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Expectations-Based Identification of Government Spending Shocks

Job Market Paper

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October 2010

Abstract

This paper addresses the econometric problems of structural vector autoregressive (SVAR) analysis of the effects of government spending when, due to anticipation effects, private agents have more information than the econometrician investigating their behavior. Using a combination of general equilibrium theory and SVAR simulations, I demonstrate how adding survey expectations to the regression not only equalizes the information sets but also makes it possible to identify structural spending shocks. In particular, I show that the econometrician can exploit natural expectations-based identifying restrictions by using survey data. In an application to U.S. data, the expectations-based approach indicates a weaker impact of government spending on output, consumption and investment than the standard fiscal SVAR approach, which does not take into account anticipation effects.

Keywords: Government spending shocks; Structural vector autoregressions; Anticipation effects; Survey data; Expectations-based identification

JEL classification: E3; E62; H3

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1 Introduction

The possibility that government spending shocks can be anticipated in advance of their realization, for instance due to pre-announcement of spending programmes or spending build-ups in the face of military conflicts, poses two significant challenges to SVAR analysis of the effects of government expenditures. First, if private agents have information which is not yet incorporated in actual spending, the equilibrium time series can have a non-fundamental moving average representation. This means that structural spending shocks cannot be recovered from current and past data on government spending by SVAR methods. Ignoring anticipation effects is therefore likely to lead to incorrect estimates, as shown in Leeper, Walker, and Yang (2009), Mertens and Ravn (2010) and Ramey (forthcoming). Second, even if the non-fundamentalness problem can be solved, the structural shocks still have to be recovered from reduced-form estimates by an appropriate identifying strategy. This strategy not only needs to distinguish government spending shocks from other economic shocks but it also needs to distinguish between anticipated and surprise spending shocks. Good identifying restrictions are therefore especially hard to come by in the presence of anticipation effects.

This paper investigates how incorporating expectations survey data in a structural vector autoregression can be useful to address both of the above econometric problems. Several recent studies suggest to exploit variation in forward-looking variables in fiscal VARs, and sometimes support this idea by simulation results (see Fisher and Peters, 2009; Sims, 2009; Ramey, forthcoming; Auerbach and Gorodnichenko, 2010; Forni and Gambetti, 2010). To obtain a deeper understanding of those proposals and to make a link to the work of Leeper et al. (2009), Mertens and Ravn (2010) and Ramey (forthcoming), I provide analytical results in a standard growth model as well as SVAR simulation evidence tightly connected to theory. Moreover, with respect to the identification problem, I exploit identifying restrictions on government spending shocks which are based on expectations formation. A surprise spending increase is identified as an increase in the expectational error between today's spending and expectations thereof formed yesterday. An anticipated shock is identified by a change in expected spending tomorrow which is orthogonal to the expectational error.

I first derive theoretical conditions under which an expectations-based approach works to solve the non-fundamentalness problem. For standard information flows, the requirement is that expectations on future spending up the anticipation horizon of private agents should be included in the VAR. Second, regarding the identification problem, I show that the success of an expectations-based approach depends on the type of endogenous reactions of government

expenditures to the state of the economy, if spending is not entirely exogenously determined. Surprise spending shocks are robustly identified provided that spending reacts with some lag to other economic shocks, which is also assumed under the short-run restrictions of the standard recursive SVAR approach (see Fatás and Mihov, 2001; Blanchard and Perotti, 2002). However, I also show that the expectations-based approach may fail to correctly recover anticipated spending shocks if expected spending picks up effects of other shocks, for example due to spending expansions out of revenue windfalls.

In a third step, the approach is applied to the U.S. using data on expected federal government spending from the Survey of Professional Forecasters. Given the restrictive requirements for the correct identification of anticipated spending shocks–namely that all other relevant exogenous shocks need to be known, observed and conditioned upon–I focus on the estimation of the effects of surprise spending shocks. The empirical results show that, indeed, the estimates obtained from the expectations-based approach differ from standard SVAR estimates. An unforeseen expansion in government spending has positive short-run effects on output but negative medium to long-run effects. The reason is that private consumption and investment both decline after the spending increase. The standard SVAR approach, on the other hand, predicts an increase in consumption and investment in the short run and larger medium to long-run multipliers on GDP. In addition, following Ramey (forthcoming), I show that the standard SVAR shocks are predictable by the survey expectations whereas the surprise shocks identified on the basis of expectational errors are truly unpredictable.

Several other recent studies have addressed the issues created by foresight on government spending. Ramey (forthcoming) applies a narrative approach which exploits military spending episodes, newspaper sources and forecast errors based on survey data. However, narrative account data sets may not be sufficiently rich to accurately estimate the effects of both anticipated and unanticipated spending changes. In addition, although she also uses expectational errors to identify surprise spending shocks, Ramey does not add expectations on future spending in the VAR, which I show does *not* lead to a fundamental moving average representation. Fisher and Peters (2009) identify spending shocks by innovations to excess stock returns of military contractors, an approach which is applicable to defense-related expenditures only. Mertens and Ravn (2010) propose an SVAR estimator for permanent spending shocks based on Blaschke matrices, following Lippi and Reichlin (1994), which can be applied when the identifying assumptions pin down the Blaschke factor. Kriwoluzky (2009) directly estimates a vector moving average model, with model-based identifying restrictions on anticipated spending shocks. Forni and Gambetti (2010) estimate a large factor model which are not affected

by the non-fundamentalness problem (see Forni, Giannone, Lippi, and Reichlin, 2009), identifying a spending shock by various sign restrictions. The latter three approaches all rely on the correct specification of the identifying theoretical model. An expectations-based approach has the advantage of applying arguably less restrictive identifying assumptions.

Finally, it is important to note that the relevance of the fiscal foresight critique is not limited to empirical research on fiscal policy, since dynamic stochastic general equilibrium (DSGE) modelling is often guided by SVAR results. For instance, Galí, Lopéz-Salido, and Vallés (2007) present SVAR evidence that private consumption in the U.S. increases following an expansion of government purchases of goods and services. They show that a New Keynesian model with non-Ricardian consumers can explain such evidence, although government spending falls under the category of 'wasteful' expenditures in this model.[1] In addition, the structural parameters of DSGE models are sometimes estimated by matching SVAR impulse response functions (see Bilbiie, Meier, and Mueller, 2006). However, if the empirical results on which the development of DSGE models is based are problematic in the first place, policy analysis based on those models may well yield misleading predictions.

The remainder of the paper is structured as follows. The next section explores the econometric problems created by foresight in a standard growth model and discusses the usefulness of an expectations-based approach in addressing those problems. Section 3 provides simulation evidence on the merits of the expectations-based approach. Section 4 investigates the robustness of the approach to, not exclusively, alternative assumptions on the structure of the spending process, and proposes possible adjustments. Section 5 discusses the results from the empirical application. Section 6 concludes.

2 Fiscal Foresight: Problems and Solutions

This section explores the problems induced by foresight on government spending in a simple analytical example, following Leeper et al. (2009) and Mertens and Ravn (2010). Leeper et al. (2009) analyze the econometric implications of foresight on future tax rates. Mertens and Ravn (2010) focus on government spending, in order to derive an SVAR estimator which is applicable in the face of permanent spending shocks. Below, I discuss potential solutions when private sector expectations can be observed, when the data is assumed to be generated

[1] Previous and subsequent studies have focused on imperfect substitutability between public and private consumption (Linnemann and Schabert, 2004), deep habits in consumption (Ravn, Schmitt-Grohé, and Uribe, 2007), small wealth effects on labor supply (Monacelli and Perotti, 2008) or spending expansions followed by reversals, which create expectations on a future fall in real interest rates (Corsetti, Meier, and Mueller, 2009; Corsetti, Kuester, Meier, and Mueller, 2010), to produce a crowding-in of private consumption.

by a version of the simple neoclassical growth model due to Hansen (1985). The main results would also hold in a larger DSGE model. The obvious advantage of using the simple growth model is that analytical results can be derived more easily.[2]

2.1 The non-fundamentalness problem in a neoclassical economy

The model economy is inhabited by a continuum of identical, infinitely lived households, whose instantaneous utility depends on consumption c_t and hours worked n_t. They provide labor services and physical capital k_t to firms and they pay lump-sum taxes τ_t to the government. Time is indexed by $t = 0, 1, 2, \ldots, \infty$. All variables are denoted in real terms. The objective of a representative household is to maximize expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t - A n_t\right), \qquad \beta \in (0,1), \qquad A > 0,$$

where E_0 is the mathematical expectations operator conditional on the information available at time 0. The household's optimization problem is subject to the flow budget constraint

$$c_t + i_t + \tau_t = w_t n_t + r_t k_{t-1},$$

where w_t denotes the hourly wage and i_t denotes investment in physical capital at the rental rate r_t. Capital accumulates according to the law of motion

$$k_t = (1-\delta) k_{t-1} + i_t, \qquad \delta \in [0,1]. \tag{1}$$

There is a continuum of identical, perfectly competitive firms which produce the final consumption good y_t using capital and labor as inputs. The production function of a representative firm is given by

$$y_t = a_t k_{t-1}^{\alpha} n_t^{1-\alpha}, \qquad \alpha \in (0,1), \tag{2}$$

where a_t is total factor productivity (TFP) which is assumed to follow the law of motion

$$\log a_t = \rho_a \log a_{t-1} + \varepsilon_{a,t}, \qquad \rho_a \in [0,1), \qquad \varepsilon_{a,t} \sim N(0, \sigma_a^2). \tag{3}$$

Profit maximization yields the factor prices $w_t = (1-\alpha)\, y_t / n_t$ and $r_t = \alpha y_t / k_{t-1}$.

Government spending g_t is assumed to be financed exclusively by lump-sum taxes, $g_t = \tau_t$,

[2] This section focuses on a simple information structure with two-period anticipation and a single news shock, the latter following Mertens and Ravn (2010) or Ramey (forthcoming). The spending process could also be extended to longer anticipation horizons without affecting the main results.

and it is modelled as an exogenous stochastic process:

$$\log(g_t/\bar{g}) = \rho_g \log(g_{t-1}/\bar{g}) + \varepsilon^u_{g,t} + \varepsilon^a_{g,t-2}, \qquad \rho_g \in [0,1), \tag{4}$$

where $\bar{g} = g$ is the deterministic steady state level of government spending, which is taken as given. This process allows for a surprise shock to government spending $\varepsilon^u_{g,t} \sim N(0, \sigma^2_{g,u})$ and a news shock $\varepsilon^a_{g,t} \sim N(0, \sigma^2_{g,a})$ which describes the pre-announced nature of spending. If there is such a shock, the associated change in spending is known (anticipated) by private agents two periods in advance of its implementation in terms of actual spending.

Combining the household's budget constraint with the government's budget constraint and the firm's first-order conditions, the feasibility constraint reads

$$c_t + i_t + g_t = y_t. \tag{5}$$

Substituting out the factor prices in the first-order conditions to the household's optimization problem yields the labor/leisure trade-off and the consumption Euler equation:

$$Ac_t = (1-\alpha)\frac{y_t}{n_t}, \tag{6}$$

$$1 = \beta E_t \frac{c_t}{c_{t+1}} R_{t+1}, \tag{7}$$

where R_t denotes the real return on capital,

$$R_t = 1 - \delta + \alpha \frac{y_t}{k_{t-1}}. \tag{8}$$

Then, a *rational expectations equilibrium* is a set of sequences $\{c_t, n_t, i_t, k_t, R_t, y_t, a_t, g_t\}_{t=0}^{\infty}$ satisfying (1) to (8) and the transversality condition for capital, for given initial values k_{-1}, a_{-1} and g_{-1} and sequences of shocks $\{\varepsilon_{a,t}, \varepsilon^u_{g,t}, \varepsilon^a_{g,t}\}_{t=0}^{\infty}$.

To obtain an analytical solution to the model, the equilibrium system is log-linearized at the deterministic steady state and the log-linearized system is solved using the method of undetermined coefficients (Uhlig, 1999). A detailed derivation is provided in the Appendix. The log-linearized system can be reduced to a two-dimensional, first-order stochastic difference equation in consumption and capital,

$$0 = E_t\left[\hat{c}_t - \phi_1 \hat{c}_{t+1} + \phi_2 \hat{a}_{t+1}\right],$$

$$0 = E_t\left[\phi_3 \hat{c}_t + \phi_4 \hat{k}_t - \phi_5 \hat{a}_t - \phi_6 \hat{k}_{t-1} + \phi_7 \hat{g}_t\right],$$

Table 1: Benchmark calibration of the model.

Parameter	*Value*	*Description*	*Source/moment*
β	0.99	Subjective discount factor	Kydland and Prescott (1982), Hansen (1985)
α	0.36	Capital share in production	Kydland and Prescott (1982), Hansen (1985)
δ	0.025	Depreciation rate (quarterly)	Kydland and Prescott (1982), Hansen (1985)
A	2.5	Disutility of labor supply	Time spent on market activities
a	1	Steady state TFP	Normalization
g/y	0.08	Government spending share in GDP	Federal spending share 1981:4 to 2010:1
g	0.1	Steady state government spending	Federal spending share 1981:4 to 2010:1
ρ_a	0.95	AR(1) parameter TFP	Hansen (1985)
ρ_g	0.85	AR(1) parameter gov. spending	Galí et al. (2007)
σ_a	0.71	Std. dev. TFP shocks (%)	Hansen (1985)
$\sigma_{g,u}$	1	Std. dev. anticip. spending shocks (%)	Benchmark
$\sigma_{g,a}$	1	Std. dev. unanticip. spending shocks (%)	Benchmark

given the exogenous processes $\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon^u_{g,t} + \varepsilon^a_{g,t-2}$ and $\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{a,t}$, where $\hat{x}_t = \log(x_t/x)$ denotes the log deviation of variable x_t from its steady state value x. The parameters ϕ_i, $i = 1, \ldots, 7$ are functions of the model parameters and steady state values. The recursive laws of motion describing the solution are

$$\hat{k}_t = \eta_{kk}\hat{k}_{t-1} + \eta_{ka}\hat{a}_t + \eta_{kg}\hat{g}_t + \eta_{k\varepsilon,2}\varepsilon^a_{g,t-1} + \eta_{k\varepsilon,1}\varepsilon^a_{g,t}, \tag{9}$$

$$\hat{c}_t = \eta_{ck}\hat{k}_{t-1} + \eta_{ca}\hat{a}_t + \eta_{cg}\hat{g}_t + \eta_{c\varepsilon,2}\varepsilon^a_{g,t-1} + \eta_{c\varepsilon,1}\varepsilon^a_{g,t}, \tag{10}$$

where the coefficients η are non-linear functions of the model parameters and the parameters ϕ. Notice that, according to (9) and (10), there is the unusual implication that the information set of private agents in period t includes the shocks $\varepsilon^a_{g,t-1}$ and $\varepsilon^a_{g,t}$ as additional states.[3]

The model is calibrated in line with the real business cycle literature (Kydland and Prescott, 1982; Hansen, 1985) and to match selected moments in U.S. quarterly data over the period 1981:4 to 2010:1. The benchmark calibration is reported in Table 1. The subjective discount factor β is set to 0.99, which implies a steady state annual real interest rate of approximately 4%.[4] The capital share in production α is set to 0.36 and the quarterly depreciation rate δ is set to 0.025, implying an annual depreciation rate of 10%. The parameter A is set to 2.5, which implies that steady state hours worked is close to l/3. The

[3]The full information set includes $\{\varepsilon_{a,j}, \varepsilon^u_{g,j}, \varepsilon^a_{g,j}\}_{j=0}^t$. The shocks $\{\varepsilon_{a,j}\}_{j=0}^t$ are incorporated in the states $\hat{a}_t$ and (up to time $t-1$) in $\hat{k}_{t-1}$. The shocks $\{\varepsilon^u_{g,j}\}_{j=0}^t$ and $\{\varepsilon^a_{g,j}\}_{j=0}^{t-2}$ are also incorporated in $\hat{g}_t$ and $\hat{k}_{t-1}$ whereas $\varepsilon^a_{g,t-1}$ and $\varepsilon^a_{g,t}$ are announced but not yet implemented innovations to spending. Therefore, the latter appear as additional state variables in the recursive laws of motion.

[4]The average annual 3-month U.S. treasury bill secondary market rate was approximately 5 percent over the period 1981:4 to 2010:1.

