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A NON-LINEAR REPRESENTATION OF THE $d=2$ so(4)-EXTENDED SUPERCONFORMAL ALGEBRA

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We present a non-linear representation of the so(4)-extended $d=2$ superconformal algebra in terms of one boson and four Majorana fermions. The matter fields and the currents can be grouped into a single $N=4$ superfield. Breaking the supersymmetry to $N=3$ or $N=2$ leads to new representations of the $N=3,2$ superconformal algebras.

1. Introduction. The study of two-dimensional models possessing conformal or superconformal symmetries [1–3] is relevant both for statistical mechanics and high-energy physics. Exponents describing the critical behaviour of certain $d=2$ statistical systems can be related to weights of unitary representations of (super)conformal algebras [4,5]. In string theory (super)conformal algebras are present as gauge algebras in the two-dimensional formulation of (super)string models [6].

In addition to the $d=2$ conformal algebra (Virasoro algebra) and the $N=1$ superconformal algebra (Neveu–Schwarz–Ramond algebra [7,8]) superconformal algebras (SCA) with extended supersymmetry exist. An N-extended superconformal algebra consists of Virasoro generators L_n, supercharges G^i_m, $m=1,2,...,N$, and additional generators that are needed to close the algebra. The most important examples are

(i) The regular series of N-extended ($N\geqslant 2$) SCA given by Ademollo et al. [9] which have the so(N) Kac–Moody algebra (KMA) as a subalgebra. For $N\leqslant 4$ these algebras admit a central extension.

(ii) An exceptional $N=4$ SCA with the su(2) KMA as a subalgebra [9,10].

The SCA's exist in different varieties which are in general not equivalent (Neveu–Schwarz, Ramond, twisted, non-twisted, see e.g. ref. [11]). A classification of all SCA's with generators of conformal spin $2\geqslant J\geqslant \frac{1}{2}$ has been given in refs. [12,13].

It is the purpose of this note to show that the so(N)-extended SCA's with $N=2,3$ or 4 can all be realized non-linearly on the Hilbert space spanned by a single real boson φ and four Majorana fermions χ^i, $i=1,...,4$, or, alternatively, on the Hilbert space spanned by six Majorana fermions χ^i, $i=1,...,4$, and ψ^a, $a=1,2$.

2. N-extended superconformal theory. In ref. [9] the so(N)-extended SCA is defined as the algebra of superconformal transformations of the coordinates $Z=(z,\theta^i)$, $i=1,...,N$, which describe one light-cone sector of a superspace extension of $d=2$ spacetime. The variation of z,θ^i under the action of a generator $G_n^{i_1...i_R}$ with parameter $\alpha_n^{i_1...i_R}$ reads

$$\delta z=\mathrm{i}^{R(R-1)/2}(2-R)\alpha_{\mathrm{n}}{}^{i_1...i_R}\theta^{i_1}...\theta^{i_R}z^{n+1-R/2}\,,$$

$$\delta\theta^i=-\mathrm{i}^{R(R-1)/2}\left(\sum_{l=1}^{R}(-1)^{R-l}\delta^{i,i_l}\alpha_n^{i_1...i_R}\theta^{i_1}...\theta^{\hat{i}_l}...\theta^{i_R}z^{n+1-R/2}-(n+1-\tfrac{1}{2}R)\alpha_{\mathrm{n}}{}^{i_1...i_R}\theta^{i_1}...\theta^{i_R}\theta^i z^{n-R/2}\right). \qquad (1)$$

Both $G_n^{i_1...i_R}$ and $\alpha_n^{i_1...i_R}$ are completely antisymmetric in $i_1...i_R$. The index n is integer (half-integer) if R is even (odd). The algebra of the transformations (1) is

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$$[G_m^{i_1\ldots i_R}, G_n^{j_1\ldots j_S}\} = \mathrm{i}^{-RS}\left([m(2-S)-n(2-R)]G_{n+m}^{i_1\ldots i_R j_1\ldots j_S} - \mathrm{i}\sum_{l=1}^{R}\sum_{k=1}^{S}(-1)^{l+k+S}\delta^{i_l j_k}G_{n+m}^{i_1\ldots\hat{i}_l\ldots i_R j_1\ldots\hat{j}_k\ldots j_S}\right). \tag{2}$$

