Downloaded from UvA-DARE, the institutional repository of the University of Amsterdam (UvA)
http://hdl.handle.net/11245/2.94900

File ID	uvapub:94900
Filename	210fulltext.pdf
Version	preprint

SOURCE (OR PART OF THE FOLLOWING SOURCE):

Type	pre-print - working paper
Title	Illinois Walls in alternative market structures
Author(s)	M.P. Schinkel, J. Tuinstra
Faculty	FEB: Amsterdam Business School Research Institute (ABS-RI), FEB: Amsterdam School of Economics Research Institute (ASE-RI), FEB
Year	2005

FULL BIBLIOGRAPHIC DETAILS:
http://hdl.handle.net/11245/1.252591

Copyright

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content licence (like Creative Commons).

UvA-DARE is a service provided by the library of the University of Amsterdam (http://dare.uva.nl)
(pagedate: 2014-11-17)

Illinois Walls in Alternative Market Structures

Maarten Pieter Schinkel*and Jan Tuinstra†

May 2005

Abstract

This note extends on our paper *Illinois Walls: How Barring Indirect Purchaser Suits Facilitates Collusion* (Schinkel, Tuinstra and Rüggeberg, 2005, henceforth STR). It presents analyses of two alternative, more competitive, market structures to conclude that when the conditions for existence of Illinois Walls derived in STR are satisfied, Illinois Walls also exist in these alternative market structures. Section 1 considers a market in which each downstream firm is able to buy and sell several varieties of the differentiated product, which increases competition at the downstream level. It is found that Illinois Walls then exist for discount factors δ with $\delta > \delta^{**}$, where δ^{**} is strictly smaller than the critical discount value found in STR. Section 2 studies the case where all wholesalers produce one and the same homogeneous input, which the downstream firms each differentiate into their own variety. In this market structure, competition is strong at the upstream level. Illinois Walls turn out to exist for *any* positive value of the discount factor. These findings suggest that Illinois Walls are robust to variations in market structure.

1 Multi-Product Downstream Competition

In the model analyzed in STR, each upstream firm deals exclusively with a single downstream firm. Such a contract grants the downstream firm some market power and, therefore, positive profits in the competitive benchmark equilibrium. Suppose instead that each variety i produced by one of the n upstream firms is distributed to at least two downstream retailers, of which we assume now that there are $m \geq 2$. An immediate consequence of this is that, since at least two different downstream firms supply variety i against constant marginal costs p_i, price competition will drive downstream prices down to $P_i = p_i$ for all i. Hence, all downstream firms make zero economic profits in competition. Below it is shown that such multi-product downstream competition increases the possibilities for erecting Illinois Walls.

* Department of Economics and ACLE, Universiteit van Amsterdam. Corresponding author at: Roetersstraat 11, 1018WB Amsterdam, The Netherlands. E-mail: m.p.schinkel@uva.nl.

† Department of Quantitative Economics and CeNDEF, Universiteit van Amsterdam.

1.1 The Competitive Benchmark

Recall that consumer demand for variety i is given by

$$Q_i\left(P_1,\ldots,P_n\right)=\frac{\left(1-e\right)\left(1-P_i\right)-e\sum_{j\neq i}\left(P_i-P_j\right)}{\left(1+\left(n-1\right)e\right)\left(1-e\right)}, \tag{1}$$

where, as before, $e\in[0,1)$ is a measure of the product differentiation and therefore of the degree of competition in the market. Consider the competitive situation. Denote by Q_{ij} the quantity of variety i sold by downstream firm j, $j=1,\ldots m$. As before, our focus throughout this note will be on symmetric equilibria.

Lemma 1 *In the multi-product downstream competitive benchmark, prices, quantities and profits are given by*

$$\begin{aligned}
p_i &= P_i=P^c=p^c=\frac{\left(2+\left(2n-3\right)e\right)\left(1-e\right)}{4+6\left(n-2\right)e+\left(2n^2-9\left(n-1\right)\right)e^2},\\
q_i &= q^c=\frac{1}{\left(1+\left(n-1\right)e\right)}\frac{2+\left(4n-7\right)e+\left(2n-3\right)\left(n-2\right)e^2}{4+6\left(n-2\right)e+\left(2n^2-9\left(n-1\right)\right)e^2},\\
Q_{ij} &= \frac{1}{m}q^c,\\
\pi_i^c &= \frac{\left(2+\left(2n-3\right)e\right)\left(1-e\right)}{\left(1+\left(n-1\right)e\right)}\frac{2+\left(4n-7\right)e+\left(2n-3\right)\left(n-2\right)e^2}{\left(4+6\left(n-2\right)e+\left(2n^2-9\left(n-1\right)\right)e^2\right)^2},\\
\Pi_i^c &= 0.
\end{aligned}$$