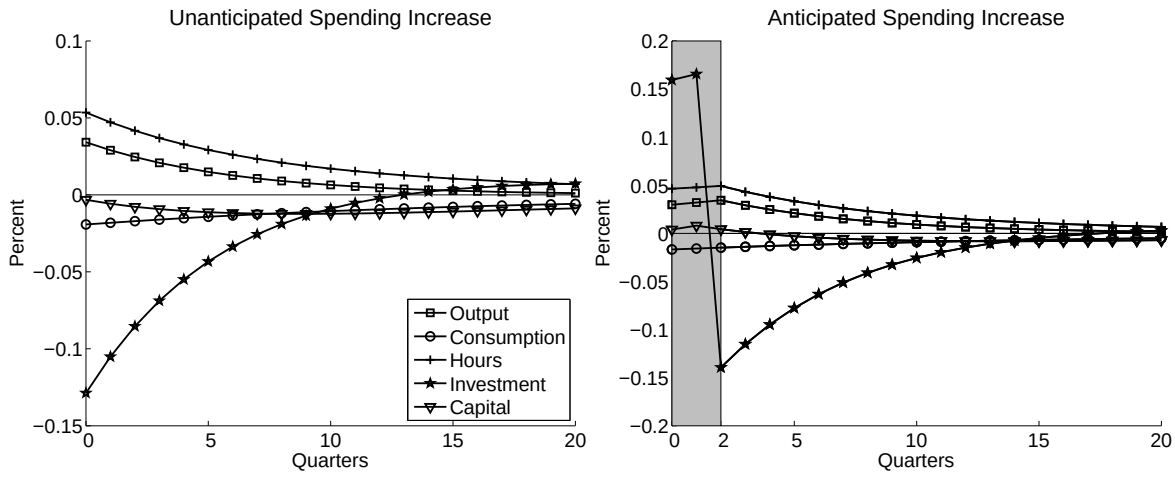

Figure 1: Model impulse responses to government spending shocks, benchmark calibration. *Notes.* Both panels show responses to one percent increases in the level of government spending; left panel: surprise spending increase in quarter 0; right panel: news in quarter 0 that spending will increase in quarter 2.

steady state share of government spending in GDP, g/y, is set to its empirical counterpart (the average federal government spending share) of 8 percent. Finally, the standard deviation of TFP shocks is set to 0.71 percent and the AR(1) parameter of TFP is set to 0.95, following Hansen (1985). The standard deviations of the two spending innovations are set to 1 percent and the AR(1) parameter of government spending to 0.85 (see e.g. Galí et al., 2007).[5]

Figure 1 shows impulse responses due to normalized spending shocks of both types. Since the model satisfies Ricardian equivalence, a surprise spending increase in quarter 0 (left panel) financed by lump-sum taxes has a negative wealth effect on the household's lifetime income. Consumption declines and, since leisure is a normal good, hours worked increase. Although the return on investment increases, the negative investment response is dictated by the feasibility constraint, under the benchmark calibration. On the other hand, if there is news in quarter 0 that spending will increase in quarter 2 (right panel) investment is positive during two quarters and then turns negative. There is an immediate negative wealth effect due to higher future taxes, so immediately consumption declines and hours worked and output increase. Investment can increase during the anticipation period since there is no government absorption of goods and services yet. To explore the non-fundamentalness problem induced by the news shock $\varepsilon^a_{g,t}$, notice that the coefficient η_{kk} is the stable root of the characteristic equation

$$0 = \phi_1\phi_4\eta_{kk}^2 - (\phi_1\phi_6 + \phi_4)\,\eta_{kk} + \phi_6.$$

[5]For the points to be made, the calibration matters insofar as it affects the relative volatilities of the structural shocks as well as the anticipation rate. Below, the robustness of the identification approach is tested when those parameters change.

In a unique saddle path solution, this equation has two real roots η_{kk}^{+} and η_{kk}^{-},

$$\eta_{kk}^{\pm} = \frac{1}{2}\left(\phi_1^{-1} + \phi_6\phi_4^{-1}\right) \pm \sqrt{\frac{1}{4}\left(\phi_1^{-1} + \phi_6\phi_4^{-1}\right)^2 - \phi_6\left(\phi_1\phi_4\right)^{-1}}, \tag{11}$$

only one of which is smaller than one in absolute value. Further, it is straightforward to show that the coefficients $\eta_{x\varepsilon,1}$ and $\eta_{x\varepsilon,2}$ $(x = k, c)$, are related according to

$$\eta_{x\varepsilon,1} = \theta\eta_{x\varepsilon,2}, \qquad \theta = \left[\phi_1^{-1} + \phi_6\phi_4^{-1} - \eta_{kk}\right]^{-1},$$

Inserting the expression for θ in (11), it follows that $|\theta| < 1$.[6] This result implies that, in forming their decisions, agents discount more recent news on government spending $\varepsilon_{g,t}^{a}$ relative to more distant news $\varepsilon_{g,t-1}^{a}$ at a constant *anticipation rate* given by θ. The reason is that recent news affects spending later than distant news (see Leeper et al., 2009; Mertens and Ravn, 2010). As noted by Mertens and Ravn (2010), the result of constant discounting generalizes to other settings (e.g. longer anticipation horizons, more control variables).

Mertens and Ravn (2010) show that the anticipation rate is, inter alia, monotonically increasing in the subjective discount factor β. Figures 2 and 3 show the impulse responses to both spending shocks when β changes from 0.8 to 0.99 from thin to thick lines, implying values for θ from 0.58 to 0.93. The initial responses of consumption and hours to the unanticipated shocks are uniformly stronger for lower discount factors (see Figure 2). The reason is that the future is then discounted at a higher rate, so households have a lower preference for consumption smoothing. For the same reason, when β decreases the anticipation rate falls and recent news receives a heavier discount. Therefore, an anticipated increase in government spending affects the economy more strongly prior to its implementation in terms of actual spending (see Figure 3). This means that, for lower anticipation rates, the differences between the impulse responses to both types of shocks *after* spending has increased become larger. Compare, for example, the investment responses which are uniformly negative from quarter 2 onwards after the anticipated spending increase but positive for some parameter values after the unanticipated spending increase. The implications of these findings are discussed in turn.

The phenomenon of constant discounting is indeed the root of the non-fundamentalness

[6] To see this, suppose that $\left|\eta_{kk}^{+}\right| = \left|\frac{1}{2}\left(\phi_1^{-1} + \phi_6\phi_4^{-1}\right) + \sqrt{\frac{1}{4}\left(\phi_1^{-1} + \phi_6\phi_4^{-1}\right)^2 - \phi_6\left(\phi_1\phi_4\right)^{-1}}\right| < 1$ such that $\left|\eta_{kk}^{-}\right| = \left|\frac{1}{2}\left(\phi_1^{-1} + \phi_6\phi_4^{-1}\right) - \sqrt{\frac{1}{4}\left(\phi_1^{-1} + \phi_6\phi_4^{-1}\right)^2 - \phi_6\left(\phi_1\phi_4\right)^{-1}}\right| > 1$. Then $\eta_{kk} = \eta_{kk}^{+}$ and, by direct calculation, $\theta = \left(\eta_{kk}^{-}\right)^{-1}$, which implies that $|\theta| < 1$. Conversely, if $\left|\eta_{kk}^{+}\right| > 1$ and $\left|\eta_{kk}^{-}\right| < 1$ then $\eta_{kk} = \eta_{kk}^{-}$ and $\theta = \left(\eta_{kk}^{+}\right)^{-1}$, which again implies that $|\theta| < 1$.

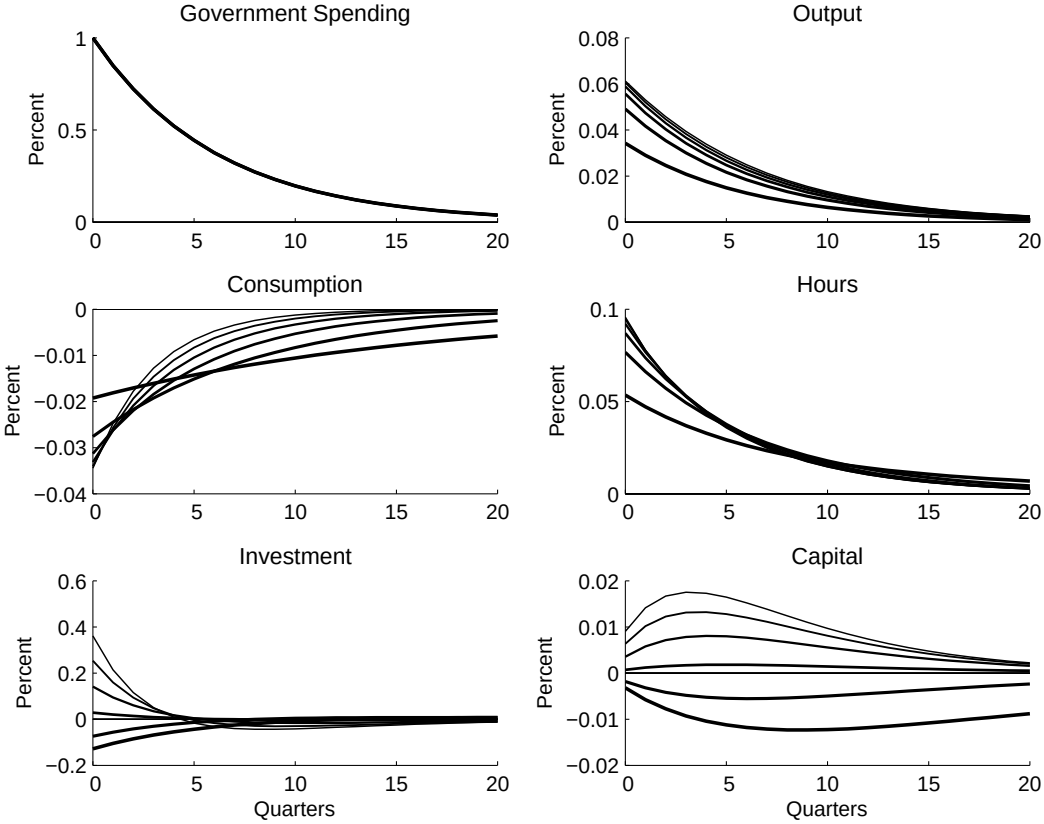

Figure 2: Model impulse responses to unanticipated spending shock, changing anticipation rate. *Notes.* From thin to thick lines: anticipation rate θ is changed from 0.58 to 0.93 by changing the discount factor β from 0.8 to 0.99; thickest line: benchmark calibration.

problem. To see this, following Leeper et al. (2009), suppose that an econometrician who is not aware of anticipation effects estimates a VAR in $\{\hat{g}_{t-j}, \hat{a}_{t-j}, \hat{k}_{t-j}\}_{t=0}^{\infty}$. According to the equilibrium representation implied by the model, the econometrician's observables can be shown to follow the multivariate moving average process

$$\begin{bmatrix} \hat{g}_t \\ \hat{a}_t \\ \hat{k}_t \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\rho_g L} & \frac{L^2}{1-\rho_g L} & 0 \\ 0 & 0 & \frac{1}{1-\rho_a L} \\ \frac{\eta_{kg}}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{kg}L^2+\eta_{k\varepsilon,2}(1-\rho_g L)(\theta+L)}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{ka}}{(1-\eta_{kk}L)(1-\rho_a L)} \end{bmatrix} \begin{bmatrix} \varepsilon_{g,t}^u \\ \varepsilon_{g,t}^a \\ \varepsilon_{a,t} \end{bmatrix},$$

or

$$\boldsymbol{y}_t = \boldsymbol{P}(L)\boldsymbol{\epsilon}_t, \tag{12}$$

where L denotes the lag operator, $L^s x_t = x_{t-s}$. If the process (12) is invertible in non-negative powers of L, then the econometrician can recover the structural shocks as a linear combination of present and past observables, i.e. $\boldsymbol{\epsilon}_t = \boldsymbol{P}^{-1}(L)\boldsymbol{y}_t$. A necessary and sufficient condition for $\boldsymbol{\epsilon}_t$ to be fundamental for $\boldsymbol{y}_t$ is that the zeroes of the determinant of $\boldsymbol{P}(z)$ do

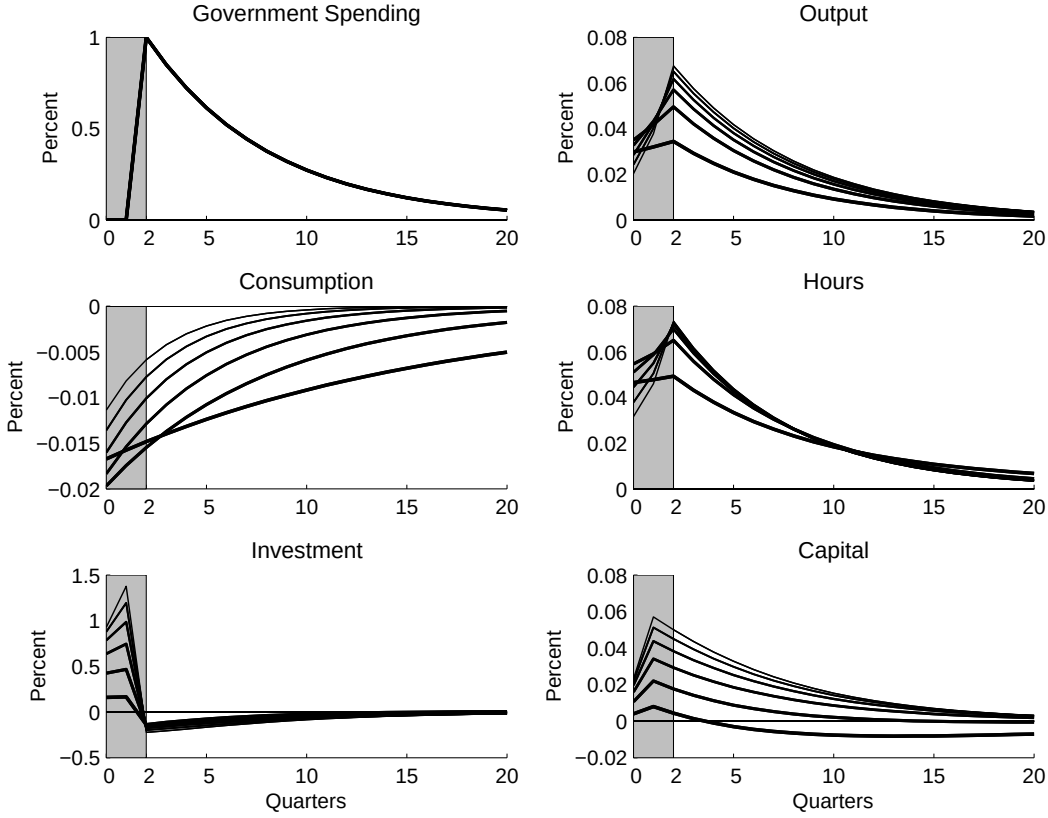

Figure 3: Model impulse responses to anticipated spending shock, changing anticipation rate. *Notes.* From thin to thick lines: anticipation rate θ is changed from 0.58 to 0.93 by changing the discount factor β from 0.8 to 0.99; thickest line: benchmark calibration.

not lie inside the unit circle (see Hansen and Sargent, 1991). In this case, the determinant of $\boldsymbol{P}(z)$ is given by

$$\det \boldsymbol{P}(z) = \frac{-\eta_{k\varepsilon,2}\,(\theta + z)}{(1 - \eta_{kk} z)\,(1 - \rho_g z)\,(1 - \rho_a z)},$$

which has a root inside the unit circle at $z = -\theta$. Thus, the structural shocks $\{\varepsilon^a_{g,t}, \varepsilon^u_{g,t}, \varepsilon_{a,t}\}_{t=0}^{\infty}$ cannot be recovered from the econometrician's information set $\{\hat{g}_{t-j}, \hat{a}_{t-j}, \hat{k}_{t-j}\}_{t=0}^{\infty}$. In other words, (13) is not a fundamental moving average (Wold) representation.

2.2 An expectations-based solution

The crux of the non-fundamentalness problem induced by foresight on government spending is the fact that, if the econometrician only observes current and past spending, his information set is misaligned with the information set of private agents who have knowledge of news on future spending over their anticipation horizon, even before this news materializes in terms of actual spending. One natural way to realign the two information sets is to incorporate private sector expectations in the econometrician's information set. Thus, suppose that instead of

present and past $\hat{a}_t$ the econometrician observes present and past expectations on spending two periods ahead, conditional on time t information, $E_t\hat{g}_{t+2}$.[7] The econometrician thus estimates a VAR in $\{\hat{g}_{t-j}, E_t\hat{g}_{t+2-j}, \hat{k}_{t-j}\}_{t=0}^{\infty}$, where

$$\begin{bmatrix} \hat{g}_t \\ E_t\hat{g}_{t+2} \\ \hat{k}_t \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\rho_g L} & \frac{L^2}{1-\rho_g L} & 0 \\ \frac{\rho_g^2}{1-\rho_g L} & \frac{1}{1-\rho_g L} & 0 \\ \frac{\eta_{kg}}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{kg}L^2+\eta_{k\varepsilon,2}(1-\rho_g L)(\theta+L)}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{ka}}{(1-\eta_{kk}L)(1-\rho_a L)} \end{bmatrix} \begin{bmatrix} \varepsilon_{g,t}^u \\ \varepsilon_{g,t}^a \\ \varepsilon_{a,t} \end{bmatrix},$$

or

$$\boldsymbol{y}_t^* = \boldsymbol{P}^*(L)\boldsymbol{\epsilon}_t. \tag{13}$$

The determinant of $\boldsymbol{P}^*(z)$ is

$$\det \boldsymbol{P}^*(z) = \frac{\eta_{ka}(1+\rho_g z)}{(1-\rho_g z)(1-\eta_{kk}z)\,(1-\rho_a z)},$$

which has one root outside the unit circle at $z = -\rho_g^{-1}$ and three poles at $z = \rho_g^{-1}$, $z = \eta_{kk}^{-1}$ and $z = \rho_a^{-1}$. Hence, (13) is an invertible moving average process, which means that the structural shocks in $\boldsymbol{\epsilon}_t$ can in principle be recovered from the econometrician's information set $\{\hat{g}_{t-j}, E_t\hat{g}_{t+2-j}, \hat{k}_{t-j}\}_{t=0}^{\infty}$, through a linear combination of present and past observables. By the inclusion of private agents' expectations on government spending, (13) is a fundamental Wold representation of the equilibrium time series.