A general superconformal transformation is parametrized by the displacement superfield

$$E(z,\theta^i) = 2\sum_n\sum_{(i)}\mathrm{i}^{R(R-1)/2}\alpha_n^{(i)}\theta^{(i)}z^{n+1-R/2}. \tag{3}$$

where (i) denotes a general multi-index $i_1\ldots i_R$, $0\leqslant R\leqslant N$. The transformation of a *primary* superfield Φ_Δ of dimension Δ reads (cf. refs. [1,14])

$$\delta\Phi_\Delta = (\delta z)\partial_z\Phi_\Delta + (\delta\theta^i)\partial_{\theta^i}\Phi_\Delta + \Delta(\partial_z E)\Phi_\Delta = E\partial_z\Phi_\Delta + \tfrac{1}{2}(\mathrm{D}^i E)(\mathrm{D}^i\Phi_\Delta) + \Delta(\partial_z E)\Phi_\Delta. \tag{4}$$

where we used the covariant derivative $\mathrm{D}^i = \partial_{\theta^i} + \theta^i\partial_z$. The components $G_n^{(i)}$ can be assembled in the *current* superfield $J^{(N)}$

$$J^{(N)} = \sum_n\sum_{(i)}\frac{1}{R!}\mathrm{i}^{R(3R+1)/2}\theta^{N-(i)}G_n^{(i)}z^{-n-2+R/2}, \tag{5}$$

where $\theta^{N-(i)}$ is shorthand for $[1/(N-R)!]\epsilon^{j_1\ldots j_{N-R}i_1\ldots i_R}\theta^{j_1}\ldots\theta^{j_{N-R}}$. This definition is such that $J^{(N)}$ transforms as a primary superfield of dimension $\Delta = \frac{1}{2}(4-N)$. In a quantum mechanical realization of the SCA its transformation will be modified due to the presence of central terms in the quantum mechanical current algebra.

Using (3) and (5) we can write the variation of a general superfield under the superconformal transformation parametrized by $E(z,\theta^i)$ very concisely as

$$\delta\Phi(Z_2) = \frac{1}{2}\oint_{C_{z_2}}\frac{\mathrm{d}Z_1}{2\pi\mathrm{i}}E(Z_1)J^{(N)}(Z_1)\Phi(Z_2), \tag{6}$$

where $Z = (z,\theta^i)$, $\oint \mathrm{d}Z/2\pi\mathrm{i} = \oint(\mathrm{d}z/2\pi\mathrm{i})\int\mathrm{d}^N\theta$ and C_z is a curve in the complex plane enclosing the point z. All information about the transformation properties of $\Phi(z)$ is encoded in the super operator product expansion (SOPE) of $J^{(N)}(Z_1)$ and $\Phi(Z_2)$.

3. A representation of the so(4)-extended SCA. Having demonstrated some general features of theories with N-extended superconformal invariance we now turn to concrete realizations of these theories in terms of bosonic and fermionic quantum fields. If these fields are described by a *free* action then the central term in the Virasoro subalgebra

$$[L_n, L_m] = (n-m)L_{n+m} + \tfrac{1}{12}cm(m^2-1)\delta_{m+n} \tag{7}$$

is given by

$$c = \#(\text{real bosons}) + \tfrac{1}{2}\#(\text{Majorana fermions}). \tag{8}$$

A number of representations of SCA's in terms of free bosonic and fermionic fields are known. The $N=0,1,2$ and the exceptional $N=4$ SCA's admit *linear* representations with $c=1$, 3/2, 3 and 6 respectively which form the basis of the $N=0,1,2$ and 4 (spinning) string models [6]. In ref. [15] a purely bosonic *non-linear* representation of the $N=2$ SCA is given in terms of vertex operators built from a single bosonic field taking values on a circle (see also ref. [16]). Also in ref. [11], where a $c=3/2$ representation of the $N=3$ SCA is presented, vertex operators are used to construct some of the superconformal generators. Another possibility, which has first been noted in refs. [14,17], is to realize superconformal symmetries *non-linearly* among free fermions only. For $N=1$ the most general construction of this type has been discussed in ref. [18]. In ref. [19] a free fermion construction has been used to construct the discrete series of $c<3$ unitary representations of the $N=2$ SCA.

Here we will present a non-linear $c=3$ representation of the so(4)-extended $N=4$ SCA which we obtain by combining the following two $c=3/2$ representations of the $N=1$ SCA.