Proof. Downstream competition will drive all downstream prices P_i down to the corresponding upstream input price levels p_i, which implies that derived upstream demand equals downstream demand, that is,

$$q_i\left(p_1,\ldots,p_n\right)=\frac{\left(1-e\right)\left(1-p_i\right)-e\sum_{j\neq i}\left(p_i-p_j\right)}{\left(1+\left(n-1\right)e\right)\left(1-e\right)}.$$

The equilibrium then follows—applying Lemma 3 from STR, with zero marginal costs—as

$$\begin{aligned}
p_i &= P_i=p^c=\frac{\left(2+\left(2n-3\right)e\right)\left(1-e\right)}{4+6\left(n-2\right)e+\left(2n^2-9\left(n-1\right)\right)e^2}\\
\text{and } q_i &= q^c=\frac{1}{\left(1+\left(n-1\right)e\right)}\frac{2+\left(4n-7\right)e+\left(2n-3\right)\left(n-2\right)e^2}{4+6\left(n-2\right)e+\left(2n^2-9\left(n-1\right)\right)e^2}.
\end{aligned}$$

Moreover, $q_i=\sum_{j=1}^m Q_{ij}$. Restricting attention to symmetric equilibria only, we have $Q_{ij}=\frac{1}{m}q_i$, for all i and j. Profits for each of the downstream firms are zero. Upstream profits are given as $\pi_i=p^cq^c$ per wholesaler. ■

1.2 Illinois Walls

A complicating factor in the analysis of multi-product downstream competition is that the Illinois Walls rationing scheme is not uniquely determined. It depends on who trades with whom and how total input production is broken down between them. Yet, consider arguably the most natural of all possible rationing schemes, in which each upstream firm produces $\overline{q}$ and distributes this evenly over all the downstream firms. This scheme remains closest to our symmetric equilibrium analysis. Each downstream firm therefore receives $\frac{\overline{q}}{m}$ units of each variety i. Market clearing downstream consumer prices then are

$$\overline{P} = 1 - (1 + (n-1)\, e)\, \overline{q},$$

for each variety. It is, however, not immediate that downstream firms will indeed purchase, produce and sell the full allocated amount. The reason for this is as follows. If all downstream firms indeed purchase $\frac{1}{m}\overline{q}$ of each commodity and quote $\overline{P}$ for every variety, then each individual downstream firm may have an incentive to deviate. First notice that unilaterally undercutting the price $\overline{P}$ is certainly not profitable, since no downstream firm will be able to produce more than it sells at $\overline{P}$, because its inputs are rationed. Undercutting will therefore decrease profits. However, each downstream firm may have an incentive to purchase less inputs and sell them at a higher price than $\overline{P}$ set by the others. If it does so, its demand will decrease, but since consumers cannot switch to any of the other firms, which are all selling at their full 'capacity' $\frac{\overline{q}}{m}$ and face the rationing constraint, a thus defecting firm can increase its price per unit. That is, the deviating firm can act as a monopolist on the residual demand it faces when all other firms sell allotted quota at which they are put on allocation. However, when the quota is set small enough, such 'overcutting' is not an option, as established below.

Proposition 2 *Suppose that every downstream firm is rationed in its demand of variety i and gets $\frac{\overline{q}}{m}$ of each variety. Denote by $\overline{P}$ the market clearing consumer price, which is given by $\overline{P} = 1 - (1 + (n-1)\, e)\, \overline{q}$. If*

$$\overline{q} \leq \frac{m}{m+1} \frac{1}{1 + (n-1)\, e} \text{ so that } \overline{P} \geq \frac{1}{m+1}, \tag{2}$$

there exists a unique pure strategy Nash equilibrium in the downstream industry with $P_i = \overline{P}$. Otherwise, a pure strategy Nash equilibrium does not exist.

Proof. The proof consists of two steps. First, we determine residual demand for a deviating firm if it quotes a price vector $(P_1, \ldots, P_n) \geq \left(\overline{P}, \ldots, \overline{P}\right)$, whereas all other downstream firms quote $\overline{P}$ for each variety. Then it is shown that it is never profitable to deviate from selling at full capacity on this residual demand as long as the condition in the proposition is satisfied.