2.3 Confronting the identification problem

Unfortunately, fundamentalness of the structural shocks with respect to the econometrician's information set is necessary but not sufficient for the correct estimation of their effects. In addition, after obtaining reduced-form estimates, the econometrician needs to recover the structural shocks through an appropriate identifying strategy. Suppose the econometrician estimates an unrestricted VAR in levels of stationarized variables of the form

$$\boldsymbol{y}_t^{**} = \boldsymbol{B}_1\boldsymbol{y}_{t-1}^{**} + \boldsymbol{B}_2\boldsymbol{y}_{t-2}^{**} + \boldsymbol{B}_3\boldsymbol{y}_{t-3}^{**} + \cdots + \boldsymbol{u}_t = \boldsymbol{C}(L)\boldsymbol{u}_t, \qquad \boldsymbol{u}_t \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}).$$

[7] Since $E_t\hat{g}_{t+1} = \rho_g\hat{g}_t + L\varepsilon_{g,t}^a = \rho_g(1-\rho_g L)^{-1}\varepsilon_{g,t}^u + \rho_g L^2(1-\rho_g L)^{-1}\varepsilon_{g,t}^a + L\varepsilon_{g,t}^a$, the two-period ahead expectation is

$$E_t\hat{g}_{t+2} = \rho_g E_t\hat{g}_{t+1} + \varepsilon_{g,t}^a = \frac{\rho_g^2 L^2}{1-\rho_g L}\varepsilon_{g,t}^a + (1+\rho_g L)\varepsilon_{g,t}^a + \frac{\rho_g^2}{1-\rho_g L}\varepsilon_{g,t}^u = \frac{1}{1-\rho_g L}\varepsilon_{g,t}^a + \frac{\rho_g^2}{1-\rho_g L}\varepsilon_{g,t}^u.$$

where $\boldsymbol{C}(L)$ is an infinite order multivariate lag polynomial with $\boldsymbol{C}(0) = \boldsymbol{I}$.[8] Assume there exists a linear mapping between reduced-form innovations and structural shocks, $\boldsymbol{u}_t = \boldsymbol{D}\boldsymbol{\epsilon}_t$, such that the moving average representation in the structural shocks is given by

$$\boldsymbol{y}_t^{**} = \boldsymbol{C}(L)\boldsymbol{D}\boldsymbol{\epsilon}_t, \qquad \boldsymbol{\epsilon}_t = \boldsymbol{D}^{-1}\boldsymbol{u}_t.$$

Normalizing $\text{cov}(\boldsymbol{\epsilon}_t) = \boldsymbol{I}$, the impact matrix $\boldsymbol{D}$ must satisfy $\boldsymbol{D}\boldsymbol{D}' = \boldsymbol{\Sigma}$ and $\boldsymbol{D} = \boldsymbol{A}\boldsymbol{R}$, where $\boldsymbol{R}$ is an orthonormal matrix, i.e. $\boldsymbol{R}\boldsymbol{R}' = \boldsymbol{I}$, and $\boldsymbol{A}$ is an arbitrary orthogonalization (achieved for example by a Cholesky decomposition of $\boldsymbol{\Sigma}$, i.e. $\boldsymbol{R} = \boldsymbol{I}$).

Furthermore, suppose that the econometrician is only interested in the effects of government spending shocks. In the example above, when observing $\{\hat{g}_{t-j}, E_t\hat{g}_{t+2-j}, \hat{k}_{t-j}\}_{t=0}^{\infty}$, the econometrician would not be able to distinguish, through restrictions on $\boldsymbol{R}$, variations in $\hat{g}_t$ and $E_t\hat{g}_{t+2}$ due to $\varepsilon_{g,t}^u$ from variations due to $\varepsilon_{g,t}^a$. When actual spending increases through an unanticipated shock, expected spending two periods ahead increases as well.

To achieve identification, the econometrician could however observe the one-period expectational error $\hat{g}_t - E_{t-1}\hat{g}_t$. Since $E_{t-1}\hat{g}_t = \rho_g\hat{g}_{t-1} + \varepsilon_{t-2}^a$, it follows that $\hat{g}_t - E_{t-1}\hat{g}_t = \varepsilon_t^u$. Then, the VAR in $\{\hat{g}_{t-j} - E_{t-1}\hat{g}_t, E_t\hat{g}_{t+2-j}, \hat{k}_{t-j}\}_{t=0}^{\infty}$ is given by

$$\begin{bmatrix} \hat{g}_t - E_{t-1}\hat{g}_t \\ E_t\hat{g}_{t+2} \\ \hat{k}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\rho_g^2}{1-\rho_g L} & \frac{1}{1-\rho_g L} & 0 \\ \frac{\eta_{kg}}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{kg}L^2+\eta_{k\varepsilon,2}(1-\rho_g L)(\theta+L)}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{ka}}{(1-\eta_{kk}L)(1-\rho_a L)} \end{bmatrix} \begin{bmatrix} \varepsilon_{g,t}^u \\ \varepsilon_{g,t}^a \\ \varepsilon_{a,t} \end{bmatrix},$$

or

$$\boldsymbol{y}_t^{**} = \boldsymbol{P}^{**}(L)\boldsymbol{\epsilon}_t. \tag{14}$$

The determinant of $\boldsymbol{P}^{**}(z)$ is

$$\det \boldsymbol{P}^{**}(z) = \frac{\eta_{ka}}{(1-\rho_g z)(1-\eta_{kk}z)\,(1-\rho_a z)},$$

such that (14) is also an invertible moving average process. The process furthermore implies that $\varepsilon_{g,t}^u$ has a contemporaneous impact on both $\hat{g}_t - E_{t-1}\hat{g}_t$ and $E_t\hat{g}_{t+2}$ whereas $\varepsilon_{g,t}^a$ has a contemporaneous impact on $E_t\hat{g}_{t+2}$ but not $\hat{g}_t - E_{t-1}\hat{g}_t$. Both shocks have an immediate impact on capital. In practice, the two structural shocks could thus be identified by a Cholesky decomposition, where the expectational error is ordered before the two-period ahead expecta-

[8] The $\boldsymbol{B}_i$, $i = 1, 2, 3, \ldots$ are matrices of coefficients and $\boldsymbol{C}(L)$ is the associated lag polynomial of the moving average representation in $\boldsymbol{u}_t$.

tion, and where capital is ordered last. The unanticipated shock is then the innovation in the equation for government spending. Conditioning on the expectational error, the remaining variation in expected spending is due to the anticipated shock. In fact, the impact matrix, obtained by setting $L = 0$ in (14), is lower triangular:

$$\boldsymbol{D} = \boldsymbol{P}^{**}(0) = \begin{bmatrix} 1 & 0 & 0 \\ \rho_g^2 & 1 & 0 \\ \eta_{kg} & \eta_{k\varepsilon,1} & \eta_{ka} \end{bmatrix},$$

which implies that the process (14) has a Cholesky structure.

3 Simulation Evidence

The model discussed in the previous section is now applied to test the usefulness of the proposed identifying strategy. That is, simulation evidence is provided for alternative calibrations, focusing on the potential problems of the standard recursive SVAR identification approach (see Fatás and Mihov, 2001; Blanchard and Perotti, 2002), which does not take into account anticipation effects, and the advantages of the expectations-based approach. Section 4 discusses modifications to the benchmark model to check the robustness of the identification approach, including alternative assumptions on the structure of government spending.

3.1 Monte Carlo set-up

The approach implemented here is a Monte Carlo exercise, following for instance Ramey (forthcoming). That is, M data samples of length T are generated from the calibrated model.[9] The identification approach is first evaluated by its asymptotic properties in large samples, setting $T = 10,000$ and $M = 100$. Small-sample results are discussed in Section 4. The estimated impulse responses for each of the M samples are ordered and the mean estimates are reported, with 90% two-sided error bands. The estimated responses are then compared to the impulse responses implied by the data-generating process.

3.2 Standard SVAR identification

First, the properties of the standard SVAR identification approach are investigated when only government spending, TFP and capital are observed and included in this order in the VAR.

[9] In this section, for convenience, the log-linearized model is solved numerically by the Gensys algorithm (Sims, 2004).

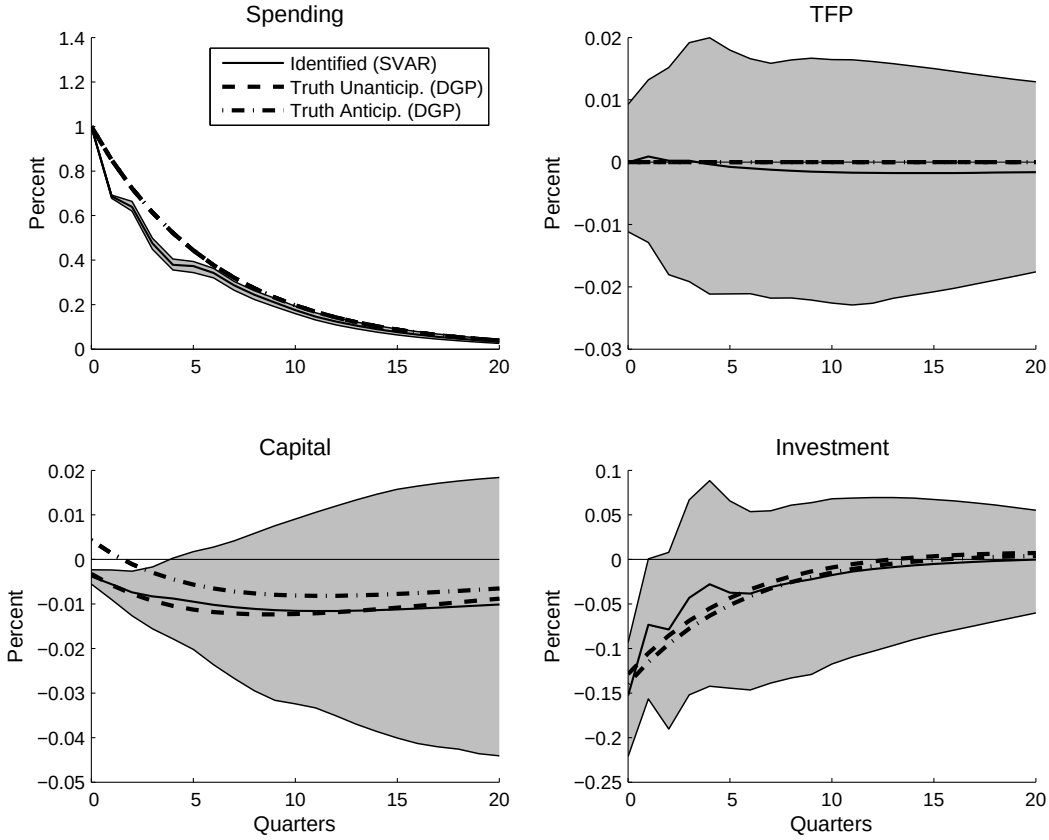

Figure 4: Monte Carlo impulse responses with standard recursive SVAR scheme, benchmark calibration ($\theta = 0.93$). *Notes.* SVAR responses (means) and 90% error bands are based on 100 samples of 10,000 observations each; a spending shock is identified by ordering government spending first in a Cholesky decomposition; DGP responses to anticipated shock are plotted from spending increase onwards.

I have shown above that the model then has a non-fundamental equilibrium representation. Furthermore, to ensure comparability with the expectations-based approach, investment is added as a fourth variable to the VAR. Since there are only three shocks in the model, to prevent stochastic singularity while avoiding distorted inference, a small measurement error on investment $\varepsilon_{i,t} \sim N(0, \sigma_i^2)$ with σ_i set to 0.01 percent is included in the DGP.

Figure 4 shows the estimated impulse responses for the benchmark calibration, where a government spending shock is identified by a Cholesky decomposition of the reduced-form covariance matrix $\boldsymbol{\Sigma}$, i.e. $\boldsymbol{\Sigma} = \boldsymbol{A}\boldsymbol{A}'$, where $\boldsymbol{A}$ is lower triangular (see Section 2.3). The figure also shows the impulse responses due to both shocks implied by the data-generating model, from the quarter when spending has increased onwards. Obviously, the impulse responses cannot pick up any variation in investment and capital due to anticipated shocks during the anticipation period. However, the results show that the error with respect to the surprise spending shock is not necessarily large. Thus, there may be cases where anticipation effects

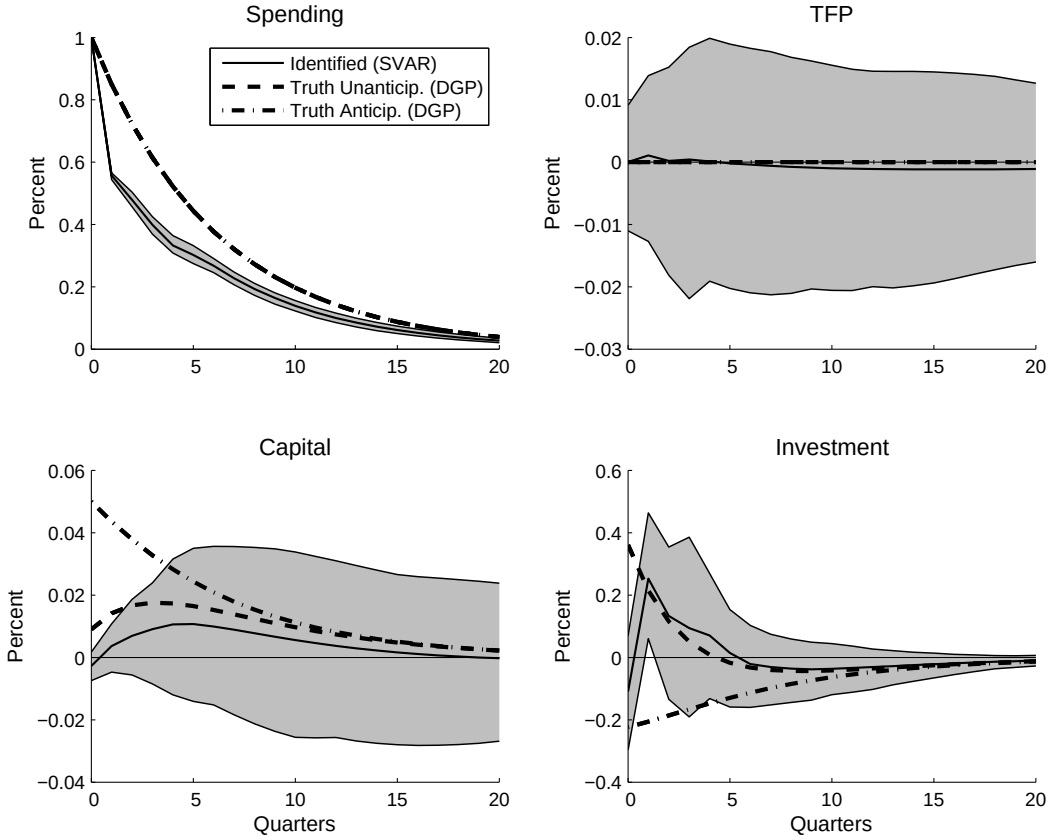

Figure 5: Monte Carlo impulse responses with standard recursive SVAR scheme, lower anticipation rate ($\theta = 0.58$). *Notes.* See Figure 4; β is set to 0.8 as in Figures 2 and 3.

do not matter much quantitatively even if they are ignored.

However, the latter is not true in general. Figure 5 reports the estimated impulse responses when the anticipation rate θ is reduced to 0.58 by reducing the subjective discount factor β to 0.8.[10] Observe that the effects on investment and capital of neither of the two shocks are correctly estimated. The SVAR responses indicate an initial decline in investment followed by an increase, whereas the 'true' investment response to the unanticipated shock is positive for several quarters while the response to the anticipated shocks is uniformly negative (after the anticipation period). Furthermore, there is a relatively strong downward bias in the estimated spending response. This means that the econometrician would overstate the overall expansionary effect of government expenditures on investment. Hence, there are realistic cases where the standard SVAR identification approach would lead an econometrician to misleading conclusions.[11]

[10]This is of course a relatively low value for β; alternatively, one could reduce the anticipation rate by increasing the intertemporal elasticity of substitution (which is equal to 1 with log utility) or lowering the capital share in production α (see Mertens and Ravn, 2010). A longer anticipation horizon would also create a stronger wedge between the effects of anticipated and unanticipated shocks.

[11]The bias in the estimated impulse responses is even larger, and more in line with Ramey's (forthcoming)

3.3 The expectations-based approach

The merits of the expectations-based identification approach are discussed next. Thus, instead of observing spending and TFP, the econometrician now observes the expectational error and the two-period ahead expectation of spending, and estimates a VAR in $\{\hat{g}_{t-j} - E_{t-1}\hat{g}_t, E_t\hat{g}_{t+2-j}, \hat{k}_{t-j}, \hat{i}^{obs}_{t-j}\}_{t=0}^{\infty}$. Notice that adding investment with a measurement error to the VAR goes without prejudice to the non-fundamentalness results discussed in the previous section. To see this, notice that the solution for investment reads

$$\hat{i}_t = \eta_{ik}\hat{k}_{t-1} + \eta_{ia}\hat{a}_t + \eta_{ig}\hat{g}_t + \eta_{i\varepsilon,2}\varepsilon^a_{g,t-1} + \eta_{i\varepsilon,1}\varepsilon^a_{g,t}, \tag{15}$$

and observed investment is $\hat{i}^{obs}_t = \hat{i}_t + \varepsilon_{i,t}$. Hence, the econometrician's VAR reads

$$\begin{bmatrix} \hat{g}_t - E_{t-1}\hat{g}_t \\ E_t\hat{g}_{t+2} \\ \hat{k}_t \\ \hat{i}^{obs}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{\rho_g^2}{1-\rho_g L} & \frac{1}{1-\rho_g L} & 0 & 0 \\ \frac{\eta_{kg}}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{kg}L^2+\eta_{k\varepsilon,2}(1-\rho_g L)(\theta+L)}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{ka}}{(1-\eta_{kk}L)(1-\rho_a L)} & 0 \\ \Theta_1(L) & \Theta_2(L) & \Theta_3(L) & 1 \end{bmatrix} \begin{bmatrix} \varepsilon^u_{g,t} \\ \varepsilon^a_{g,t} \\ \varepsilon_{a,t} \\ \varepsilon_{i,t} \end{bmatrix},$$

where the $\Theta_s(L)$, $s = 1, 2, 3$ follow from substituting out the capital, TFP and spending processes in (15). The determinant of the lag matrix is again equal to $\frac{\eta_{ka}}{(1-\rho_g z)(1-\eta_{kk}z)(1-\rho_a z)}$ as in process (14), implying that also the modified process is fundamental.[12] Notice that one could include more variables (e.g. output, consumption) in the VAR in a similar way.

Figure 6 reports the estimated SVAR impulse respones due to both types of spending shocks. The results show that the estimated effects fairly closely match those from the data-generating model. Under the unanticipated shock, the expectational error (the difference of spending in quarter t with expected spending in quarter t conditional on quarter $t-1$ information) increases in quarter 0. Investment declines and the impact response of the two-period ahead expectation (expected spending in quarter $t+2$ conditional on quarter t information) is close to the theoretical value of $\rho_g^2 = 0.72$. Under the anticipated shock, the two-period ahead expectation of spending increases by one percent in quarter 0, orthogonally to the expectational error. Importantly, investment increases during the anticipation period but it is negative from quarter 2 onwards, as the theory predicts.

