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A note on chaotic behavior in simple neural networks.

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A note on chaotic behavior in simple neural networks.

Abstract.

Local dynamics in a neural network are described by a two-dimensional (backpropagation or Hebbian) map of network activation and coupling strength. Adiabatic reduction leads to a non-linear one-dimensional map of coupling strength, suggesting the presence of a period-doubling route to chaos. It is shown that smooth variation of one of the parameters of the original map, -learning rate-, gives rise to period-doubling bifurcations of total coupling strength. Firstly, the associated bifurcation diagrams are given which indicate the presence of chaotic regimes and periodic windows. Secondly, pseudo-phase space diagrams and the Lyapunov exponents for alleged chaotic regimes are presented. Finally, spectral plots associated with these regimes are shown.

Keywords:
Autoassociator, Backpropagation, Hebbian learning, Chaotic behavior, Non-linearity, Periodic windows, Bifurcation, Deterministic chaos.

1.Dynamics of the autoassociator.

A large part of neural network modelling is based on the one-layer autoassociator. In this type of network two kinds of processes can be distinguished. The first process, which is described by the slow equation, represents the update of the synaptic weights on recursion k+1. The second process, which is described by the fast equation, is nested within the first. This process represents the way the activation values of the units develop within each cycle. Update of the weights takes place after the activation values approach stable values. Based on the above made distinction the autoassociator can be defined as follows:

The slow equation:

Backpropagation (e.g.McClelland & Rumelhart, 1986):

(1) $W_{ij}(k+1) = (1-D_W)\, W_{ij}(k) + \eta\, a_j(k)\, I_i(k) - \eta\, a_j(k)\, \Sigma a_l(k)\, W_{il}(k).$

Hebbian (e.g. Hopfield, 1982):

(2) $W_{ij}(k+1) = (1-D_W)\, W_{ij}(k) + \eta\, a_i(k)\, a_j(k).$

where W_{ij} is the weight constant modulating the activation of unit i by unit j, a_i the activation of unit i, I_i the external input of unit i, and k indexes the recursion step. D_W is a decay parameter of the weights and η notates the learning rate.

The fast equation:

(3) $a_i(t+1)=(1-D_a)\, a_i(t) + E\, (\Sigma\, [a_l(t)\, W_{il}(t)] + I_i(t))\, (1-a_i(t)),$
if $\Sigma_l\, [a_l(t)\, W_{il}(t)] + I_i(t) > 0.$

(4) $a_i(t+1)=(1-D_a)\, a_i(t) + E\, (\Sigma\, [a_l(t)\, W_{il}(t)] + I_i(t))\, (1+a_i(t)),$
if $\Sigma_l\, [a_l(t)\, W_{il}(t)] + I_i(t) \leq 0.$

where D_a is the decay parameter of the activations, E is the excitation parameter and t denotes the iteration step within each recursion.

The fast equation is repeated several times before the slow equation is updated.
Repeated application of equation (3) and (4) within each recursion k shows that a_i is a non-linear function of W_{ij}, while W_{ij} is a nonlinear function of all W. Specifically, substitution in equation (1) and (2) of the equilibrium values of a_i obtained after many iterations yields schematically:

(5) $W_{ij}(k+1) = (1-D_W)\, W_{ij}(k) + \eta\, f^e(W_{ij}(k)^p), \qquad p>1.$

where $f^e(.)$ denotes the value of the second and third term of the right hand side of equation (1) and (2) after substitution. η, the learning rate parameter, can be conceived of as a control parameter of the non-linear effects of the autoassociator.

2.Bifurcation diagrams.

Investigation of the non-linear effects of the autoassociator occurred by registrating the sum of absolute values of the weights for different values of η. Absolute values have been used in order to avoid redundant diagrams for sums of negative and positive weights separately.