Step 1. The representative consumer maximizes

$$U\left(Q_{0,}Q_1,Q_2,\ldots,Q_n\right)=Q_0+\sum_{i=1}^{n}Q_i-\frac{1}{2}\left(\sum_{i-1}^{n}Q_i^2+e\sum_{i=1}^{n}\sum_{j\neq i}Q_iQ_j\right),$$

restricted by

$$\begin{aligned}Q_0+\overline{P}\left(Q_1^R+\ldots+Q_n^R\right)+P_1Q_1^d+\ldots+P_nQ_n^d &\leq M\\ Q_i^R+Q_i^d &= Q_i,\quad i=1,\ldots,n\\ Q_i^R &\leq \frac{m-1}{m}\overline{q},\quad i=1,\ldots,n,\end{aligned}$$

where Q_i^R is the total quantity of variety i purchased from the $m-1$ firms that remain at full allotted capacity, and Q_i^d is the quantity of variety i purchased from the firm that deviates by selling less. Obviously, the deviating firm's price for variety i satisfies $P_i\geq\overline{P}$. Since $\overline{P}$ is defined such that the consumer will then want to consume $\overline{q}$ of each variety, we can take $Q_i^R=\frac{m-1}{m}\overline{q}$. The utility function can thus be rewritten as

$$U\left(Q_{0,}Q_1,Q_2,\ldots,Q_n\right)=Q_0+\sum_{i=1}^{n}\left(\frac{m-1}{m}\overline{q}+Q_i^d\right)$$

$$-\frac{1}{2}\left(\sum_{i-1}^{n}\left(\frac{m-1}{m}\overline{q}+Q_i^d\right)^2+e\sum_{i=1}^{n}\sum_{j\neq i}\left(\frac{m-1}{m}\overline{q}+Q_i^d\right)\left(\frac{m-1}{m}\overline{q}+Q_j^d\right)\right),$$

and the budget constraint as

$$Q_0+P_1\left(Q_1^d+\frac{m-1}{m}\overline{q}\right)+\ldots+P_n\left(Q_n^d+\frac{m-1}{m}\overline{q}\right)\leq M+\sum_{i=1}^{n}\left(P_i-\overline{P}\right)\frac{m-1}{m}\overline{q}.$$

Residual demand for the deviating firm then follows as

$$Q_i^d\left(P_1,\ldots,P_n\right)=Q_i\left(P_1,\ldots,P_n\right)-\frac{m-1}{m}\overline{q}\text{ for each }i.$$

This type of demand rationing (of consumers) is commonly known as 'efficient' or 'parallel' rationing.[1]

Step 2. Given that all the other $m-1$ firms ask $\overline{P}$ for each variety, a downstream firm's profit from deviating is

$$\sum_{i=1}^{n}P_i\left(Q_i\left(P_1,\ldots,P_n\right)-\frac{m-1}{m}\overline{q}\right)-\frac{n}{m}\overline{p}\ \overline{q}.$$

$$=\sum_{i=1}^{n}P_i\left(\frac{1-e-\left(1+\left(n-2\right)e\right)P_i+e\sum_{j\neq i}P_j}{\left(1+\left(n-1\right)e\right)\left(1-e\right)}-\frac{m-1}{m}\overline{q}\right)-\frac{n}{m}\overline{p}\ \overline{q}.$$

[1] For a discussion of different demand rationing schemes, see Levitan and Shubik (1972) or Davidson and Deneckere (1986).

Notice that we assume that each firm buys $\frac{\overline{q}}{m}$ units of each variety whether or not it rations consumers. In other words, the deviating firm can choose to stock up, but has to purchase the full amount of allotted inputs if it wants to try raising output prices. This is reasonable to assume, given that the rationing scheme is not uniquely determined. If the deviating firm would not purchase its full allotted capacity, namely, the upstream cartel would want to allocate its unsold inputs over the other firms. That would subsequently allow them to accept all the demand that the deviating retailer is trying to increase its consumer price on, so that deviation cannot be successful.

