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Part V

$\mathcal{N} = 4$ Dyons

In this last part of the thesis we focus on one single theory, namely the heterotic string compactified on six-torus, or equivalently the type II string theory compactified on $K3 \times T^2$. An introduction of the basic properties of K3 manifolds and the basic features of the low-energy effective theory of the IIA$/K3 \times T^2$ compactification can be found in chapter 3.

There are two chapters in this part of the thesis. In the first one we discuss the microscopic degeneracies of the BPS states of this theory. In section 7.1 we review the counting of 1/2-BPS and 1/4-BPS states using various duality frames and in particular the derivation of the dyon-counting formula. This counting formula turns out to have various seemingly unrelated mathematical properties. In section 7.2 we will review them using the theme of a generalised (Borcherds-) Kac-Moody algebra as the connecting point of these different properties.

After introducing the mathematical background we need, in chapter 8 we will present our study of the BPS spectrum of the theory. This chapter is based on the publications [128, 129]. First we address the issue of the moduli dependence of the spectrum. The main tool used here is the $\mathcal{N} = 4$, $d = 4$ supergravity effective theory and the walls of marginal stability of certain multi-centered solutions in this theory. Second we study the ambiguity of choosing a contour of integration, when one attempts to retrieve the actual BPS degeneracies from its generating function. In section 8.4 we show that the contour-dependence of the dyon degeneracies is related to its moduli-dependence, and show how an appropriate choice of contour can incorporate the moduli-dependence of the spectrum into the counting formula. After that we turn to the role of the Borcherds-Kac-Moody algebra in the BPS spectrum of the theory. First we argue that, with an appropriate identification of the simple roots and the highest weight, the counting formula is related to a certain character formula for the Verma module of the algebra. Second we argue that the Weyl group of the algebra plays the role of the group of a discretised version of attractor flows of the theory, with the walls of marginal stability identified with the walls of the Weyl chambers. Finally we comment on some arithmetic properties of this discrete attractor flow.

In this part of the thesis we will focus on the cases which are relatively well-understood, namely we will assume that the total charges of the states, given by two vectors P and Q in the 28-dimensional charge lattice (3.3.1), satisfies the following "co-prime condition"

$$\text{g.c.d}(P^a Q^b - P^b Q^a) = 1 \quad , \quad a, b = 1, \cdots, 28 \ , \tag{6.6.7}$$

which ensures that the degeneracies are completely determined by the set of three T-duality invariants $(P^2/2, Q^2/2, P \cdot Q)$. Please see [120, 130, 131,

132, 133] for discussions about the cases in which the above condition is not satisfied.