(i) The linear $N=1$ representation defined in terms of a real boson $\varphi(z)$ and a Majorana fermion $\psi(z)$. Using the elementary contractions

$$\langle\varphi(z)\varphi(w)\rangle=-\log(z-w)\,,\quad \langle\psi(z)\psi(w)\rangle=\frac{1}{z-w} \tag{9}$$

and the Wick theorem it is easily shown that the currents

$$L(z)=-\tfrac{1}{2}:(\partial_z\varphi)^2:-\tfrac{1}{2}:\psi\partial_z\psi:\,,\quad G(z)=-\mathrm{i}\psi\partial_z\varphi \tag{10}$$

satisfy the following short distance operator product expansions (OPE) with $c=3/2$:

$$L(z)L(w)=\frac{\frac{1}{2}c}{(z-w)^4}+\frac{2L(w)}{(z-w)^2}+\frac{\partial_w L(w)}{z-w}+\dots\,,$$

$$L(z)G(w)=\frac{\frac{3}{2}G(w)}{(z-w)^2}+\frac{\partial_w G(w)}{z-w}+\dots\,,\quad G(z)G(w)=\frac{\frac{3}{2}c}{(z-w)^3}+\frac{2L(w)}{z-w}+\dots\,. \tag{11}$$

These OPE's are equivalent to the component algebra (2), which for $N=1$ is just the familiar NSR algebra.

(ii) The purely fermionic $N=1$ representation defined in terms of three Majorana fermions $\chi^i(z)$, $i=1,2,3$. The currents

$$L(z)=-\tfrac{1}{2}:\chi^i\partial_z\chi^i:\,,\quad G(z)=\mathrm{i}\chi^1\chi^2\chi^3 \tag{12}$$

satisfy the same OPE's (11) with $c=3/2$. Supersymmetry is realized non-linearly on the fields $\chi^i(z)$

$$\delta_Q\chi^i(z)=\tfrac{1}{2}\,\mathrm{i}\epsilon^{ijk}\epsilon(z)\chi^j(z)\chi^k(z)\,. \tag{13}$$

The important observation is now that the combination (φ,ψ,χ^i) of both $c=3/2$ systems, which trivially realizes the $N=1$ SCA with $c=3$, has a much richer symmetry structure which turns out to be as large as the so(4)-extended SCA! In order to see this we write (χ^i,ψ) as χ^i, $i=1,\dots,4$, and we define

$$L(z)=-\tfrac{1}{2}:(\partial_z\varphi)^2:-\tfrac{1}{2}:\chi^i\partial_z\chi^i:\,,\quad G^i(z)=-\tfrac{1}{6}\mathrm{i}\epsilon^{ijkl}\chi^j\chi^k\chi^l-\mathrm{i}\chi^i\partial_z\varphi\,,$$

$$T^{ij}(z)=\mathrm{i}\chi^i\chi^j\,,\quad \Gamma^i(z)=\chi^i\,,\quad \Delta(z)=\varphi\,. \tag{14}$$

These operators satisfy the following OPE's with $c=3$:

$$L(z)L(w)=\frac{\frac{1}{2}c}{(z-w)^4}+\frac{2L(w)}{(z-w)^2}+\frac{\partial_w L(w)}{z-w}+\dots\,,\quad L(z)G^i(w)=\frac{\frac{3}{2}G^i(w)}{(z-w)^2}+\frac{\partial_w G^i(w)}{z-w}+\dots\,, \tag{15}$$

$$L(z)T^{ij}(w)=\frac{T^{ij}(w)}{(z-w)^2}+\frac{\partial_w T^{ij}(w)}{z-w}+\dots\,,$$

$$L(z)\Gamma^i(w)=\frac{\frac{1}{2}\Gamma^i(w)}{(z-w)^2}+\frac{\partial_w\Gamma^i(w)}{z-w}+\dots\,,\quad L(z)\Delta(w)=\frac{\partial_w\Delta(w)}{z-w}+\dots\,,$$

$$G^i(z)G^j(w)=\frac{\frac{2}{3}c\delta^{ij}}{(z-w)^3}+\frac{2\mathrm{i}T^{ij}(w)}{(z-w)^2}-\frac{\mathrm{i}\partial_w T^{ij}(w)}{z-w}+\frac{2\delta^{ij}L(w)}{z-w}+\dots\,,$$

$$G^i(z)T^{jk}(w)=-\epsilon^{ijkl}\left(\frac{\Gamma^l(w)}{(z-w)^2}+\frac{\partial_w\Gamma^l(w)}{z-w}\right)-\frac{\mathrm{i}[\delta^{ik}G^j-\delta^{ij}G^k(w)]}{z-w}+\dots\,, \tag{15}$$