Next, the previous exercise is repeated when the anticipation rate θ is reduced to 0.58

findings on the effects of anticipated shocks, if the standard deviation of anticipated shocks is increased relative to the standard deviation of unanticipated shocks (not reported).

[12] Notice that the determinant of a triangular matrix is equal to the product of the diagonal entries.

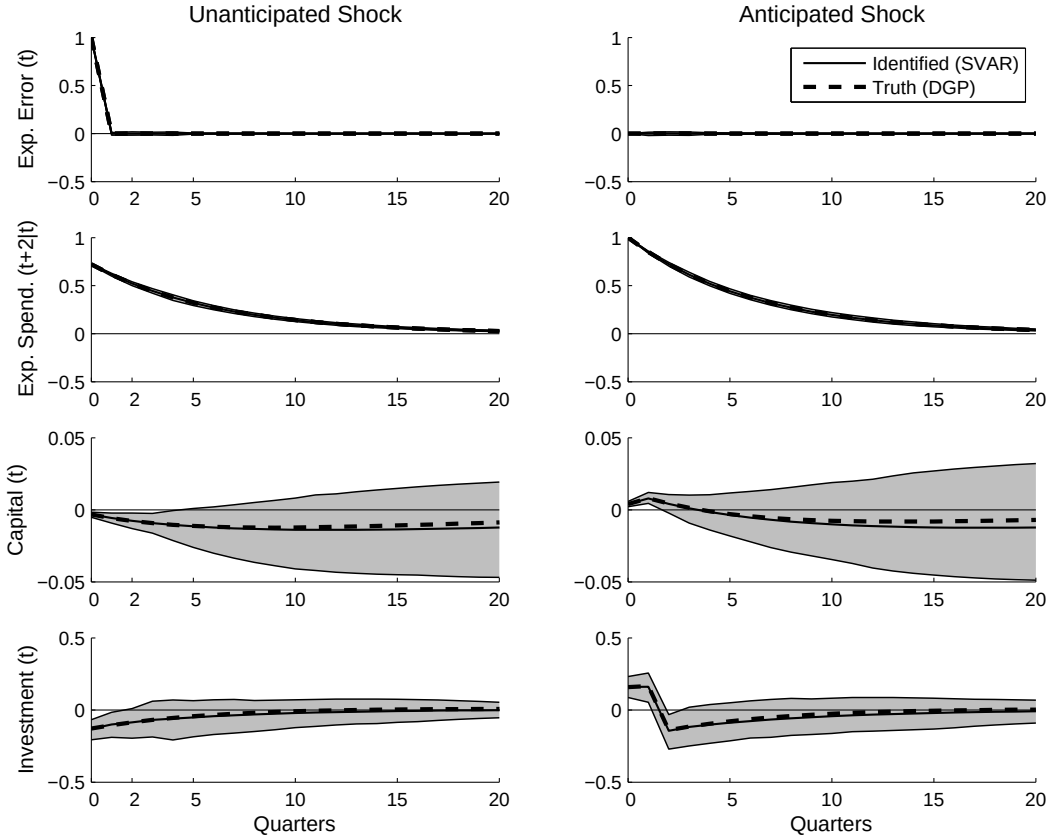

Figure 6: Monte Carlo impulse responses with expectations-based identification scheme, benchmark calibration ($\theta = 0.93$). *Notes.* See Figure 4; an unanticipated spending shock is identified by ordering the expectational error first in a Cholesky decomposition, the shock is a one percent increase in the expectational error in quarter 0; an anticipated spending shock is identified by ordering the two-period ahead expectation of spending second after the expectational error, the shock being a one percent increase in the two-period ahead expectation in quarter 0.

by reducing β to 0.8, which was seen to increase the relevance of the non-fundamentalness problem of the standard recursive SVAR approach. However, Figure 7 shows that for the expectations-based approach the results are robust to changes in θ: the estimated effects of both shocks are similarly close to the 'truth' as in the benchmark calibration. The identification approach is also robust to changes in the relative volatility of the two types of spending shocks (not shown).[13]

4 Robustness and Adjustments

This section discusses the results of three types of robustness exercises. First, the Monte Carlo experiment of the previous section is repeated for a smaller sample size. Second, the spending

[13]Results are available from the author.

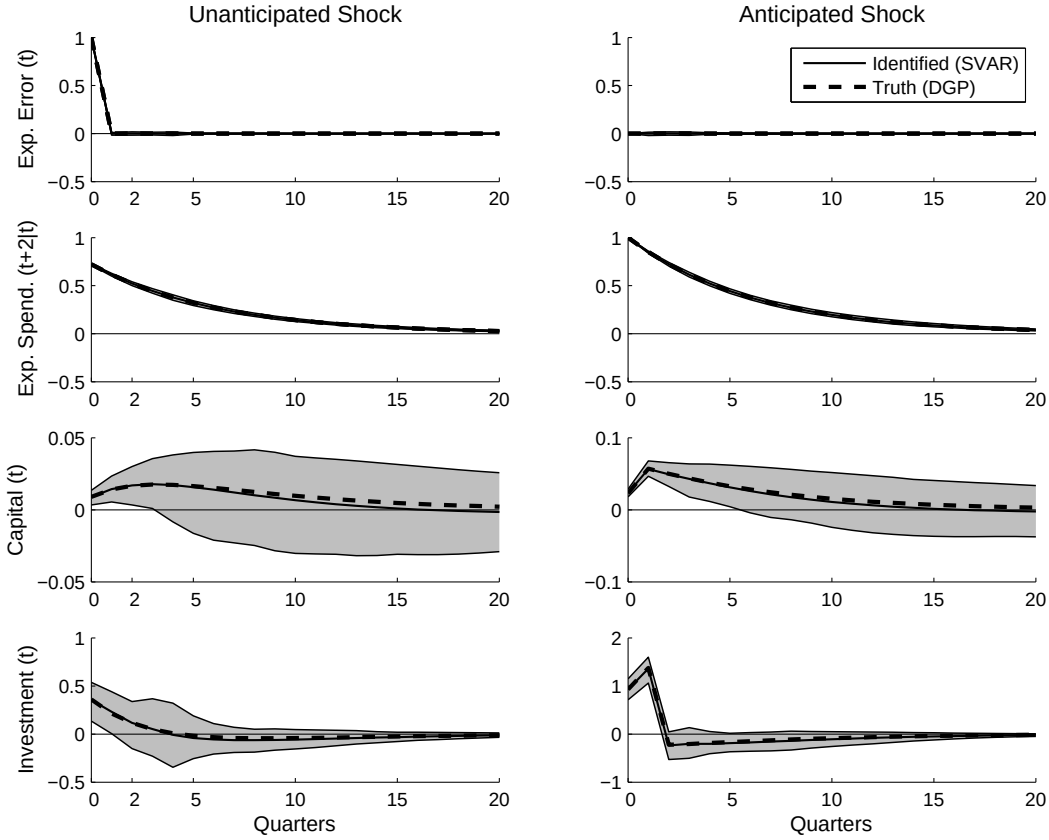

Figure 7: Monte Carlo impulse responses with lower anticipation rate ($\theta = 0.58$). *Notes.* See Figure 6; β is set to 0.8 to reduce θ.

process is modified in the sense that spending is allowed to react to other economic shocks–TFP shocks in the model considered here–both contemporaneously and with a lag. Based on the results, I propose some adjustments to the standard expectations-based approach. Third, I check whether surprise spending shocks can also be correctly identified in a VAR which does not include expectations on future spending but only expectational errors, as in Ramey (forthcoming) and Auerbach and Gorodnichenko (2010). The implications for the empirical application are discussed at the end of this section.

4.1 Small sample results

The results reported so far were based on large samples ($T = 10,000$). For Figure 8, the Monte Carlo exercise is repeated for an empirically realistic sample size of $T = 114$ and $M = 10,000$.[14] The reduction in the sample size implies that the data contains less information, so the error bands become wider. However, the point estimates remain close to the impulse responses of the data-generating process. Although estimates can be rather imprecise, the

[14]The sample size is equal to the data sample below, i.e. 114 quarters from 1981:4 to 2010:1.

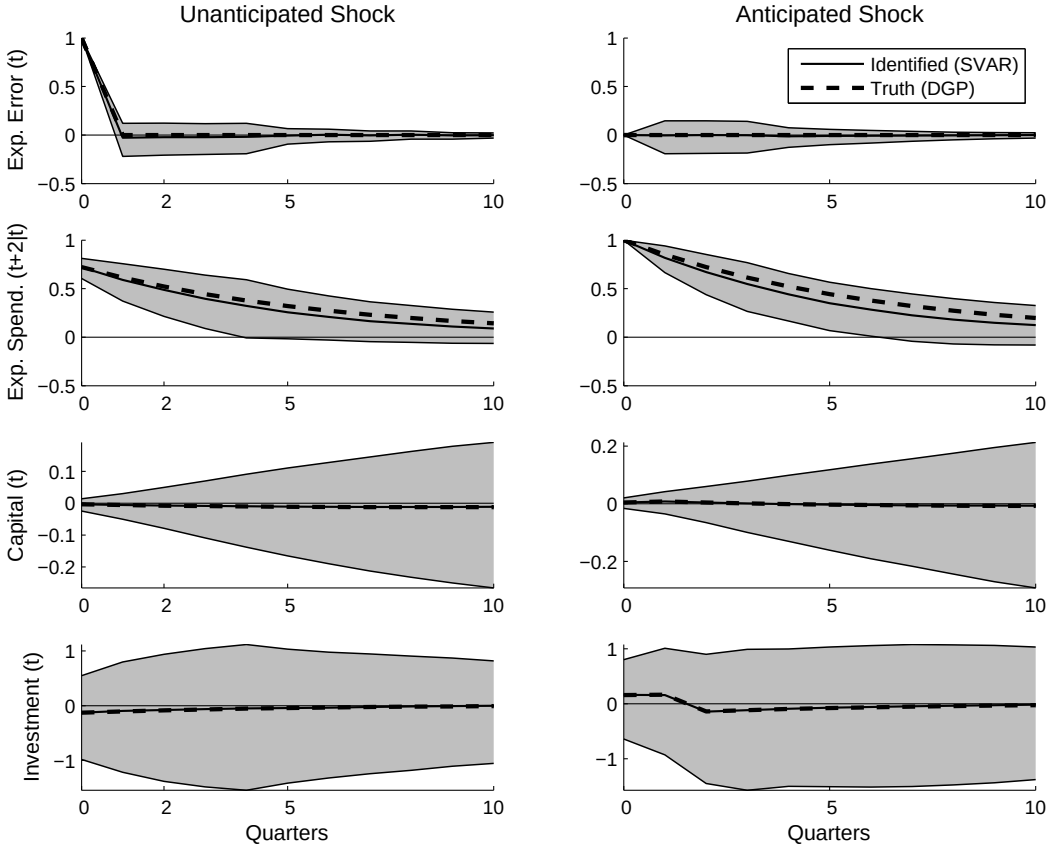

Figure 8: Monte Carlo impulse responses in small samples. *Notes.* See Figure 6; impulse responses are in percent; sample size is $T = 114$.

bias of the standard SVAR approach is still eliminated by the expectations-based approach.

4.2 Spending reaction to lagged TFP

Consider now an alternative spending process of the form

$$\log(g_t/\bar{g}) = \rho_g \log(g_{t-1}/\bar{g}) + \rho_{ga} \log a_{t-1} + \varepsilon^u_{g,t} + \varepsilon^a_{g,t-2}, \quad \rho_g \in [0,1), \quad \rho_{ga} \in \mathbb{R}. \tag{16}$$

According to (16), government spending can react with a one-period lag to changes in the state of productivity, through the coefficient ρ_{ga}. This modification of the spending process is a convenient short-cut for more complicated reactions of government spending to the state of the business cycle (e.g. movements in output or government revenues), which allows to obtain simple and easily tractable analytical expressions. Impulse responses from the modified model are shown in Figure 9. Without a spending reaction to TFP, i.e. when $\rho_{ga} = 0$, the productivity shock leads to an expansion in hours, output, consumption and investment (the investment response is shown below). Consumption and hours worked increase simultaneously

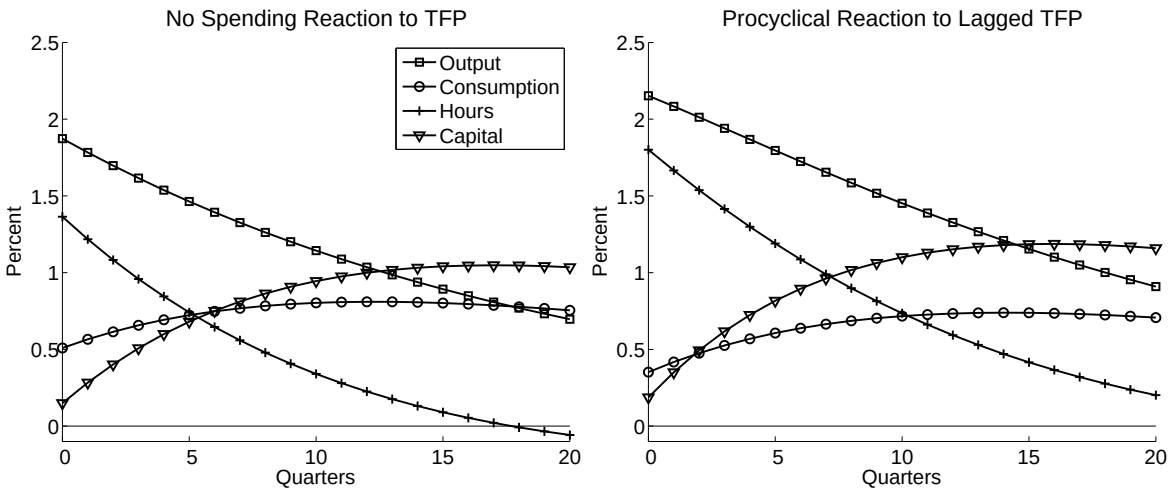

Figure 9: Model impulse responses to unanticipated productivity shocks, spending reaction to lagged TFP. *Notes.* Both panels show responses to one percent surprise increases in TFP; left panel: no spending reaction to TFP ($\rho_{ga} = 0$); right panel: procyclical spending reaction to lagged TFP ($\rho_{ga} = 1$).

since the substitution effect due to higher productivity is larger than the positive wealth effect on lifetime income. If spending reacts to lagged TFP in a procyclical manner, setting $\rho_{ga} = 1$, there is a smaller wealth effect due to government absorption of goods and services, so the increase in hours is stronger and the consumption increase is weaker.

For the modified spending process, the one-period expectational error is still given by $E_{t-1}\hat{g}_t - \hat{g}_t = \varepsilon^u_{g,t}$, since spending only reacts with a lag to productivity shocks. However, two-period ahead expected spending becomes

$$E_t\hat{g}_{t+2} = \frac{\rho_g^2}{1-\rho_g L}\varepsilon^u_{g,t} + \frac{1}{1-\rho_g L}\varepsilon^a_{g,t} + \frac{\rho_{ga}\rho_a}{1-\rho_a L}\varepsilon_{a,t},$$

implying that expected spending is affected by the current state of productivity. Suppose that the econometrician estimates a similar VAR as above:

$$\begin{bmatrix} \hat{g}_t - E_{t-1}\hat{g}_t \\ E_t\hat{g}_{t+2} \\ \hat{k}_t \\ \hat{i}_t^{obs} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{\rho_g^2}{1-\rho_g L} & \frac{1}{1-\rho_g L} & \frac{\rho_{ga}\rho_a}{1-\rho_a L} & 0 \\ \frac{\eta_{kg}}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{kg}L^2+\eta_{k\varepsilon,2}(1-\rho_g L)(\theta+L)}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta^*_{ka}}{(1-\eta_{kk}L)(1-\rho_a L)} & 0 \\ \Theta_1(L) & \Theta_2(L) & \Theta^*_3(L) & 1 \end{bmatrix} \begin{bmatrix} \varepsilon^u_{g,t} \\ \varepsilon^a_{g,t} \\ \varepsilon_{a,t} \\ \varepsilon_{i,t} \end{bmatrix}.$$

The determinant of the lag matrix is equal to $\frac{\eta^*_{ka}}{(1-\rho_g z)(1-\eta_{kk}z)(1-\rho_a z)}$, such that the process is fundamental.[15] However, the spending reaction to TFP shocks through the term

[15]Notice the presence of η^*_{ka}, which is equal to η_{ka} only for $\rho_{ga} = 0$. The coefficient η_{ca} also changes to η^*_{ca}, and similarly for $\Theta_3(L)$ which becomes $\Theta^*_3(L)$. Details are provided in the Appendix.

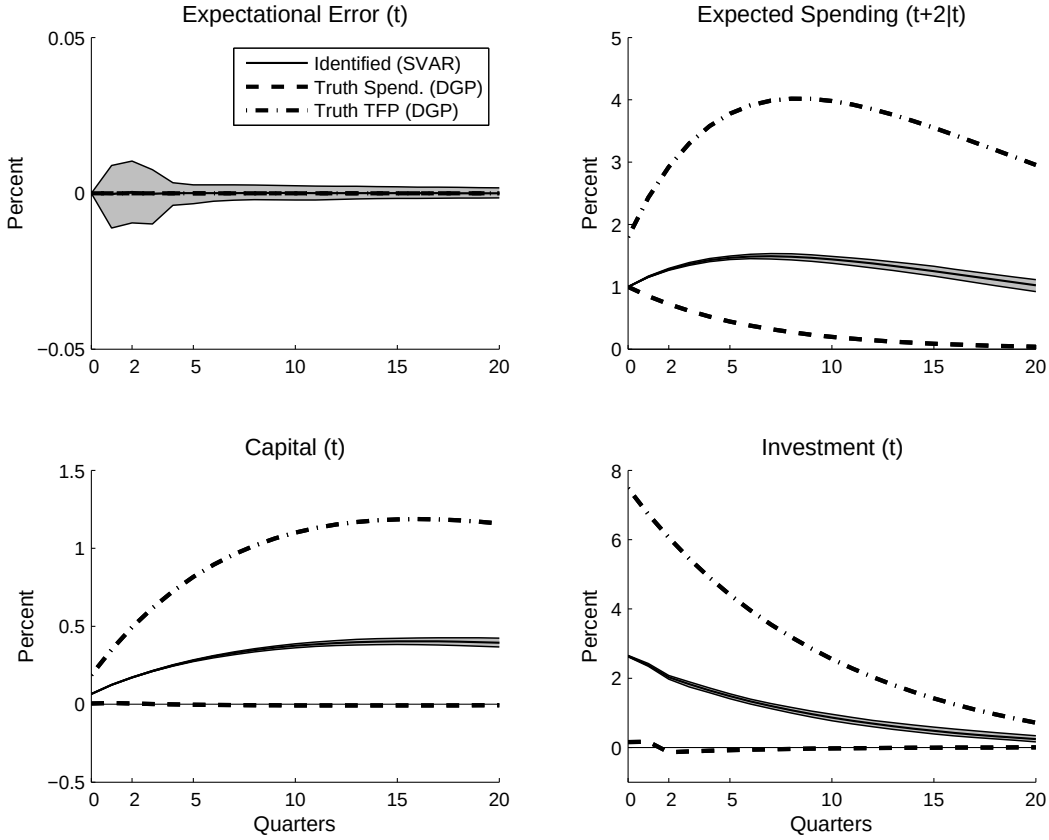

Figure 10: Monte Carlo impulse responses to anticipated spending shock when spending reacts to lagged productivity. *Notes.* See Figure 6; spending reacts procyclically to lagged TFP, setting $\rho_{ga} = 1$.