The deviating firm's first-order condition with respect to variety i reads

$$Q_i\left(P_1,\ldots,P_n\right)-\frac{m-1}{m}\overline{q}+P_i\frac{\partial Q_i\left(P_1,\ldots,P_n\right)}{\partial P_i}+\sum_{j\neq i}P_j\frac{\partial Q_i\left(P_1,\ldots,P_n\right)}{\partial P_i}=0.$$

Since these first-order conditions are linear in prices and all the same, a unique and symmetric optimum for the deviating firm exists. It is given implicitly by

$$P=\frac{1}{2}-\frac{1}{2}\frac{m-1}{m}\left(1+\left(n-1\right)e\right)\overline{q}=\frac{1}{2}-\frac{1}{2}\frac{m-1}{m}\left(1-\overline{P}\right).$$

We therefore have that

$$P\leq\overline{P}\Leftrightarrow\overline{P}\geq\frac{1}{m+1}.$$

Hence, if $\overline{P}\geq\frac{1}{m+1}$, no firm has an incentive to quote prices different from $\overline{P}$. Suppose $\overline{P}<\frac{1}{m+1}$. Then it is obvious that no pure strategy Nash equilibrium exists: if some prices are above $\overline{P}$, then undercutting these increases profits. The condition on $\overline{q}$ given in (2) follows straightforwardly from the market-clearing condition. ■

With this additional 'pure strategy equilibrium constraint' (2) on $\overline{q}$, we are now ready to construct an Illinois Wall, which is a bit more subtle now. Essentially, downstream firms need to be sufficiently rationed to keep $\overline{P}$ high enough, as otherwise a Nash equilibrium in pure strategies does not exist in the downstream industry. Yet, Illinois Walls can still easily be erected, as follows.

Theorem 3 *If each consumer goods variety is traded by two or more retailers, then as long as $\delta>\delta^{**}$, with*

$$\delta^{**}=\begin{cases}\frac{\varphi(\mu,\beta,T^s)p^c}{\varphi(\mu,\beta,T^s)p^c+(1-2p^c)} & p^c\geq\frac{1}{m+1}\\ \frac{\varphi(\mu,\beta,T^s)p^c}{\varphi(\mu,\beta,T^s)p^c+\left(\frac{m}{m+1}-p^c\right)} & p^c<\frac{1}{m+1}\end{cases},\tag{3}$$

Illinois Brick *sustains the upstream cartel. Moreover, for all values of φ, n and e, we have $\delta^{**}<\delta^{*}$ in which δ^{*} is the critical discount factor established in Theorem* 1 *of STR. Hence, multi-product downstream competition enhances the scope for Illinois Walls.*

Proof. The proof proceeds in 4 steps. The first three steps respectively isolate rationed prices from the incentive constraint, identify two Illinois Wall candidates, depending on the value of $\overline{P}$ for which pure strategy equilibria exist, and show that these candidates are profitable for the upstream cartel to install over competing. In step 4, the relationship with the Illinois Wall in the text is laid.

Step 1. The incentive constraint for the downstream firms is

$$\frac{1}{1-\delta}\overline{\Pi}\left(\overline{p},\overline{q}\right) \geq \overline{\Pi}\left(\overline{p},\overline{q}\right) + \varphi\left(\mu,\beta,T\right)\left(\overline{p}-p^{c}\right)\overline{q}. \tag{4}$$

Observe that the constraint is weaker than the one in the text, because the downstream firms make zero profits in the competitive benchmark. The condition can be rewritten as

$$\overline{p} \leq \frac{\overline{P}+\varphi\frac{1-\delta}{\delta}p^{c}}{1+\varphi\frac{1-\delta}{\delta}} = \frac{1-\left(1+\left(n-1\right)e\right)\overline{q}+\varphi\frac{1-\delta}{\delta}p^{c}}{1+\varphi\frac{1-\delta}{\delta}}.$$

Step 2. Maximizing per firm profit of the upstream cartel $\overline{pq}$ using the constraint returns

$$\overline{q} = \frac{1+\varphi\frac{1-\delta}{\delta}p^{c}}{2\left(1+\left(n-1\right)e\right)},\ \overline{p} = \frac{1}{2}\frac{1+\varphi\frac{1-\delta}{\delta}p^{c}}{1+\varphi\frac{1-\delta}{\delta}} \text{ and } \overline{P} = \frac{1}{2}-\frac{1}{2}\varphi\frac{1-\delta}{\delta}p^{c},$$

as candidate values for the wall. This candidate solution is only feasible if condition (2) holds, that is, if $\overline{P} \geq \frac{1}{m+1}$, which translates into