Table 1
The so(4)-extended superconformal algebra. A $c=3$ realization of this algebra is provided by the Laurent coefficients $L_n = \oint(\mathrm{d}z/2\pi\mathrm{i})L(z)z^{n+1}$, $G^i_n = \oint(\mathrm{d}z/2\pi\mathrm{i})G^i(z)z^{n+1/2}$, $T^{ij}_n = \oint(\mathrm{d}z/2\pi\mathrm{i})T^{ij}(z)z^n$, $\Gamma^i_n = \oint(\mathrm{d}z/2\pi\mathrm{i})\Gamma^i(z)z^{n-1/2}$, $\Delta_n = \oint(\mathrm{d}z/2\pi\mathrm{i})\Delta(z)z^{n-1}$ of the currents (14). Up to central terms this algebra is identical to the algebra (2) for $N=4$ by the identifications $L_n = \frac{1}{2}G_n$, $G^i_n = G^i_n$, $T^{ij}_n = G^{ij}_n$, $\tilde{T}^{ij}_n = \frac{1}{2}\epsilon^{ijkl}G^{kl}_n$, $\Gamma^i_n = (-1/3!)\epsilon^{ijkl}G^{jkl}_n$, $\Delta_n = (1/4!)\epsilon^{ijkl}\Delta^{ijkl}_n$.

$[L_m,L_n] = (m-n)L_{m+n} + \frac{1}{12}cm(m^2-1)\delta_{m+n}$
$[L_m,G^i_n] = (\frac{1}{2}m-n)G^i_{m+n}$, $[L_m,T^{ij}_n] = -nT^{ij}_{m+n}$, $[L_m,\Gamma^i_n] = (-\frac{1}{2}m-n)\Gamma^i_{n+m}$, $[L_m,\Delta_n] = (-m-n)\Delta_{m+n}$
$\{G^i_m,G^j_n\} = 2\delta^{ij}L_{m+n} - \mathrm{i}(m-n)T^{ij}_{m+n} + \frac{1}{3}c(m^2-\frac{1}{4})\delta^{ij}\delta_{m+n}$
$[G^i_m,T^{jk}_n] = n\epsilon^{ijkl}\Gamma^l_{m+n} + 2\mathrm{i}\delta^{i[j}G^{k]}_{m+n}$, $\{G^i_m,\Gamma^j_n\} = \mathrm{i}(m+n)\delta^{ij}\Delta_{m+n} - \tilde{T}^{ij}_{m+n}$, $[G^i_m,\Delta_n] = \mathrm{i}\Gamma^i_{n+m}$
$[T^{ij}_m,T^{kl}_n] = -\mathrm{i}(\delta^{ik}T^{jl}_{m+n} - \delta^{il}T^{jk}_{m+n} - \delta^{jk}T^{il}_{m+n} + \delta^{jl}T^{ik}_{m+n}) + \frac{1}{3}c(\delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk})m\delta_{m+n}$
$[T^{ij}_m,\Gamma^k_n] = -2\mathrm{i}\delta^{k[i}\Gamma^{j]}_{m+n}$, $[T^{ij}_m,\Delta_n] = 0$, $\{\Gamma^i_m,\Gamma^j_n\} = \frac{1}{3}c\delta^{ij}\delta_{m+n}$, $[\Gamma^i_m,\Delta_n] = 0$, $[\Delta_m,\Delta_n] = \frac{1}{3}c(1/m)\delta_{m+n}$

$$G^i(z)\Gamma^j(w) = -\frac{\tilde{T}^{ij}(w)}{z-w} - \frac{\mathrm{i}\delta^{ij}\partial_w\Delta(w)}{z-w} + \dots, \quad G^i(z)\Delta(w) = \frac{\mathrm{i}\Gamma^i(w)}{z-w} + \dots,$$

$$T^{ij}(z)T^{kl}(w) = \frac{\frac{1}{3}c(\delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk})}{(z-w)^2} - \frac{-\mathrm{i}[\delta^{ik}T^{jl}(w) - \delta^{il}T^{jk}(w) - \delta^{jk}T^{il}(w) + \delta^{jl}T^{ik}(w)]}{z-w} + \dots,$$

$$T^{ij}(z)\Gamma^k(w) = \frac{-\mathrm{i}[\delta^{ki}\Gamma^j(w) - \delta^{jk}\Gamma^i(w)]}{z-w} + \dots,$$

$$\Gamma^i(z)\Gamma^j(w) = \frac{\frac{1}{3}c\delta^{ij}}{z-w} + \dots, \quad \Delta(z)\Delta(w) = -\frac{1}{3}c\log(z-w) + \dots. \tag{15cont'd}$$