$\rho_{ga}\rho_a/(1-\rho_a L)$ makes the identification problem worse. The econometrician now cannot distinguish changes in expected spending due to anticipated spending shocks and TFP shocks, by conditioning on $\varepsilon^u_{g,t}$ only, since the modified process does not have a Cholesky structure.

Figure 10 shows the implications of the missing Cholesky structure, when the econometrician nevertheless attempts to estimate the effects of the anticipated spending shock. Since expected spending is now also driven by productivity shocks, the shock identified by an increase in expected spending which is orthogonal to the expectational error does not have the effects implied by the theoretical model. Instead, the estimated effects are located in between the responses to spending and TFP shocks from the data-generating process. Of course, the bias becomes smaller with a smaller reaction of spending to the state of productivity. However, it has been verified that even for relatively small feedbacks ρ_{ga} the SVAR identification produces a bias; for negative ρ_{ga} the bias turns negative.[16]

A natural way to address the issues caused by the reaction of spending to the state of

[16]The results are available from the author.

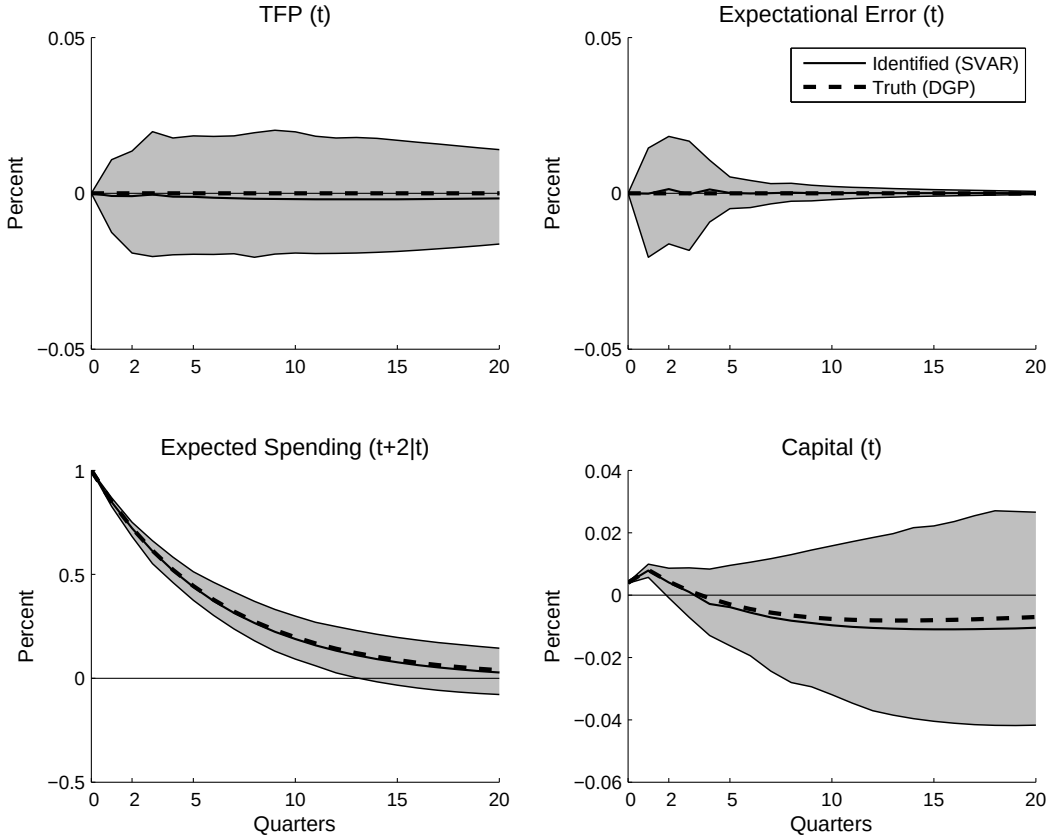

Figure 11: Monte Carlo impulse responses to anticipated spending shock when spending reacts to lagged productivity, TFP observed. *Notes.* See Figure 6; spending reacts procyclically to lagged TFP, setting $\rho_{ga} = 1$; identified shock is orthogonal to TFP.

productivity is to condition on TFP. That is, suppose the econometrician includes $\hat{a}_t$ as the first variable in the VAR:

$$\begin{bmatrix} \hat{a}_t \\ \hat{g}_t - E_{t-1}\hat{g}_t \\ E_t\hat{g}_{t+2} \\ \hat{k}_t^{obs} \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\rho_a L} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\rho_{ga}\rho_a}{1-\rho_a L} & \frac{\rho_g^2}{1-\rho_g L} & \frac{1}{1-\rho_g L} & 0 \\ \frac{\eta_{ka}^*}{(1-\eta_{kk}L)(1-\rho_a L)} & \frac{\eta_{kg}}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{kg}L^2+\eta_{k\varepsilon,2}(1-\rho_g L)(\theta+L)}{(1-\eta_{kk}L)(1-\rho_g L)} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{a,t} \\ \varepsilon_{g,t}^u \\ \varepsilon_{g,t}^a \\ \varepsilon_{k,t} \end{bmatrix},$$

Notice that investment has been dropped and instead there is a measurement error on capital, $\varepsilon_{k,t} \sim N(0, \sigma_k^2)$ with σ_k set to 0.01 percent. Furthermore, the surprise spending shock is now ordered second and the news shock third. This is again a fundamental process, the determinant of the lag matrix being equal to $[(1-\rho_g L)(1-\rho_a L)]^{-1}$, and the impact matrix

has a Cholesky structure:

$$\boldsymbol{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \rho_{ga}\rho_a & \rho_g^2 & 1 & 0 \\ \eta_{ka}^* & \eta_{kg} & \eta_{k\varepsilon,1} & 1 \end{bmatrix}.$$

Figure 11 shows that, by conditioning on TFP, the anticipated spending shock is well-identified again. Notice, however, that the requirements on the econometrician's information set have become more stringent under this modified identification scheme, since now also the level of TFP needs to be available as an observable variable.

4.3 Spending reaction to current TFP

If spending reacts contemporaneously to the state of productivity, the expectational error of spending is a weighted average of unanticipated spending and TFP shocks, the weight on TFP shocks given by the strength of the spending reaction. That is, when

$$\log(g_t/\bar{g}) = \rho_g \log(g_{t-1}/\bar{g}) + \rho_{ga} \log a_t + \varepsilon_{g,t}^u + \varepsilon_{g,t-2}^a, \quad \rho_g \in [0,1), \quad \rho_{ga} \in \mathbb{R}, \tag{17}$$

the expectational error of spending is a mix of TFP shocks and surprise spending shocks: $\hat{g}_t - E_{t-1}\hat{g}_t = \rho_{ga}\varepsilon_{a,t} + \varepsilon_{g,t}^u$.[17] If TFP remains unobserved, the identification of surprise spending shocks based on the expectational error would therefore fail. This is documented in Figure 12, which shows the identified responses to the unanticipated spending shock, based on the standard expectations-based scheme. Similarly as before, the estimated effects are located in between the responses to spending and TFP shocks from the data-generating process.

However, if TFP can be observed, the previous results go through. The econometrician could estimate the VAR

$$\begin{bmatrix} \hat{a}_t \\ \hat{g}_t - E_{t-1}\hat{g}_t \\ E_t\hat{g}_{t+2} \\ \hat{k}_t^{obs} \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\rho_a L} & 0 & 0 & 0 \\ \rho_{ga} & 1 & 0 & 0 \\ \frac{\rho_{ga}\rho_a^2}{1-\rho_a L} & \frac{\rho_g^2}{1-\rho_g L} & \frac{1}{1-\rho_g L} & 0 \\ \frac{\eta_{ka}^{**}}{(1-\eta_{kk}L)(1-\rho_a L)} & \frac{\eta_{kg}}{(1-\eta_{kk}L)(1-\rho_g L)} & \frac{\eta_{kg}L^2+\eta_{k\varepsilon,2}(1-\rho_g L)(\theta+L)}{(1-\eta_{kk}L)(1-\rho_g L)} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{a,t} \\ \varepsilon_{g,t}^u \\ \varepsilon_{g,t}^a \\ \varepsilon_{k,t} \end{bmatrix},$$

and apply the expectations-based identification scheme, conditioning on TFP when estimat-

[17] Under process (17), the two-period ahead expectation of spending is given by

$$E_t\hat{g}_{t+2} = \frac{\rho_{ga}\rho_a^2}{1-\rho_a L}\varepsilon_{a,t} + \frac{\rho_g^2}{1-\rho_g L}\varepsilon_{g,t}^u + \frac{1}{1-\rho_g L}\varepsilon_{g,t}^a.$$

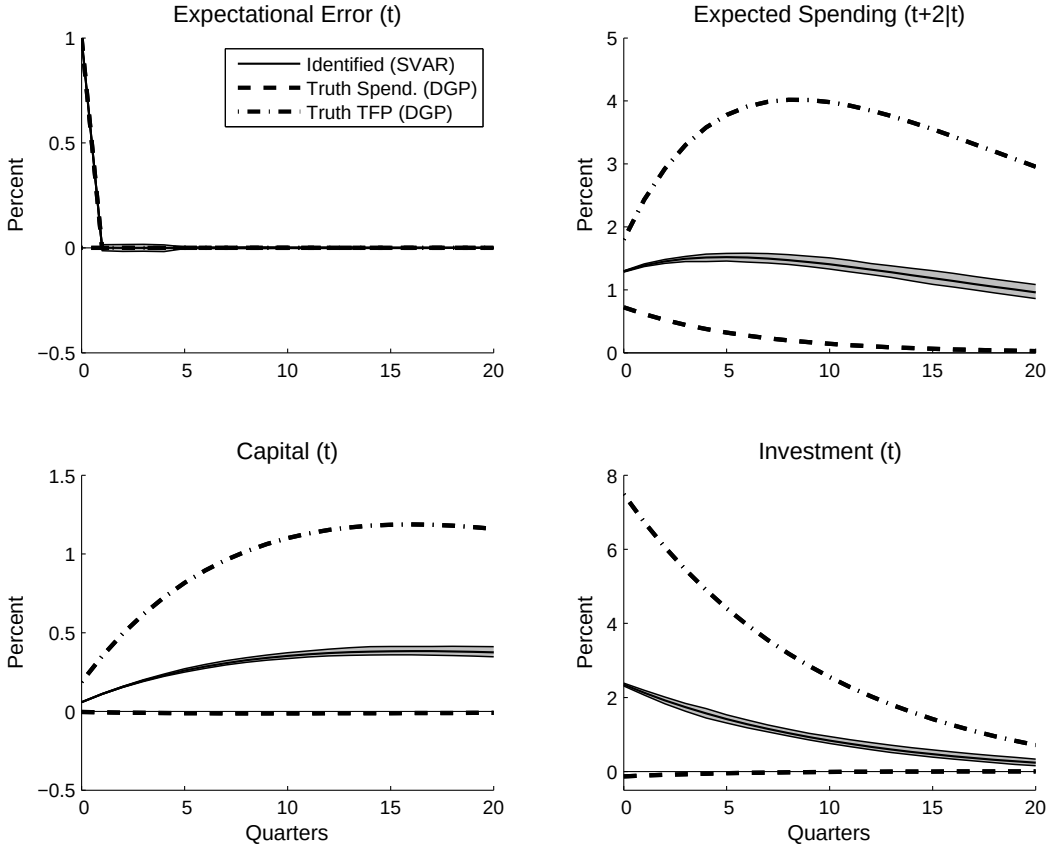

Figure 12: Monte Carlo impulse responses to unanticipated spending shock when spending reacts to current productivity. *Notes.* See Figure 6; spending reacts procyclically to current TFP, setting $\rho_{ga} = 1$.

ing the effects of spending shocks. The resulting impulse responses to a surprise spending shock are shown in Figure 13, indicating that the shock is well-identified by the adjusted scheme. Even if government expenditures react contemporaneously (within a quarter) to the level of productivity, if TFP can be observed the econometrician would correctly recover the structural spending shocks.

4.4 Surprise shocks under non-fundamentalness

Ramey (forthcoming) and Auerbach and Gorodnichenko (2010) suggest to extract the unanticipated component of exogenous movements in government spending through expectational errors. Similarly as above, they interpret an innovation to the forecast error as an unanticipated shock. However, they do not include expectations on future spending in the regression equation. It can be shown, similarly as above, that the VAR specified in this way does not have a fundamental moving average representation. The question is whether an econometrician who applies a 'non-fundamental' identification strategy of this type would still correctly

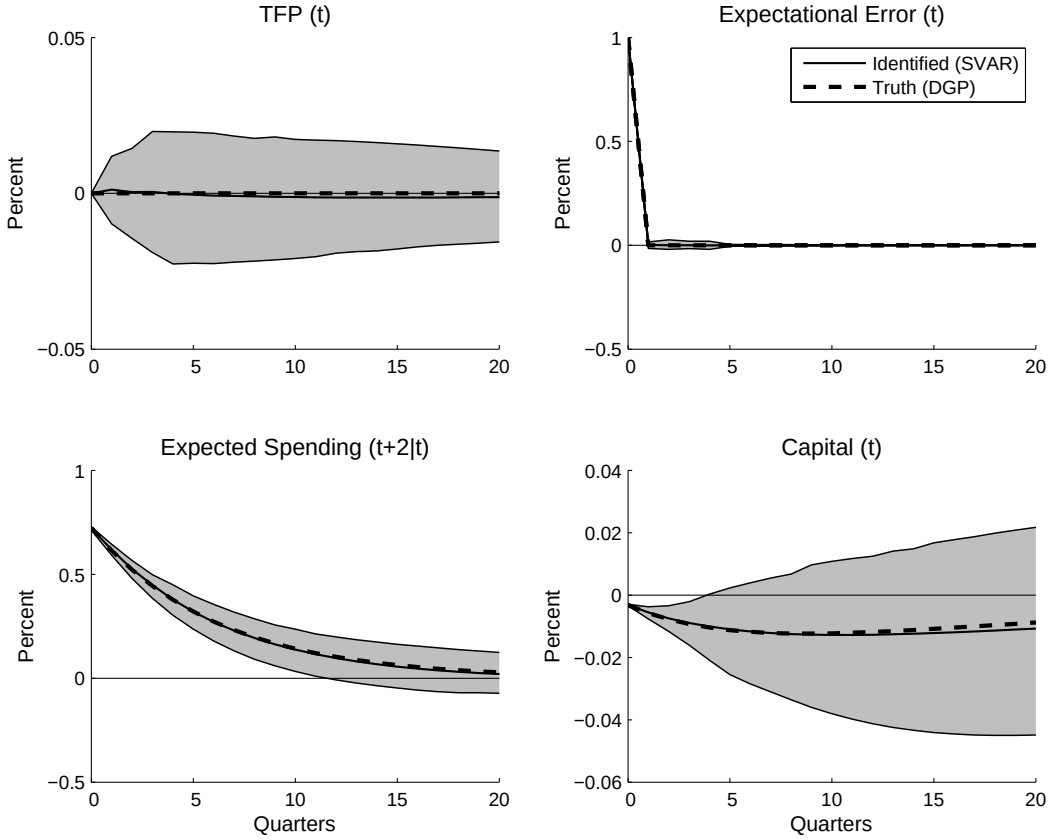

Figure 13: Monte Carlo impulse responses to unanticipated spending shock when spending reacts to current productivity, TFP observed. *Notes.* See Figure 6; spending reacts procyclically to current TFP, setting $\rho_{ga} = 1$; identified shock is orthogonal to TFP.

estimate the effects of unanticipated spending shocks.

Figure 14 compares point estimates for ten simulated data sets, for the benchmark specification of the model (as in Figure 6). The left-hand panels show results from the expectations-based identification, where the two-period ahead expectation of spending is included in the VAR. The right-hand panels show results from the non-fundamental identification, where expected future spending is not included. The estimated VARs thus have identical variables and simulated data except for expected future spending. In both cases, without affecting the effects on the remaining variables, consumption is added as an additional observed variable through a small measurement error of 0.1 percent on observed consumption in the model. Unlike the 'full' expectations-based identification, the non-fundamental identification produces a downward bias in the estimated responses of investment, capital and consumption, especially at longer horizons. These results indicate that an SVAR identification strategy based on expectational errors, but where the VAR has a non-fundamental representation, may not be appropriate to estimate the effects of surprise spending shocks.