$$\varphi\frac{1-\delta}{\delta} \leq \frac{m-1}{m+1}\frac{1}{p^{c}} \tag{5}$$

If (5) does not hold for the optimum, then the upstream cartel will select $\overline{q}$ such that $\overline{P}$ will be equal to $\frac{1}{m+1}$. The corresponding value for $\overline{p}$ would follow from the incentive constraint (4), so that the second-best Illinois Wall solution would be

$$\overline{q} = \frac{m}{m+1}\frac{1}{1+\left(n-1\right)e},\ \overline{p} = \frac{\frac{1}{m+1}+\varphi\frac{1-\delta}{\delta}p^{c}}{1+\varphi\frac{1-\delta}{\delta}} \text{ and } \overline{P} = \frac{1}{m+1}.$$

Step 3. Having completely characterized the profit maximizing rationing scheme for the upstream cartel when it takes both incentive constraint (4) and 'pure strategy constraint' (2) into account, we are in position to determine under which conditions this arrangement is more profitable for the upstream cartel than the competitive benchmark.

First, consider the case for which (5) is satisfied. Then $\overline{p} \geq p^{c}$ for

$$\varphi\frac{1-\delta}{\delta} \leq \frac{1-2p^{c}}{p^{c}}.$$

It is easily seen, moreover, that this upper-bound on $\varphi(1-\delta)/\delta$ does not violate condition (5) if and only if $p^c \geq \frac{1}{m+1}$.

If $p^c < \frac{1}{m+1}$, which happens when the number of downstream firms is relatively small , but competition is strong otherwise, that is, when e and n are large, the lower upper-bound violates condition (5). We therefore have to compare profits in the competitive benchmark with the upstream cartel profits resulting from the rationing profile $\overline{q}$ in which consumer prices equal $\overline{P} = \frac{1}{m+1}$. Notice that it is not sufficient to compare prices $\overline{p}$ and p^c, since the price $\overline{p}$ has not yet been chosen to maximize profits. Yet, if $p^c < \frac{1}{m+1}$ it is easily checked that $\overline{P} > \overline{p} > p^c$, as required. Some basic computations furthermore show that $\overline{\pi} \geq \pi^c$, if and only if

$$\varphi \frac{1-\delta}{\delta} \leq \frac{1}{p^c}\left(\frac{m}{m+1} - p^c\right), \tag{6}$$

so that condition (3) holds for this case. Notice further that, since $p^c < \frac{1}{2}$—as $p^c = \frac{1}{2}$ is the monopoly price, which obviously does not obtain for $e > 0$ and $n \geq 2$—the left hand side of (6) is positive and therefore δ^{**} always exists.

Step 4. The expression $\varphi\frac{1-\delta}{\delta}$ is decreasing in δ. We therefore need to show that $\varphi\frac{1-\delta^{**}}{\delta^{**}} > \varphi\frac{1-\delta^{*}}{\delta^{*}}$ for all n and e. Using the expressions for competitive prices from Lemma 1, we have

$$\varphi\frac{1-\delta^*}{\delta^*} = \frac{(4+n-4n^2+n^3)\,e^3 + (n^2+6n-15)\,e^2 - 2\,(2n-7)\,e - 4}{(1-e)\,((2n-3)\,e+2)\,((n-3)\,e+2)},$$

and

$$\varphi\frac{1-\delta^{**}}{\delta^{**}} = \begin{cases} \frac{\left(4+6(n-2)e+\left(2n^2-9(n-1)\right)e^2\right)-2(2+(2n-3)e)(1-e)}{(2+(2n-3)e)(1-e)} & p^c \geq \frac{1}{m+1} \\ \frac{m}{m+1}\frac{4+6(n-2)e+\left(2n^2-9(n-1)\right)e^2}{(2+(2n-3)e)(1-e)} - 1 & p^c < \frac{1}{m+1} \end{cases}.$$

First, consider the case with $p^c \geq \frac{1}{m+1}$. After some simplifications, we find that for that case $\varphi\frac{1-\delta^{**}}{\delta^{**}} > \varphi\frac{1-\delta^{*}}{\delta^{*}}$ is equivalent to

$$f(n,e) = 4 + 2\,(4n-9)\,e + (5n-9)\,(n-3)\,e^2 + \left(-13+17n-7n^2+n^3\right)e^3 > 0.$$

All four terms in this expression are nonnegative (and the first is strictly positive) for $n \geq 3$ and all e. We therefore only have to check the inequality for $n = 2$. In that case, we have $f(2,e) = 4 - 2e - e^2 + e^3$, which clearly is strictly positive for all $e \in [0,1]$.

