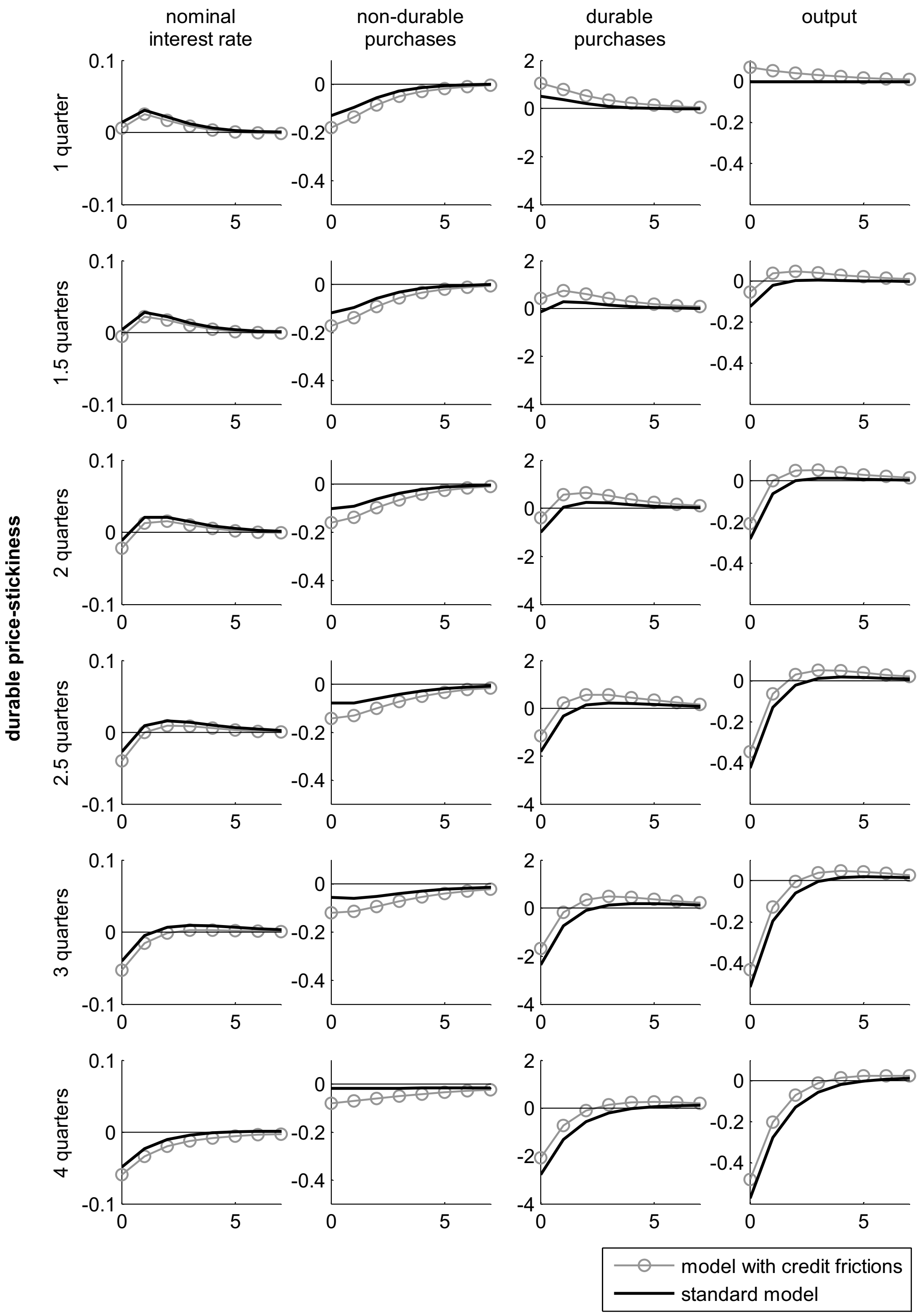
Note: See Table 2.2.

Appendix 2.B Alternative monetary policy rules

Durable inflation in the monetary policy rule Following Monacelli (2009), the benchmark results are generated under the assumption that monetary policy attaches no weight to inflation in the durable goods sector.[23] In this appendix, the more realistic assumption is adopted that monetary policy responds to a composite index of inflation in the durable and non-durable sectors. The weight attached to durable inflation is chosen to reflect the share of durable purchases in total expenditures in the steady state, that is, τ is set to 0.2. Figure 2·5 plots the IRFs for both models with this monetary policy rule. The main difference with the benchmark rule is that the response of the nominal interest rate is more negative. With the more realistic monetary policy rule, the range of values for the durable price-stickiness parameter ϑ_d for which durable purchases, non-durable purchases, and the nominal interest rate all move in the desired directions right after

[23] The numerical results in Monacelli (2009) are generated by setting τ equal to zero. In his model description, Monacelli (2009) proposes to set τ equal to α but provides no rationale for this choice. Recall that α is the parameter that determines the relative importance of the *stock* of durables in the CES consumption basket. The parameter α is, thus, not equal to the steady state expenditure share of durables.

Figure 2·5: Responses to a monetary tightening in the model with credit frictions and without credit frictions (standard model), both with a composite inflation index in the monetary policy rule ($\tau = 0.2$). The rows correspond to different degrees of durable price-stickiness. Responses are plotted as percentage deviations from the steady state.



the shock becomes very small for the standard model. The model with credit frictions performs even worse in the sense that there is no value for ϑ_d for which it generates positive comovement between durables and non-durables and a positive response of the nominal interest rate during the monetary tightening.

Output gap in the monetary policy rule I now consider a monetary policy rule that includes an output term. This does not change the result that the introduction of credit frictions makes the comovement problem more severe. I considered the following alternative monetary policy rule:

$$\frac{R_t}{R} = \left(\frac{\widetilde{\pi}_t}{\widetilde{\pi}}\right)^{\xi_\pi} \left(\frac{Y_t^{agg}}{Y^{agg}}\right)^{\xi_y} \exp\left(\varepsilon_t\right),$$

where R_t is the nominal interest rate, $\widetilde{\pi}_t$ is the inflation index, Y_t^{agg} is total aggregate output as defined in the main text of this chapter, ε_t is the shock, and variables without time index denote steady state values.[24] I considered the following (standard) parameter values: $\xi_\pi = 1.5$ and $\xi_y = 0.5/4$. I left the remainder of the model, including the calibration, the same as in the main text of the chapter. The IRFs are shown in Figure 2·6. The IRFs show that, as under the benchmark rule without an output term, it is the case that in the model with credit frictions (i) the IRF of non-durables is more negative, (ii) the IRF of durable purchases is more positive, and (iii) total output responds more positively to a monetary tightening.

[24] With only monetary policy shocks, the natural level of output is constant, and deviations of output from its steady state level equal the output gap in this model.

Figure 2·6: Responses to a monetary tightening in the model with credit frictions and without credit frictions under the alternative monetary policy rule with output gap.