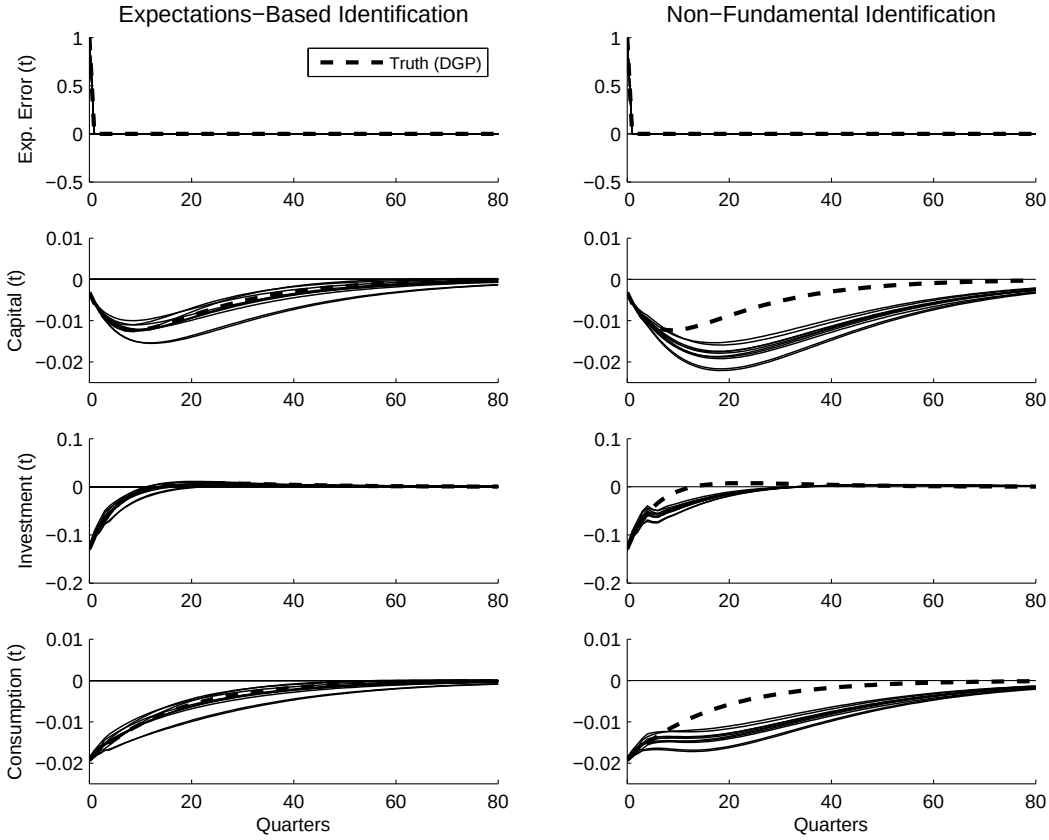

Figure 14: Monte Carlo impulse responses to unanticipated spending shock when expected future spending is not included in the VAR, benchmark specification. *Notes.* See Figure 6; left-hand panels: two-quarter ahead expectation of spending included in the VAR; right-hand panels: two-period ahead expectation of spending not included in the VAR.

4.5 Implications for applied research

What are the implications of those results for applied research on the effects of government expenditures? First, the results show that an expectations-based identification approach can help to solve the non-fundamentalness problem which distorts econometric inference based on the standard recursive SVAR identification approach. For the simple information flows considered above, at least, the requirement is that expectations on future spending up the anticipation horizon of private agents should be included in the VAR.

Second, with respect to the identification problem of distinguishing anticipated spending shocks from surprise spending shocks and other economic shocks–on TFP in the example above–the results are mixed. If future spending is affected by other shocks, the econometrician needs to know, observe and condition upon those shocks. Since there is uncertainty on which shocks affect government spending and/or revenues, the expectations-based approach is therefore likely to fail. In addition, most structural shocks are unobserved state variables, so

they cannot be included in the econometrician's information set.[18] Hence, the expectations-based identification of anticipated spending shocks is prone to significant problems.[19]

The good news is that, by exploiting variation in expectational errors, surprise spending shocks can be robustly identified-if at the same time expected future spending is controlled for in the VAR unlike in Ramey (forthcoming) and Auerbach and Gorodnichenko (2010). This is the case as long as spending reacts with some lag to other economic shocks. If spending reacts contemporaneously to other shocks, the econometrician again needs to condition on those shocks. However, since government spending is usually defined as government final consumption expenditures plus government investment (see Blanchard and Perotti, 2002), the assumption that spending does not react within a quarter to other shocks seems justified. If spending is defined net of transfer and interest payments it is arguably acyclical, so there is no automatic reaction of spending to movements in the business cycle. Further, due to implementation lags in the policy process, a discretionary fiscal response to economic shocks is unlikely to occur within a quarter.

5 Empirical Application

This section discusses empirical results for the U.S., when survey data on federal government expenditures obtained from the Survey of Professional Forecasters is taken as a measure of private sector expectations. Given the above results, the empirical application focuses on the effects of surprise spending shocks. The discussion below focuses on a comparison of the estimates from the expectations-based approach with those implied by the standard recursive SVAR identification approach which does not take into account anticipation effects.

5.1 Data description

Figure 15 shows the percentage deviations of real federal government spending from the predictions of the respondents to the Philadelphia Fed's Survey of Professional Forecasters, over the period 1981:4 to 2010:1. Government spending is defined as the sum of government consumption and gross investment, following for instance Blanchard and Perotti (2002). Details on data definitions are provided in the Appendix. The expectational errors are computed on

[18] Few exceptions such as multifactor productivity estimates are available from official sources. Even so, the Bureau of Labor Statistics (the main source of productivity data on the U.S.) only provides productivity estimates on an annual basis since some data on inputs is not available at a higher frequency.

[19] Of course, things are even worse for the econometrician if there is not only news on future fiscal variables but also on future economic shocks (e.g. productivity news, see Beaudry and Portier, 2006) to which spending might react in the future. Then the econometrician would need to condition on *expectations* of future unobserved state variables.

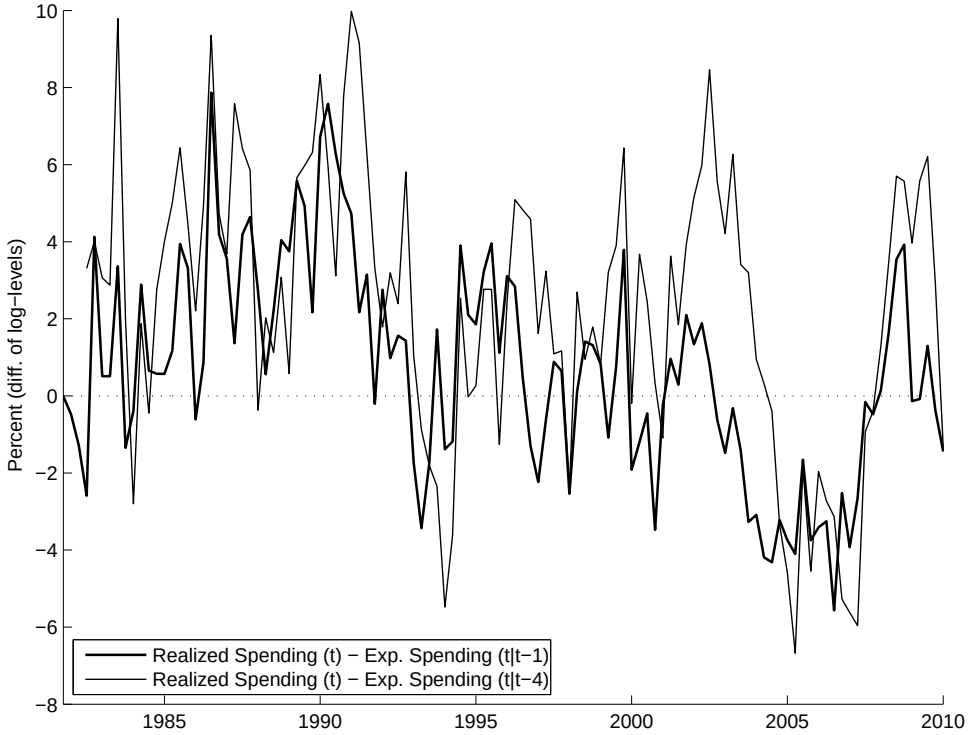

Figure 15: Expectational errors of U.S. federal government spending. *Notes.* Quarterly data, 1981:4 to 2010:1; expectational errors are computed as log differences (in %) of real spending in quarter t and the prediction thereof made in quarters $t-1$ or $t-4$.

the basis of the average predictions across all panelists made one and four quarters earlier. The forecasts submitted in quarter t are also taken conditional on quarter t information, although officially the survey takes forecasts made in t conditional on $t-1$ information.[20] The reason is that the questionnaires are sent out right after the advance report of the Bureau of Economic Analysis (BEA) is released, which contains the first estimate of GDP and its components for the previous quarter. However, the forecasters form their expectations conditional on all information they have available in period t, which is not necessarily restricted to the BEA report, so conditioning on the information set at the time the forecast is made seems reasonable.[21]

5.2 Expectations-based identification

The expectational errors shown in Figure 15 indicate the presence of a pronounced unanticipated component in federal government expenditures. Thus, a natural next step is to exploit

[20] The official documentation on the survey is available from `http://www.philadelphiafed.org`.

[21] Note that the forecasts for levels are originally scaled to the national accounts base year in effect at the time the survey questionnaire was sent to the forecasters. Over time, as benchmark revisions to the data occur, the scale changes. Since there have been a number of changes of base year in U.S. national accounts since the survey began, the forecasts were therefore scaled to the current base year, 2005, through backcasting by the actual growth rates and imposing the average growth rate over the sample at the break points.

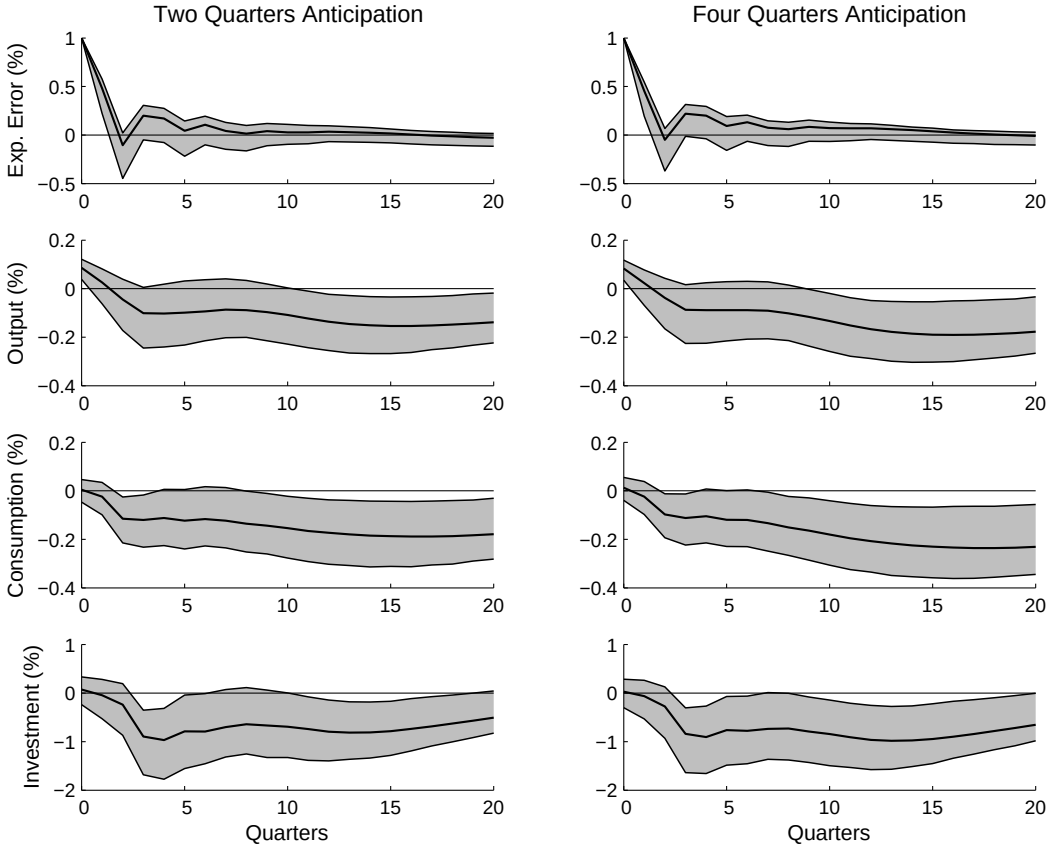

Figure 16: Empirical impulse response functions due to surprise spending shock identified by expectations-based approach. *Notes.* Normalized one percent increase in federal government spending; surprise spending increase in quarter 0; the VAR includes the one-quarter expectational error and the two-quarter (left panels) and four-quarter (right panels) ahead expectations of spending; 90% two-sided confidence bands are calculated by 1,000 bootstrap replications.

the variation in those expectational errors, using SVAR analysis, to estimate the effects of surprise spending changes, by applying the identifying strategy discussed above. In order to achieve fundamentalness, expectations on future government spending are included in the VAR. However, the precise anticipation horizon of private agents is uncertain. I therefore estimate two reduced-form VARs by ordinary least squares which include the one-quarter expectational error of spending and, respectively, the two- and four-quarter ahead expectations of spending. Real GDP, private consumption and private investment are added as additional variables, measured in log levels. Both VARs include four lags of the endogenous variables, a constant and a quadratic time trend.[22]

A surprise spending shock is identified as a one-percent increase in the expectational error which is orthogonal to all other innovations by a Cholesky decomposition of the reduced-form

[22]The results are robust to the use of a linear trend and three or five lags (not reported).

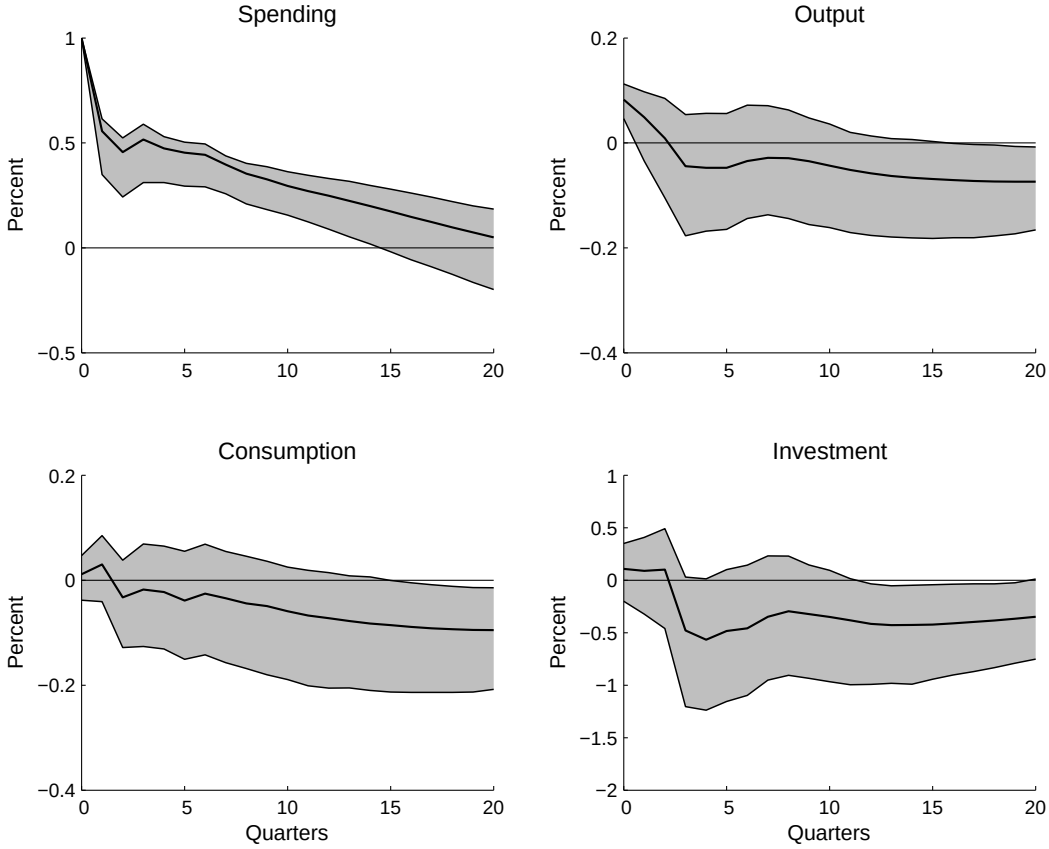

Figure 17: Empirical impulse response functions due to spending shock identified by standard recursive approach. *Notes.* See Figure 16; the VAR does not include expectations data; a government spending shock is identified by ordering spending growth before the remaining variables in a Cholesky decomposition.

covariance matrix. Figure 16 shows the estimated mean impulse responses of the expectational error, output, consumption and investment to this shock, together with their 90% two-sided bootstrap confidence bands. According to both VARs, the spending shock leads, on average, to an initial increase in output. Consumption and investment hardly react on impact, but start to decline shortly after the unexpected spending increase. After some time, both components of private demand turn significantly negative leading to a reversal of the output effect in the medium to long run.