Second, we consider the case for which $p^c < \frac{1}{m+1}$. For that case, $\varphi\frac{1-\delta^{**}}{\delta^{**}}$ is decreasing in m, so if we can show that $\varphi\frac{1-\delta^{**}}{\delta^{**}} > \varphi\frac{1-\delta^{*}}{\delta^{*}}$ for $m = 2$, then it will hold for any m. This last inequality, with $m = 2$, is equivalent to

$$g(n,e) = 16 + 2\,(13n-33)\,e + \left(90-69n+11n^2\right)e^2 + \left(-39+42n-12n^2+n^3\right)e^3 > 0.$$

All four terms in this expression are strictly positive for $n \geq 7$ and all e. We therefore need to check that $g(n,e)$ is positive for $n = 2$, 3, 4, 5 and 6 and for every possible e, with $0 \leq e \leq 1$. We have $g(2,e) = 16-14e-4e^2+5e^3$, $g(3,e) = 16+12e-18e^2+6e^3$, $g(4,e) = 16+38e-10e^2+e^3$, $g(5,e) = 16+64e+20e^2-4e^3$ and $g(6,e) = 16+90e+72e^2-3e^3$, all of which are indeed strictly positive for all admissible values of e. Hence, $\delta^{**} < \delta^{*}$ for all values of φ, n and e. ■

The critical discount factor δ^{**}, at least when constraint (2) is not binding, is very similar to δ^{*} from Theorem 1 from STR. Note that in the present market structure, we have $P^c = p^c$. Moreover, since downstream competition is stronger than in the market structure analyzed in STR, the competitive price in the present market will be lower.

2 Homogeneous Wholesale Products

Assume that the production of the n varieties for which consumer demand is given in equation (1), is done at the level of the n downstream retailers out of a homogeneous input. That is, let there be m upstream firms producing a homogeneous commodity at constant marginal cost c—assumed to be positive in the following to show that assuming $c = 0$ is without loss of generality—which they sell at a uniform price p. This input is purchased by the n downstream firms, who each create their own variety i out of this homogeneous input, at no additional costs. Firms in the downstream industry are involved in Bertrand price competition with differentiated commodities.

2.1 The Competitive Benchmark

We have the following benchmark results for this market structure. Note that, since the upstream firms produce a homogeneous commodity and compete on prices, we know that $p_1 = \ldots p_n = p$.

Lemma 4 *Given the input price p, the following symmetric Bertrand-Nash equilibrium prices establish in the downstream industry*

$$P = \frac{1}{(n-3)e+2}\left(1-e+(1+(n-2)e)p\right). \tag{7}$$

Furthermore, the implied demand for the inputs from the upstream industry is given by

$$q(p) = \frac{n(1+(n-2)e)}{((n-3)e+2)(1+(n-1)e)}(1-p). \tag{8}$$

Proof. Downstream firm i sets P_i in order to maximize profits $(P_i - p)\, Q_i\,(P_1, \ldots, P_n)$. Using (1), the first-order condition for firm i is

$$2\,(1 + (n-2)\,e)\,P_i - e \sum_{j \neq i} P_j = \xi \equiv 1 - e + (1 + (n-2)\,e)\,p.$$

Solving the system of n first-order conditions, the symmetric Bertrand-Nash equilibrium prices are found to be

$$\begin{aligned}
\begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{pmatrix} &= \begin{pmatrix} 2\,(1+(n-2)\,e) & -e & \cdots & -e \\ -e & \ddots & & \vdots \\ \vdots & & \ddots & -e \\ -e & \cdots & -e & 2\,(1+(n-2)\,e) \end{pmatrix}^{-1} \begin{pmatrix} \xi \\ \xi \\ \vdots \\ \xi \end{pmatrix} \\
&= A_{e,n} \begin{pmatrix} 2+(n-2)\,e & e & \cdots & e \\ e & \ddots & & e \\ \vdots & & \ddots & \vdots \\ e & e & \cdots & 2+(n-2)\,e \end{pmatrix} \begin{pmatrix} \xi \\ \xi \\ \vdots \\ \xi \end{pmatrix},
\end{aligned}$$

where

$$A_{e,n} = \frac{1}{4 + 6\,(n-2)\,e + (2n^2 - 9\,(n-1))\,e^2}.$$

The expression for the Bertrand-Nash equilibrium prices can be simplified to the one in (8). The uniform input price p gives rise to the equilibrium $(P\,(p), \ldots, P\,(p))$. Consumer demand is given by

$$Q_i\,(P, \ldots, P) = \frac{1}{1 + (n-1)\,e}\,(1 - P) = Q,$$

which implies that the input demand of the downstream industry for the product of the upstream industry is given by

$$q\,(p) = nQ = \frac{n\,(1 + (n-2)\,e)}{((n-3)\,e + 2)\,(1 + (n-1)\,e)}\,(1 - p)\,.$$