As an illustration , we used a simple case, a three unit system where the external input, I , was given by the values: +1, -1, +1. The values of the parameters D_a, D_W, E were respectively 0.15, 0.5, 0.15 (McClelland & Rumelhart, 1986).

Figure 1, obtained with the Hebbian-rule, is found by plotting the sum of weights for the recursions 100 - 200; η was consecutively increased by .00666.

insert figure 1 about here.

Until η ♠ 5 the system reaches one state of equilibrium. Notice that the usual value of η (usually determined by 1/ N, in which N denotes the number of units) is within this stable range. Around η=5 a bifurcation appears: Between 5 and 6 the system is characterized by two equilibrium states. Between 6.05 and 6.25 additional bifurcations occur. At η= 6.25 a chaotic regime appears, followed by a periodic window consisting of 6 equilibrium values. From 6.3 upwards the system remains in an unpredictable chaotic regime. For η greater than 6.7 the activities, the weights and the sum of weights grow exponentially.

Across the total range from 0 to 6.7 the representation-task is successfully performed, the system produces the original prototype when solely the external input of the first unit is given.

The system with the backpropagation rule gives us a much richer diagram. For this rule η is consecutively updated with 0.01. Interestingly, in the periodic window between η=2.9 and η=3.2, the bifurcations visibly reappear.

insert figure 2 about here.

Also in this case across the total range from 0 to 3.3 the representation-task is successfully performed.

3.Chaotic regimes.

The bifurcation diagrams suggest chaotic behavior for certain intervals of the values of the learning rate parameter η. As a further test we will present three additional indices of chaotic behavior: the pseudo phase space diagrams, Lyapunov exponents and power spectra.

Pseudo phase space:

The backpropagation rule, for η=2.9 (a chaotic regime), produces a pseudo phase space diagram which is plotted in three dimensions. Figures 3A and 3B show two-dimensional projections (frontal and perpendicular view). Figures 4A and 4B gives the projections for the Hebbian rule.

These figures suggest chaos (Moon,1987).

insert figure 3 about here.

insert figure 4 about here.

Lyapunov exponents:

The Lyapunov exponents indicate chaos when the largest exponent is positive. These exponents were computed for the 3-dimensional pseudo phase space trajectories (Wolf et al., 1984). The results are given in table 1.:

insert table 1 about here.

Power spectra:

Power spectra for chaos regimes of both the backpropagation rule (η=2.9) and the Hebbian rule (η=6.4) are shown in Figures 5 and Figure 6.

insert figure 5 about here.

insert figure 6 about here.

Both figures 5 and 6 show that power is distributed across all frequencies, indicating chaos (Moon, 1987).

4.Discussion.

The case described in the foregoing sections is a simple one, only one prototype is used as external input in a system of three units. For more complex systems, the range between equilibrium and the exponential growth of the sum of weights decreases. Whithin this range, however, the same chaotic regimes have been found to accur. Yet, in the light of recent publications on chaotic neural activity (Skarda & Freeman, 1987; Harth, 1983), it is intriguing that a simple neural network already shows such complex behavior.

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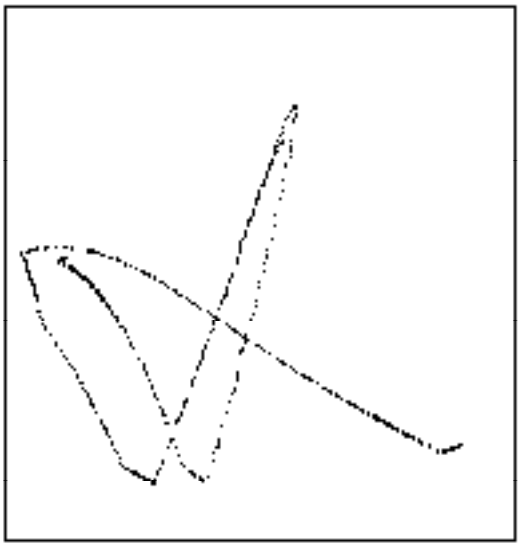
figure 1: Bifurcation diagram of the Hebbian rule.

figure 2: Bifurcation diagram of the Backpropagation rule.

figure 3A: Pseudo phase space diagram for the backpropagation rule. η=2.9 (frontal view).

figure 3B: Pseudo phase space diagram for the backpropagation rule. η=2.9 (perpendicular view).