5.3 Comparison with standard SVAR identification

In order to compare the expectations-based estimates with the standard SVAR estimates, Figure 17 reports results obtained from an application of the standard recursive identification approach (see Fatás and Mihov, 2001; Blanchard and Perotti, 2002; Perotti, 2005), where a government spending shock is identified as a one-percent increase in spending which

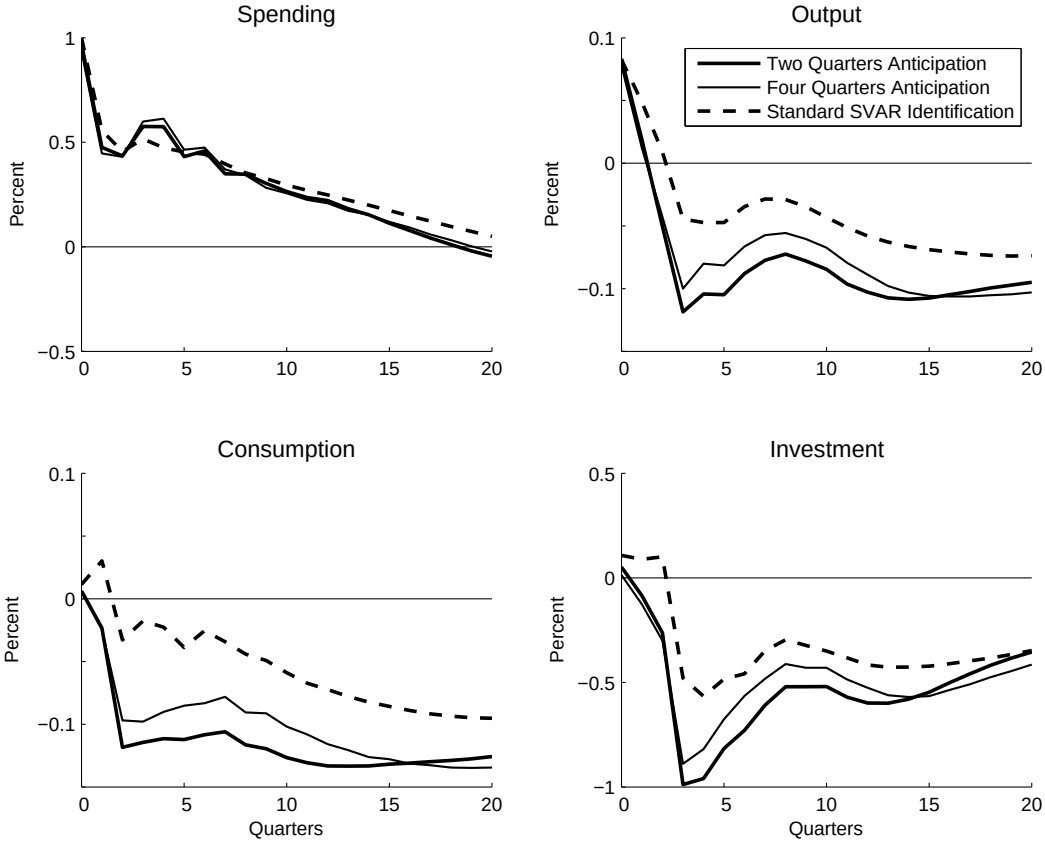

Figure 18: Empirical impulse response functions due to (unanticipated) spending shocks identified by expectations-based approach and standard recursive approach. *Notes.* See Figures 16 and 17; the expectations-based VARs include two-/four-quarter ahead expectations of spending (ordered third) and realized spending (ordered second).

is orthogonal to the innovations in output, consumption and investment by a Cholesky decomposition. In contrast to the previous results, a shock which is identified in this way leads to increases in consumption and investment during two quarters, which are however not significant at the 90% level. In addition, both impulse responses as well as the output response are more persistent than under the expectations-based approach, and do not turn significantly negative until towards the end of the horizon considered (five years).

The expectations-based VARs considered above do not include the level of government spending as an endogenous variable but the standard VAR does. Thus, to make the results comparable, I estimate two additional regressions where the level of spending is added (ordered second) next to the expectational error and the expectations of future spending. The point estimates from these VARs are compared to the point estimates from the standard VAR in Figure 18. The results show that, although the responses of spending are very similar in terms of size and persistence, the responses of output, consumption and investment are, on average, much smaller under the expectations-based identification scheme.

Table 2: Multipliers due to government spending shocks of size 1% of GDP.[a]

	Impact	4 qrts.	8 qrts.	12 qrts.	20 qrts.	Maximum
Two quarters anticipation[b]						
GDP	1.10	-1.57	-1.02	-1.28	-1.29	1.10 (impact)
Spending	1.00	0.59	0.36	0.24	-0.02	
Four quarters anticipation[b]						
GDP	1.07	-1.37	-0.79	-1.09	-1.44	1.07 (impact)
Spending	1.00	0.64	0.40	0.24	0.00	
Standard SVAR identification						
GDP	1.06	-0.57	-0.37	-0.66	-0.95	1.06 (impact)
Spending	1.00	0.52	0.40	0.27	0.07	

[a] The multipliers on GDP are computed according to the following formula: multiplier in quarter t = GDP response in quarter t/(spending response in quarter 0 times average share of spending over GDP over the sample).
[b] For the expectations-based identification, the multipliers are computed from two different VARs which include the two-quarter and four-quarter ahead expectations of government spending, respectively.

Following Blanchard and Perotti (2002), Table 2 compares the dollar change in GDP due to the dollar change in government spending at different horizons, for the three estimated VARs. The table entries can be interpreted as multipliers for GDP after a fiscal shock leading to an initial increase in the level of government spending of size one percent of GDP. The results show that both the expectations-based approach and the standard recursive approach yield multipliers on GDP of approximately 1.1 on impact. However, the expectations-based approach yields multipliers smaller than minus 1 at longer horizons, whereas the estimated multipliers from the standard recursive approach are uniformly larger than that.

5.4 Predictability of shocks

One possible explanation for the differences between the results from the two approaches is that the impulse responses to the standard SVAR shocks may incorporate some of the effects of anticipated spending shocks. The standard recursive approach may then pick up the upward-sloping paths of the responses of consumption and investment to shocks which were anticipated for some quarters in advance, whereas the econometrician treats the spending increase as if it was unanticipated. In fact, Ramey (forthcoming) shows that the standard SVAR shocks for federal government spending are predictable by professional forecasts made one to four quarters earlier. A perverse but likely implication of this result is that, due to anticipation effects, the econometrician would not capture the 'true' economic impact of government expenditures, even if surprise spending shocks are the only object of interest.

Thus, Figure 19 compares the identified shocks from the two approaches for the period

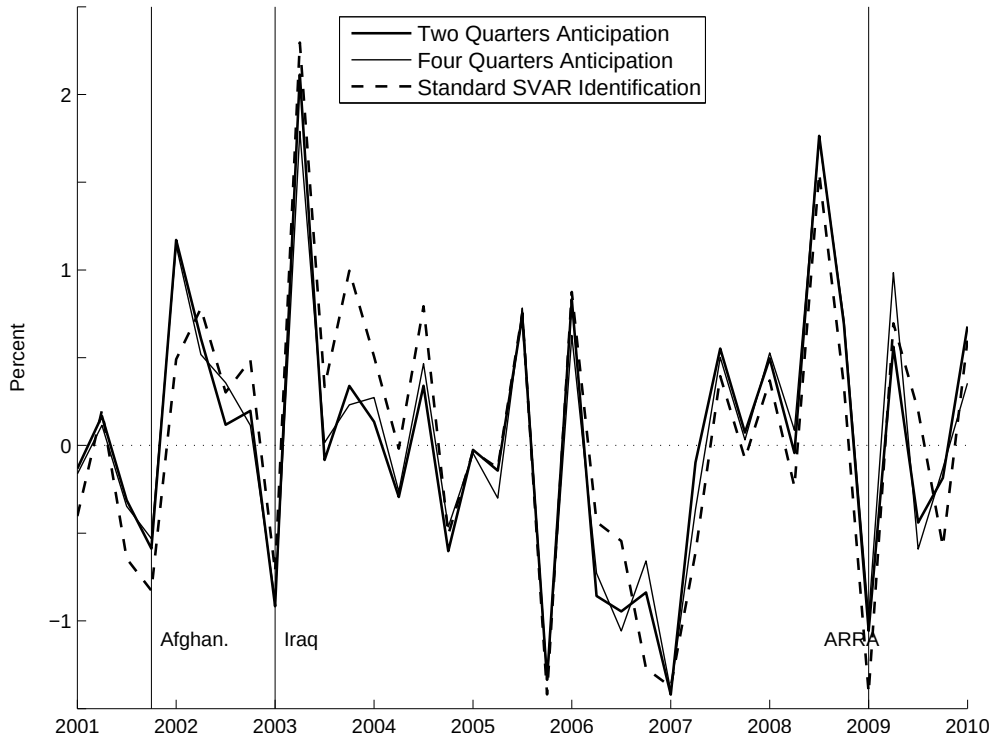

Figure 19: Surprise spending shocks identified by expectations-based approach and standard SVAR shocks, 2001:1 to 2010:1. *Notes.* See Figures 16 and 17; the expectations-based VAR includes two-quarter ahead expectation of spending.

2001:1 to 2010:1. During this period, three easily identified events have affected U.S. federal expenditures. The first two are the wars in Afghanistan and Iraq which began, respectively, on October 7, 2001 and March 20, 2003. The third event is the American Recovery and Reinvestment Act (ARRA) which was signed into law by President Obama on February 17, 2009. These events are marked by the vertical lines in Figure 19. The figure shows that spending shocks are identified immediately after all three events. However, the standard SVAR approach identifies positive shocks during about two years after the beginning of the war in Iraq, whereas the expectations-based approach does not identify any large surprise spending shocks during this time. Hence, one may suspect that some of the standard SVAR shocks were indeed anticipated by private agents.

In order to check this, following Ramey (forthcoming), I check whether the professional forecasts Granger-cause the identified shocks from the two approaches. In particular, I perform a series of F-tests where the unrestricted test equation has the form

$$x_t = a_0 + a_1 x_{t-1} + a_2 x_{t-2} + \cdots + a_p x_{t-p} + b_1 f_{t|t-1} + b_2 f_{t|t-2} + \cdots + b_h f_{t|t-h},$$

Table 3: Granger causality tests on identified shocks.

Independent variable[a,b]	F-statistic		5% critical value		P-value	
	$p = 1$	$p = 4$	$p = 1$	$p = 4$	$p = 1$	$p = 4$
Standard SVAR identification						
One-quarter ahead forecasts	1.87	3.36*	3.93	3.93	0.17	0.07
Two-quarter ahead forecasts	2.02	2.21	3.93	3.93	0.16	0.14
Three-quarter ahead forecasts	5.52**	5.77**	3.93	3.93	0.02	0.02
Four-quarter ahead forecasts	3.50*	3.07*	3.93	3.93	0.06	0.08
All forecasts simultaneously	4.60**	4.39**	3.93	3.93	0.03	0.04
Two quarters anticipation[c]						
Two-quarter ahead forecasts	0.00	0.10	3.93	3.93	0.95	0.75
All forecasts simultaneously	0.04	0.35	3.93	3.94	0.85	0.56
Four quarters anticipation[c]						
Four-quarter ahead forecasts	0.96	1.19	3.93	3.93	0.33	0.28
All forecasts simultaneously	0.80	1.24	3.93	3.94	0.38	0.27

[a] The dependent variable collects the identified shocks in quarter t. They are regressed on a constant, p own lags and the log difference of forecasted spending for quarter t made one to four quarters earlier.
[b] The null hypothesis is that the forecasts do not Granger-cause the shocks. ** indicates rejection of the null hypothesis at the 5% significance level, * at the 10% significance level.
[c] For the expectations-based identification, the identified shocks from the VARs which include the two-quarter and four-quarter ahead expectations of government spending are taken as dependent variables, respectively.

and where the restricted equation is

$$x_t = a_0 + a_1 x_{t-1} + a_2 x_{t-2} + \cdots + a_p x_{t-p},$$

where x_t denotes the identified shocks in quarter t and $f_{t|t-1}, \ldots, f_{t|t-h}$ the log first differences of forecasts on real federal spending made up to h quarters earlier. The null hypothesis is thus that the forecasts $f_{t|t-1}, \ldots, f_{t|t-h}$ do not Granger-cause the shocks; that is, the shocks are not actually forecastable by the professional forecasters' predictions.

Table 3 reports the test results when the forecasts are added individually and jointly as independent variables in the unrestricted regression, for both $p = 1$ and $p = 4$ lags of the dependent variables. The results show that, indeed, the standard SVAR shocks are predictable on average by the professional forecasters' predictions. When the forecasts are added jointly in the unrestricted regression, the null hypothesis that the forecasts do not Granger-cause the shocks is clearly rejected at the 5% significance level. On the other hand, the null hypothesis cannot be rejected for the shocks identified on the basis of expectational errors. These results indicate that the expectations-based approach is successful in extracting the unpredictable part of exogenous spending changes. Any bias of the standard SVAR approach should therefore be eliminated by the expectations-based identification scheme.

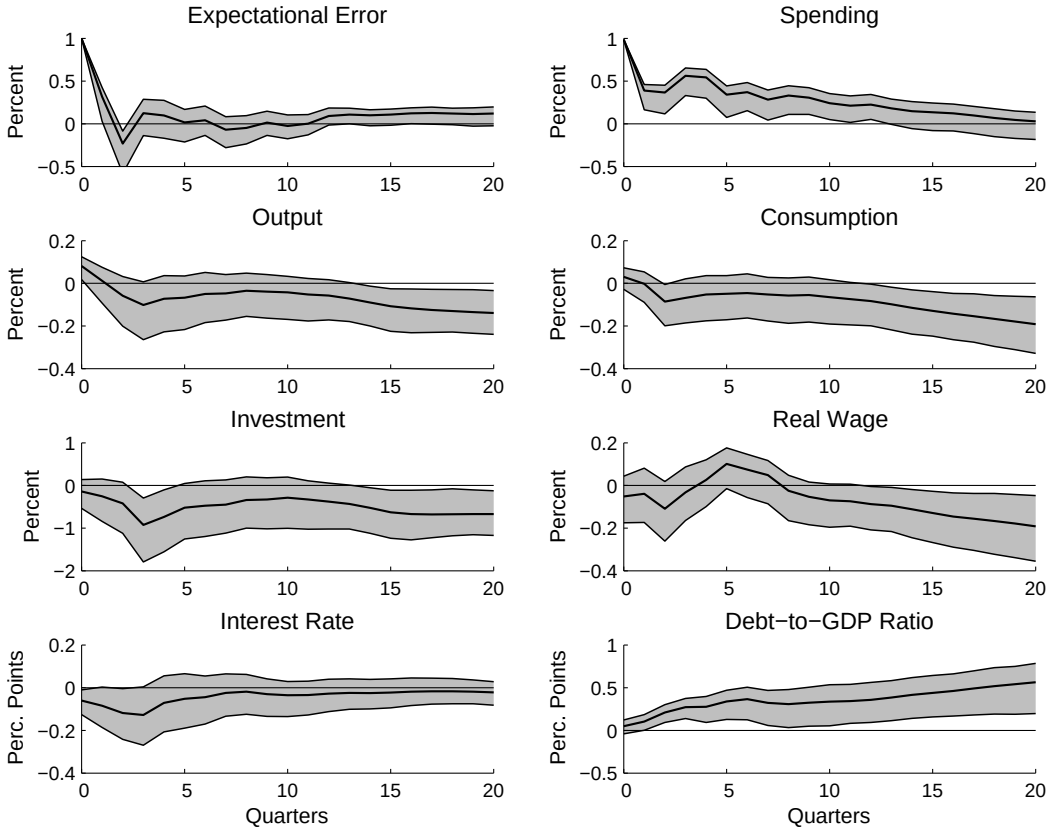

Figure 20: Empirical impulse response functions due to surprise spending shock identified by expectations-based approach, extended regression specification. *Notes.* See Figure 16; the VAR includes the two-quarter ahead expectation of spending.

5.5 Extended regression specification

As a final step of the analysis, I investigate the impact of surprise spending shocks on a broader set of indicators. That is, the real wage, the 3-month treasury bill rate and the federal government debt-to-GDP ratio are added as additional endogenous variables in the VAR. The real wage is added since it is an important variable in the controversy on the effects of government spending shocks.[23] The T-bill rate is added to assess the impact of spending shocks on interest rates. The debt-to-GDP ratio is included in order to capture the financing aspect of spending changes, and also to address the problem of omitted state variables of SVAR analysis (see Chari, Kehoe, and McGrattan, 2005).

Figure 20 shows the results. The estimated VAR includes the two-quarter ahead expectation of spending and the level of spending, as before.[24] Also in the extended specification, the spending increase has small effects on output and it leads to a decline in consumption and investment. The response of the real wage is insignificant in the short run and it is neg-

[23]See, for instance, Linnemann and Schabert (2003), Perotti (2007), or Ramey (forthcoming).
[24]The results are again robust to using the four-quarter ahead expectation of spending instead.

ative at longer horizons. The interest rate declines immediately. In terms of financing, the spending increase leads to a sustained increase in the debt-to-GDP ratio. Hence, also in the extended specification a surprise spending shock is estimated to have contractionary effects in the medium to long run, in contrast to most of the previous SVAR literature but in line with the findings of, for instance, Mountford and Uhlig (2009) and Ramey (forthcoming).

6 Conclusions

This paper has demonstrated how the econometric problems created by foresight on government spending can be addressed when private agents' expectations on future spending are included, through survey data, in a structural VAR data set. By a combination of theory and SVAR simulations, I have shown that adding survey expectations to the regression equation not only solves the non-fundamentalness problem created by foresight but also makes it possible to identify structural shocks. Even in the presence of endogenous spending feedbacks from other structural shocks, surprise spending shocks are robustly identified if spending reacts with some lag to those other shocks and if expectations on future spending are also included in the VAR. However, the standard expectations-based approach may fail to correctly recover the impact of anticipated spending shocks.

Therefore, I have focused on the effects of surprise spending shocks in an application of the approach to U.S. data. I have used the Survey of Professional Forecasters as a source of private sector expectations on federal government spending, including expectational errors. The empirical results suggest that the main findings of previous SVAR studies may indeed need to be qualified due to the presence of anticipation effects, in line with the concerns raised by Leeper et al. (2009) and Ramey (forthcoming). Using the expectations-based approach, I estimate positive short-run output effects of federal government expenditures, but negative medium to long-run effects due to declining private demand. The standard SVAR approach, on the other hand, predicts an increase in consumption and investment in the short run and larger medium to long-run multipliers for GDP.

In addition to anticipation effects, several alternative explanations for the differences to previous studies are conceivable. For example, the post-1980 period is often argued to have smaller fiscal multipliers than the pre-1980 period on average.[25] In addition, the structure of spending is likely to matter, given that more than 70% of federal spending falls on defense-related expenditures. An investigation of the effects of other types of expenditures such as

[25] See, for instance, Blanchard and Perotti (2002) and Bilbiie, Meier, and Mueller (2006).

state and local government spending, for which expectations data is also available from the Survey of Professional Forecasters, would thus be a useful extension of this paper.