This establishes our central result: the currents (14) form a closed $N=4$ superconformal algebra. The commutator algebra of the Laurent coefficients L_n, G^i_n, T^{ij}_n, Γ^i_n, Δ_n is listed in table 1.

From the representation (14) we can also obtain a purely fermionic representation of the so(4)-extended SCA. By introducing two fermions $\psi^1(z)$, $\psi^2(z)$ and making the replacements

$$L(z) \to -\frac{1}{2}:\psi^a\partial_z\psi^a: - \frac{1}{2}:\chi^i\partial_z\chi^i:, \quad \partial_z\Delta(z) \to \psi^1\psi^2, \quad G^i(z) \to -\frac{1}{6}\mathrm{i}\epsilon^{ijkl}\chi^j\chi^k\chi^l - \mathrm{i}\chi^i\psi^1\psi^2,$$

we obtain a realization of the so(4)-extended SCA in terms of fermionic fields only. The presence of six fermionic fields in this representation suggests that it could be possible to have a so(6) symmetry among these fermions and to define as much as $\binom{6}{3} = 20$ supercharges trilinear in the fermionic fields. However, upon closer inspection it turns out that in order to have a closed algebra without four-fermion terms (as we have in (15)) it is necessary to break the symmetry to so(4) and to keep only $\binom{4}{3} = 4$ of the supercharges.

In the representation (14) the superconformal transformations are realized *non-linearly* on the matter fields (φ,χ^i) but (if we discard the central terms for a moment) they are realized *linearly* on the currents $(L,G^i,T^{ij},\Gamma^i,\Delta)$. The current superfield, which for general N is given in (5) reads as follows for $N=4$:

$$J^{(4)} = -\Delta + \mathrm{i}\theta^i\Gamma^i - \frac{1}{4}\mathrm{i}\epsilon^{ijkl}\theta^i\theta^jT^{kl} - \frac{1}{6}\epsilon^{ijkl}\theta^i\theta^j\theta^kG^l + \frac{1}{12}\epsilon^{ijkl}\theta^i\theta^j\theta^k\theta^lL. \tag{16}$$

Apart from central terms, $J^{(4)}$ transforms as a primary superfield of dimension 0. The OPE's (15) for the currents $(L,G^i,T^{ij},\Gamma^i,\Delta)$ are equivalent to the following SOPE for the current superfield $J^{(4)}$

$$J^{(4)}(Z_1)J^{(4)}(Z_2) = \left(\sum_i \frac{\theta_{12}^{4-i}}{Z_{12}}\mathrm{D}_2^i + 2\frac{\theta_{12}^4}{Z_{12}}\partial_{z_2}\right)J^{(4)}(Z_2) - \log(Z_{12}), \tag{17}$$

where $Z_{12}=z_1-z_2-\Sigma_i\theta_1^i\theta_2^i$ and $\theta_{12}^i=\theta_1^i-\theta_2^i$. Note that this single formula summarizes all the commutation relations of the so(4)-extended SCA as listed in table 1. Since $\Delta(z)=\varphi(z)$ and $\Gamma^i(z)=\chi^i(z)$ we have the amusing situation that the $N=4$ *current* superfield is at the same time the $N=4$ *matter* superfield, or, stated differently, that the currents and the matter fields are in the same $(8+\bar{8})$ component chiral multiplet.