■

Having thus determined the input demand for the firms in the upstream industry, we can establish the Bertrand-Nash equilibrium in the upstream industry. Obviously, it has $p^c = c$. These prices determine the downstream output prices P^c. With all prices known, sales and profits can be determined. The next result describes the full equilibrium of the upstream-downstream price competition game.

Lemma 5 *The Bertrand-Nash equilibrium is given by*

$$\begin{aligned}
p^c &= c, \\
P^c &= \frac{1-e+(1+(n-2)\,e)\,c}{(n-3)\,e+2}, \\
Q^c &= \frac{(1+(n-2)\,e)\,(1-c)}{((n-1)\,e+1)\,((n-3)\,e+2)} \quad \textit{and } q^c = nQ^c, \\
\pi^c &= 0, \\
\Pi^c &= \frac{(1-e)\,(1-c)^2\,(1+(n-2)\,e)}{((n-1)\,e+1)\,((n-3)\,e+2)^2}.
\end{aligned}$$

Proof. Everything follows straightforwardly from the fact that the price in the upstream industry is determined by marginal costs. Using (7) and (8) gives the expressions for the price P^c and individual production Q^c, and these two together give profits $\Pi^c = (P^c - p^c)\,Q^c$. ■

2.2 Illinois Walls

Let $(\overline{p}, \overline{q})$ be the rationing scheme of the upstream cartel. Instantaneous damages for each downstream firm are

$$D\,(\overline{p}, \overline{q}) = (\overline{p} - p^c)\,\overline{q} = (\overline{p} - c)\,\overline{q}$$

So the incentive constraint becomes

$$\frac{1}{1-\delta}\overline{\Pi}\,(\overline{p}, \overline{q}) \geq \overline{\Pi}\,(\overline{p}, \overline{q}) + \varphi\,(\mu, \beta, T)\,(\overline{p} - c)\,\overline{q} + \frac{\delta}{1-\delta}\Pi^c,$$

under which the upstream cartel maximizes its profits. Our Illinois Wall then is very robust, as summarized in the following result.

Theorem 6 *For all $\delta > 0$,* Illinois Brick *sustains the upstream cartel.*

Proof. The proof proceeds again in four steps.

Step 1. From the binding incentive constraint

$$\frac{\delta}{1-\delta}\overline{\Pi}\,(\overline{p}, \overline{q}) = \varphi\,(\overline{p} - c)\,\overline{q} + \frac{\delta}{1-\delta}\Pi^c$$

follows

$$\overline{p} = \frac{1+\frac{1-\delta}{\delta}\varphi c}{1+\frac{1-\delta}{\delta}\varphi} - \frac{1+(n-1)\,e}{1+\frac{1-\delta}{\delta}\varphi}\overline{q} - \frac{\Pi^c}{\left(1+\frac{1-\delta}{\delta}\varphi\right)\overline{q}}.$$

Step 2. The cartel seeks to maximize $\pi(\overline{p},\overline{q}) = (\overline{p}-c)\,\overline{q}$, constrained by this relation between $\overline{p}$ and $\overline{q}$. That is, it solves

$$\max_{\overline{q}} = \frac{1-c}{1+\frac{1-\delta}{\delta}\varphi}\overline{q} - \frac{1+(n-1)\,e}{1+\frac{1-\delta}{\delta}\varphi}\overline{q}^2 - \frac{\Pi^c}{\left(1+\frac{1-\delta}{\delta}\varphi\right)},$$

which returns

$$\overline{q}^w = \frac{1-c}{2\,(1+(n-1)\,e)},$$

and the associated price can be written as

$$\overline{p}^w = c + \frac{1}{2}\frac{1}{1+\frac{1-\delta}{\delta}\varphi}\left(1-c-\frac{4\,(1+(n-1)\,e)\,\Pi^c}{1-c}\right).$$

Consumer prices follow as $\overline{P}^w = \frac{1}{2}\,(1+c)$.