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A Analytical Solution

This appendix provides a detailed derivation of the analytical solution to the model. The steps are as follows. First, the deterministic steady state solution is derived from the non-linear equilibrium conditions. The equilibrium system is then log-linearized and reduced to a two-dimensional first-order linear difference equation in capital and consumption, given the stochastic processes for TFP and government expenditures. Finally, the parameters in the recursive laws of motion for consumption and capital are derived using the method of undetermined coefficients (Uhlig, 1999).

Non-linear equilibrium. The non-linear equilibrium conditions are

$$\begin{aligned}
\text{Labor/leisure} \quad &: \quad Ac_t = (1-\alpha)\,\frac{y_t}{n_t}, \\
\text{Euler equation} \quad &: \quad 1 = \beta E_t \frac{c_t}{c_{t+1}} R_{t+1}, \\
\text{Real return} \quad &: \quad R_t = 1 - \delta + \alpha \frac{y_t}{k_{t-1}}, \\
\text{Production} \quad &: \quad y_t = a_t k_{t-1}^{\alpha} n_t^{1-\alpha}, \\
\text{Feasibility} \quad &: \quad y_t = c_t + k_t - (1-\delta)k_{t-1} + g_t, \\
\text{TFP} \quad &: \quad \log a_t = \rho_a \log a_{t-1} + \varepsilon_{a,t}, \\
\text{Gov. expenditures} \quad &: \quad \log\left(g_t/\bar{g}\right) = \rho_g \log\left(g_{t-1}/\bar{g}\right) + \varepsilon^{a}_{g,t-2}.
\end{aligned}$$

where investment has been eliminated in the feasibility constraint by using the capital accumulation equation.

Steady state. Let variables without time subscript denote deterministic steady state values. The TFP process implies $a = 1$ since $\rho_a \in (0, 1]$. The Euler equation yields $R = \beta^{-1}$. The real return equation can be solved for the output-to-capital ratio:

$$\frac{y}{k} = \frac{R - 1 + \delta}{\alpha} = \frac{\beta^{-1} - 1 + \delta}{\alpha}.$$

The production function implies that

$$\frac{y}{n} = \frac{k^{\alpha} n^{1-\alpha}}{n} = \left(\frac{k}{n}\right)^{\alpha}, \qquad \frac{y}{k} = \frac{k^{\alpha} n^{1-\alpha}}{k} = \left(\frac{k}{n}\right)^{\alpha-1}.$$

From the second equation, $\frac{k}{n} = \left(\frac{y}{k}\right)^{\frac{1}{\alpha-1}} = \left(\frac{y}{k}\right)^{-\frac{1}{1-\alpha}}$. Substituting this expression into the first equation yields an expression for the output-to-labor ratio

$$\frac{y}{n} = \left(\frac{y}{k}\right)^{-\frac{\alpha}{1-\alpha}} = \left(\frac{\beta^{-1} - 1 + \delta}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}.$$

The labor/leisure tradeoff then yields $c = \left(\frac{1-\alpha}{A}\right)\frac{y}{n}$. Dividing the feasibility constraint by y and re-writing yields

$$n = \frac{c}{\left(1 - \delta\frac{k}{y} - \frac{g}{y}\right)\frac{y}{n}}.$$

Taking $s_g = \frac{g}{y}$ as given, the government spending equation implies $g = \bar{g}$, since $\rho_g \in [0, 1)$. The remaining steady state solutions are

$$y = \frac{y}{n}n, \qquad k = \frac{k}{y}y, \qquad g = \bar{g} = s_g y.$$

Log-linearized system. The log-linearized system is given by

$$\begin{aligned}
\text{Labor/leisure} &: \hat{n}_t = \hat{y}_t - \hat{c}_t, && \text{(A.1)}\\
\text{Euler equation} &: 0 = E_t\left[\hat{c}_t - \hat{c}_{t+1} + \widehat{R}_{t+1}\right], && \text{(A.2)}\\
\text{Production} &: \hat{y}_t = \hat{a}_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}_t, && \text{(A.3)}\\
\text{Feasibility} &: c\hat{c}_t = y\hat{y}_t - k\hat{k}_t + (1-\delta)k\hat{k}_{t-1} - g\hat{g}, && \text{(A.4)}\\
\text{Real return} &: \widehat{R}_t = \frac{\alpha}{R}\frac{y}{k}\left(\hat{y}_t - \hat{k}_{t-1}\right), && \text{(A.5)}\\
\text{TFP} &: \hat{a}_t = \rho_a\hat{a}_{t-1} + \varepsilon_{a,t}, && \text{(A.6)}\\
\text{Gov. expenditures} &: \hat{g}_t = \rho_g\hat{g}_{t-1} + \varepsilon^a_{g,t-2}. && \text{(A.7)}
\end{aligned}$$

where $\hat{x}_t = \log(x_t/x)$ such that $x_t = x\exp(\hat{x}_t) \approx x(1+\hat{x}_t)$ for $\hat{x}_t \approx 0$.

Difference equations. The log-linearized system is now reduced to the two first-order linear difference equations reported in the main text. Substituting (A.1) into (A.3) and leading the result by one period yields, after re-arranging terms,

$$\hat{y}_{t+1} = \frac{\hat{a}_{t+1} + \alpha\hat{k}_t - (1-\alpha)\hat{c}_{t+1}}{\alpha}. \tag{A.8}$$

Substituting (A.5) into (A.2) yields

$$0 = E_t\left[\hat{c}_t - \hat{c}_{t+1} + \frac{\alpha}{R}\frac{y}{k}\left(\hat{y}_{t+1} - \hat{k}_t\right)\right].$$

Combining the latter two expressions gives the first difference equation:

$$\begin{aligned}
0 &= E_t\left[\hat{c}_t - \hat{c}_{t+1} + \frac{\alpha}{R}\frac{y}{k}\left(\frac{1}{\alpha}\hat{a}_{t+1} + \hat{k}_t - \frac{1-\alpha}{\alpha}\hat{c}_{t+1} - \hat{k}_t\right)\right],\\
&= E_t\left[\hat{c}_t - \left(1 + \frac{y}{k}\frac{1-\alpha}{R}\right)\hat{c}_{t+1} + \frac{y}{k}\frac{1}{R}\hat{a}_{t+1}\right].
\end{aligned}$$

Using (A.3) in (A.4) yields

$$c\hat{c}_t = y\hat{a}_t + \left[\alpha y + (1-\delta)k\right]\hat{k}_{t-1} + (1-\alpha)y\hat{n}_t - k\hat{k}_t - g\hat{g}_t.$$

Substituting out (A.1) in the last expression, using (A.8) lagged by one period, re-arranging terms and taking expectations yields the second difference equation:

$$0 = E_t \left[\left(c + y \frac{1-\alpha}{\alpha} \right) \hat{c}_t + k\hat{k}_t - \frac{y}{\alpha} \hat{a}_t - (y + (1-\delta)k) \hat{k}_{t-1} + g\hat{g}_t \right].$$

The reduced system is thus given by

$$\begin{aligned} 0 &= E_t \left[\hat{c}_t - \phi_1 \hat{c}_{t+1} + \phi_2 \hat{a}_{t+1} \right], && \text{(A.9)} \\ 0 &= E_t \left[\phi_3 \hat{c}_t + \phi_4 \hat{k}_t - \phi_5 \hat{a}_t - \phi_6 \hat{k}_{t-1} + \phi_7 \hat{g}_t \right], && \text{(A.10)} \end{aligned}$$

where $\phi_1 = 1 + \frac{y}{k}\frac{1-\alpha}{R}$, $\phi_2 = \frac{y}{k}\frac{1}{R}$, $\phi_3 = c + y\frac{1-\alpha}{\alpha}$, $\phi_4 = k$, $\phi_5 = \frac{y}{\alpha}$, $\phi_6 = y + (1-\delta)k$ and $\phi_7 = g$.

Recursive laws of motion. Next, guess the laws of motion

$$\begin{aligned} \hat{k}_t &= \eta_{kk}\hat{k}_{t-1} + \eta_{ka}\hat{a}_t + \eta_{kg}\hat{g}_t + \eta_{k\varepsilon,1}\varepsilon^a_{g,t} + \eta_{k\varepsilon,2}\varepsilon^a_{g,t-1}, \\ \hat{c}_t &= \eta_{ck}\hat{k}_{t-1} + \eta_{ca}\hat{a}_t + \eta_{cg}\hat{g}_t + \eta_{c\varepsilon,1}\varepsilon^a_{g,t} + \eta_{c\varepsilon,2}\varepsilon^a_{g,t-1}, \end{aligned}$$

Repeatedly substituing those into (A.9) and (A.10) and using $E_t\hat{g}_{t+1} = \rho_g\hat{g}_t + \varepsilon^a_{g,t-1}$ and $E_t\hat{a}_{t+1} = \rho_a\hat{a}_t$ yields, after some tedious but straightforward algebra:

$$\begin{aligned} 0 &= \left[(1 - \phi_1\eta_{kk})\, \eta_{ck} \right] \hat{k}_{t-1} \\ &\quad + \left[\eta_{ca}\,(1 - \phi_1\rho_a) + \phi_2\rho_a - \phi_1\eta_{ck}\eta_{ka} \right] \hat{a}_t + \left[\eta_{cg}\,(1 - \phi_1\rho_g) - \phi_1\eta_{ck}\eta_{kg} \right] \hat{g}_t \\ &\quad + \left[\eta_{c\varepsilon,1} - \phi_1\,(\eta_{c\varepsilon,2} + \eta_{ck}\eta_{k\varepsilon,1}) \right] \varepsilon^a_{g,t} + \left[\eta_{c\varepsilon,2} - \phi_1\,(\eta_{cg} + \eta_{ck}\eta_{k\varepsilon,2}) \right] \varepsilon^a_{g,t-1}, && \text{(A.11)} \\ 0 &= \left[\phi_3\eta_{ck} + \phi_4\eta_{kk} - \phi_6 \right] \hat{k}_{t-1} \\ &\quad + \left[\phi_3\eta_{ca} + \phi_4\eta_{ka} - \phi_5 \right] \hat{a}_t + \left[\phi_3\eta_{cg} + \phi_4\eta_{kg} + \phi_7 \right] \hat{g}_t \\ &\quad + \left[\phi_3\eta_{c\varepsilon,1} + \phi_4\eta_{k\varepsilon,1} \right] \varepsilon^a_{g,t} + \left[\phi_3\eta_{c\varepsilon,2} + \phi_4\eta_{k\varepsilon,2} \right] \varepsilon^a_{g,t-1}. \end{aligned}$$

Solving for the dynamics. Finally, one can solve for the coefficients η in the recursive laws of motion. Both of the above equations must hold with equality for all values of the state variables. First, set $\hat{a}_t = \hat{g}_t = \varepsilon^a_{g,t} = \varepsilon^a_{g,t-1} = 0$:

$$\begin{aligned} 0 &= \left[(1 - \phi_1\eta_{kk})\, \eta_{ck} \right] \hat{k}_{t-1}, \\ 0 &= \left[\phi_3\eta_{ck} + \phi_4\eta_{kk} - \phi_6 \right] \hat{k}_{t-1}. \end{aligned}$$

Since both equations also need to hold for any value of $\hat{k}_{t-1}$, it must be that

$$\begin{aligned} 0 &= (1 - \phi_1\eta_{kk})\, \eta_{ck}, \\ 0 &= \phi_3\eta_{ck} + \phi_4\eta_{kk} - \phi_6. \end{aligned}$$

The second equation implies

$$\eta_{ck} = \frac{\phi_6}{\phi_3} - \frac{\phi_4}{\phi_3}\eta_{kk},$$

and the first equation implies

$$0 = \phi_1\phi_4\eta_{kk}^2 - (\phi_1\phi_6 + \phi_4)\,\eta_{kk} + \phi_6.$$

with solutions

$$\eta_{kk}^{\pm} = \frac{\frac{1}{\phi_1} + \frac{\phi_6}{\phi_4}}{2} \pm \sqrt{\left(\frac{\frac{1}{\phi_1} + \frac{\phi_6}{\phi_4}}{2}\right)^2 - \frac{\phi_6}{\phi_1\phi_4}}.$$

Similarly, comparing coefficients on $\hat{a}_t$ gives

$$\eta_{ka} = \frac{\phi_5}{\phi_4} - \frac{\phi_3}{\phi_4}\eta_{ca}, \qquad \eta_{ca} = \frac{\phi_1\frac{\phi_5}{\phi_4}\eta_{ck} - \phi_2\rho_a}{1 - \phi_1\rho_a + \phi_1\frac{\phi_3}{\phi_4}\eta_{ck}}.$$

Comparing coefficients on $\hat{g}_t$ yields

$$\eta_{kg} = -\left(\frac{\phi_7}{\phi_4} + \frac{\phi_3}{\phi_4}\eta_{cg}\right), \qquad \eta_{cg} = \frac{-\phi_1\frac{\phi_7}{\phi_4}\eta_{ck}}{1 + \phi_1\left(\frac{\phi_3}{\phi_4}\eta_{ck} - \rho_g\right)}.$$

Further, comparing coefficients on $\varepsilon_{g,t-1}^a$ gives

$$\eta_{c\varepsilon,2} = -\frac{\phi_4}{\phi_3}\eta_{k\varepsilon,2}, \qquad \eta_{k\varepsilon,2} = \frac{-\eta_{cg}}{\eta_{ck} + \frac{\phi_4}{\phi_1\phi_3}}.$$

Finally, comparing coefficients on $\varepsilon_{g,t}^a$ yields

$$\eta_{c\varepsilon,1} = -\frac{\phi_4}{\phi_3}\eta_{k\varepsilon,1}, \qquad \eta_{k\varepsilon,1} = \frac{-\eta_{c\varepsilon,2}}{\eta_{ck} + \frac{\phi_4}{\phi_1\phi_3}}.$$

Modifications. If spending reacts to lagged TFP, the coefficient on $\hat{a}_t$ in equation A.11 changes to $\eta_{ca}(1-\phi_1\rho_a) + \phi_2\rho_a - \phi_1(\eta_{ck}\eta_{ka} + \eta_{cg}\rho_{ga})$. Thus, the only coefficients in the recursive laws of motion which are affected are η_{ca} and η_{ka}. For $\rho_{ga} \in \mathbb{R}$, they are

$$\eta_{ka}^* = \frac{\phi_5}{\phi_4} - \frac{\phi_3}{\phi_4}\eta_{ca}^*, \qquad \eta_{ca}^* = \frac{\phi_1\frac{\phi_5}{\phi_4}\eta_{ck} - \phi_2\rho_a + \rho_{ga}\phi_1\eta_{cg}}{1 - \phi_1\rho_a + \phi_1\frac{\phi_3}{\phi_4}\eta_{ck}}.$$

When $\rho_{ga} = 0$, therefore, $\eta_{ka}^* = \eta_{ka}$ and $\eta_{ca}^* = \eta_{ca}$.

If spending reacts to current TFP, the coefficient on $\hat{a}_t$ in equation A.11 changes to $\eta_{ca}(1-\phi_1\rho_a) + \phi_2\rho_a - \phi_1(\eta_{ck}\eta_{ka} + \eta_{cg}\rho_{ga}\rho_a)$. In this case,

$$\eta_{ka}^{**} = \frac{\phi_5}{\phi_4} - \frac{\phi_3}{\phi_4}\eta_{ca}^{**}, \qquad \eta_{ca}^{**} = \frac{\phi_1\frac{\phi_5}{\phi_4}\eta_{ck} - \phi_2\rho_a + \rho_{ga}\rho_a\phi_1\eta_{cg}}{1 - \phi_1\rho_a + \phi_1\frac{\phi_3}{\phi_4}\eta_{ck}}.$$

When $\rho_{ga} = 0$, it follows that $\eta_{ka}^{**} = \eta_{ka}^* = \eta_{ka}$ and $\eta_{ca}^{**} = \eta_{ca}^* = \eta_{ca}$.

B Data Definitions

This appendix provides details on data sources and data definitions. Throughout, NIPA refers to the National Income and Product Accounts of the Bureau of Economic Analysis, BLS to the Bureau of Labor Statistics, ALFRED to the Archival Federal Reserve Economic Data of the Federal Reserve Bank of St. Louis and SPF to the Survey of Professional Forecasters of the Federal Reserve Bank of Philadelphia. All time series are provided in seasonally adjusted terms from the original sources, except the data on federal debt and the T-bill rate which are not seasonally adjusted.

- *Government spending, realization:* Real federal government consumption and gross investment; the nominal series is taken from NIPA Table 1.1.5. Line 22; the series is then scaled to constant 2005 prices by its deflator (NIPA Table 1.1.4. Line 22) and converted into natural logarithms.
- *Government spending, forecasts:* One to three-quarter ahead forecasts of real federal government consumption and gross investment; the level series is the mean prediction of SPF variable RFEDGOV; given breaks in levels due to NIPA base year changes, the forecasts are scaled to constant 2005 prices by backcasting the actual growth rates and imposing the average growth rate over the sample at the break points.
- *Government spending, expectational error:* First difference of natural logarithms of realized spending and the prediction thereof made one quarter earlier.
- *Output:* Real gross domestic product; the nominal series is taken from NIPA Table 1.1.5. Line 1; the series is then scaled to constant 2005 prices by its deflator (NIPA Table 1.1.4. Line 1) and converted into natural logarithms.
- *Consumption:* Real personal consumption expenditure; the nominal series is taken from NIPA Table 1.1.5. Line 2; the series is then scaled to constant 2005 prices by its deflator (NIPA Table 1.1.4. Line 2) and converted into natural logarithms.
- *Investment:* Real gross private investment; the nominal series is taken from NIPA Table 1.1.5. Line 7; the series is then scaled to constant 2005 prices by its deflator (NIPA Table 1.1.4. Line 7) and converted into natural logarithms.
- *Real wage:* Real hourly compensation, business sector; BLS series ID: PRS84006153; the original series is converted into natural logarithms.
- *Interest rate:* 3-month treasury bill rate, secondary market rate; series TB3MS in ALFRED database; the interest rate is expressed in annual terms.
- *Debt-to-GDP ratio:* total end-of-period federal government debt divided by nominal GDP; public debt data: series GFDEBTN in ALFRED database.