4. Reduction to $N=3,2$. Since the so(N)-extended SCA's with $N=3,2$ are subalgebras of the $N=4$ algebra, the fields (φ,χ^i) automatically provide representations of these smaller algebras, which are of some interest in their own right. Upon reducing $N=4\to N=3$ we only keep the superconformal generators $L(z)$, $G^i(z)$, $T^i(z)=-\frac{1}{2}\epsilon^{ijk}T^{jk}(z)$ and $\Gamma^4(z)$, $i,j,k=1,2,3$. Their Laurent coefficients $L_n, G_n^i, T_n^i, \Gamma_n^4$ form the so(3)-extended SCA with central charge $c=3$ which is a subalgebra of the $N=4$ SCA listed in table 1. The superfield $J^{(4)}$ splits as $J^{(4)}=-\Phi^{(3)}-\theta^4 J^{(3)}$, where $J^{(3)}$ is the $N=3$ current superfield (cf. (5)) and $\Phi^{(3)}$ is an $N=3$ matter superfield,

$$\Phi^{(3)}=\varphi-\mathrm{i}\theta^i\chi^i-\tfrac{1}{2}\epsilon^{ijk}\theta^i\theta^j\chi^k\psi+\tfrac{1}{6}\mathrm{i}\epsilon^{ijk}\theta^i\theta^j\theta^k(\tfrac{1}{6}\epsilon^{lmn}\chi^l\chi^m\chi^n-\psi\partial_z\varphi)\,. \tag{18}$$

Interestingly, this construction automatically provides an answer to the question of how to obtain an invariant action density for the components $(\varphi,\chi^i,F^i,\lambda)$ of a chiral $N=3$ multiplet, i.e. a $N=3$ scalar superfield $\Phi(z)$. It has been noted in ref. [9] that the traditional way to obtain an invariant action from a scalar superfield (by constructing a superspace density of the appropriate dimension from Φ and its covariant derivatives $\mathrm{D}^i\Phi$) fails for the N-extended algebras with $N\geqslant 3$. From the analysis above we see that for $N=3$ this problem can be solved quantum mechanically by expressing the "auxiliary fields" F^i and λ in φ, χ^i and an additional field ψ according to

$$F^k=\chi^k\psi\,,\quad \lambda=\tfrac{1}{6}\epsilon^{ijk}\chi^i\chi^j\chi^k-\psi\partial_z\varphi\,. \tag{19}$$

A free action for the fields (φ,χ^i,ψ) is then invariant under all superconformal transformations. The price one has to pay is that supersymmetry is realized non-linearly on (φ,χ^i,ψ),

$$\delta\varphi=\mathrm{i}\epsilon^i\chi^i\,,\quad \delta\chi^i=-\mathrm{i}\epsilon^i\partial_z\varphi+\mathrm{i}\epsilon^{ijk}\epsilon^j\chi^k\psi\,,\quad \delta\psi=-\tfrac{1}{2}\mathrm{i}\epsilon^{ijk}\epsilon^i\chi^j\chi^k\,. \tag{20}$$

A similar analysis for the breaking $N=4\to N=2$ leads to a non-linear $c=3$ representation of the $N=2$ SCA with one real boson and four Majorana fermions, in contrast with the linear $c=3$ representation which has a complex scalar field and one single Dirac fermion.

5. Comments.

(i) Superstring compactification. By applying fermionization to compactified coordinates the degrees of freedom of the $N=1$ superstring can be organized as follows [20–22]:

$$(x^\mu,\psi^\mu)\quad \mu=1,\ldots,4\,,\quad \chi^I\quad I=1,\ldots,18\,.$$

Since the number of internal fermions is a multiple of 6 it is possible to define a global $N=4$ superconformal symmetry on the internal fermionic degrees of freedom. It would be interesting to see whether this symmetry has any consequences for superstring dynamics.

(ii) Highest weight representations. A representation of the so(4)-extended SCA automatically provides a representation of the so(4) Kac–Moody subalgebra, which has level $k=c/3$. In a unitary highest weight representation the level k is a positive integer and consequently c is a positive multiple of 3. These values for c can all be realized by taking tensor products of copies of the basic $c=3$ representation (14).

(iii) $N=3,4$ string models. In principle the field φ in the multiplet (φ,χ^i) can be interpreted as a string-coordinate. However, due to the non-linearity of the supersymmetry transformations the formulation of "so(3)- or so(4)-strings" is not a straightforward extension of the results for strings with u(1)- or su(2)-extended supersymmetry [23,10,24].

(iv) Critical central charge. If a SCA appears as the algebra of constraints of a quantum system then the

theory is consistent only for one special value of the central charge. In ref. [25] it is shown that in the BRST-quantization of a system with constraints corresponding to the $N=3$ SCA the ghost contribution to Q^2_{BRST} vanishes and that accordingly the value of the critical central charge for this algebra is 0. It turns out that also the $N=4$ so(4)-extended SCA has critical central charge $c=0$ [#1]. This value differs from the critical value of c for the $N=4$ su(2)-extended SCA which is -12.

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