Step 3. Now note that from this it follows that the requirement that $\overline{p}^w \geq p^c = c$, is equivalent with

$$\Pi^c \leq \frac{(1-c)^2}{4\,(1+(n-1)\,e)},$$

which does not depend on the value of δ or φ. Using the expression for Π^c, this condition reduces to

$$((n-3)\,e+2)^2 \geq 4\,((n-2)\,e+1)\,(1-e)\,,$$

which in turn reduces to

$$(n-1)^2\,e^2 \geq 0.$$

This latter inequality is trivially always satisfied, so that the Illinois Wall example is not conditional on specific parameter values. Note however from the expression for $\overline{p}^w$ that $\overline{p}^w$ will approach c as $\delta \to 0$ or as $\varphi \to \infty$.

Step 4. The last step is to show that $\overline{p}^w$ and $\overline{q}^w$ satisfy condition (19) from Lemma 7 in STR, so that the Nash equilibrium in the downstream industry is properly defined. Using the expressions from above for $\overline{p}^w$ and $\overline{q}^w$ this condition can be rewritten as

$$\left((1-e)-(n-1)\,e\frac{1-\delta}{\delta}\varphi\right)(1-c)^2 \leq 4\,(1+(n-2)\,e)\,(1+(n-1)\,e)\,\Pi^c.$$

We want to establish that this holds for all possible values of $\frac{1-\delta}{\delta}\varphi$. Therefore, we need to show that it holds for $\frac{1-\delta}{\delta}\varphi = 0$ and we are done. Using that value and substituting for Π^c we find that

$$1 \leq 4\left(\frac{(n-2)\,e+1}{(n-3)\,e+2}\right)^2$$

which holds for all $n \geq 1$ and $e \in [0,1]$. ■

The reason why homogenous inputs allow for colluding under *Illinois Brick*, irrespective of the discount factor, is that within the Illinois Wall arrangement the wholesalers, who made zero economic profits to begin with, can, if such is necessary in order to keep the retailers in the collusive arrangement, be pushed back to sell their homogeneous inputs (almost) at the competitive marginal cost price level, whilst still supplying each of the retailers with the rationed quantity required for them to have a high output price. In particular, as can be seen in the proof of Theorem 6, the upstream cartel always sets $\overline{q}$ such that the consumer price equals $\overline{P}^w = \frac{1}{2}(1+c)$ and profits in the whole production chain, $(P-c)\,q$, are maximized. If the upstream cartel sells the output $\overline{q}$ at $\overline{p} = c$, the downstream firms cannot claim any damages, but all of the maximized chain profits accrue to the downstream firm. It is obvious then that, independent of the values of δ and φ, it is always possible to increase $\overline{p}$ above c in such a way that the downstream firms are indifferent between accepting the Illinois Walls side-payment arrangement or competing. Such a price increase always benefits cartel, as all wholesalers earn a positive profit.

An alternative way of seeing this is that when inputs are homogeneous, the upstream cartel is always able to enforce the optimal cartel outcome and maximize chain profits. The fraction of these chain profits the upstream cartel can claim for itself then depends on the discount factor δ and the damage multiple φ. Notice that when δ approaches 1 or when φ goes to infinity, the Illinois Wall price $\overline{p}^w$ approaches c and almost all of these chain profits accrue to the downstream firms.[2]

References

[1] Davidson, C. and R. Deneckere, "Long-Run Competition in Capacity, Short-Run Competition in Price, and the Cournot Model," *Rand Journal of Economics* 17, 1986, 404-415;

[2] Levitan, R. and M. Shubik, "Price Duopoly and Capacity Constraints," *International Economic Review*, 13, 1972, 111-121;

[3] Schinkel, M.P., J. Tuinstra and J. Rüggeberg, "Illinois Walls: How Barring Indirect Purchaser Suits Facilitates Collusion,." *CeNDEF Working Paper Series, Universiteit van Amsterdam* 05-10, 2005.

[2] Interpreted in terms of Figure 3 in STR, for the homogeneous input market structure the marginal chain profit curve and the marginal compensation curve—which is less steep—always intersect at the point where chain profits are maximized, that is, at $q = q^m$. This, again, implies that the upstream cartel will always choose $\overline{q}$ to maximize chain profits, and that such a mechanism exists for all $\delta > 0$ and φ.