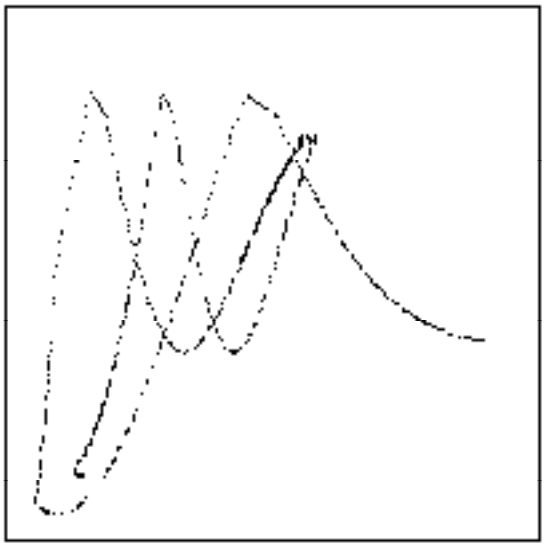
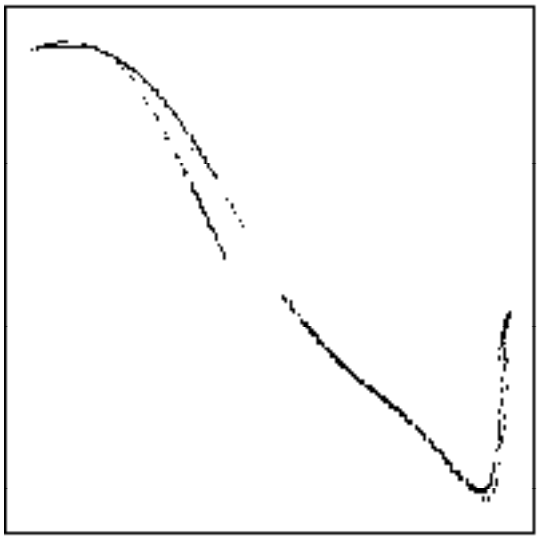


figure 4A: Pseudo phase space diagram for the Hebbian rule. η=6.4 (frontal view).

figure 4B: Pseudo phase space diagram for the Hebbian rule. η=6.4 (perpendicular view).



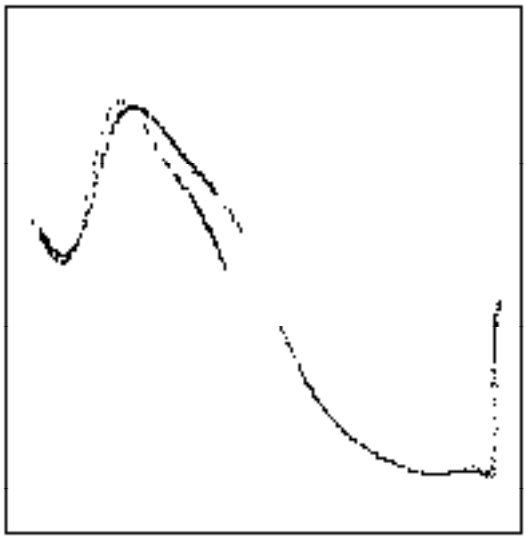


table 1: Lyapunov exponents.

Backpropagation rule			Hebbian rule		
η	Lyap. exp.	behavior	η	Lyap. exp.	behavior
1.00	-1.354	equilibrium	1.00	-0.850	equilibrium
2.28	-0.819	period	6.29	-0.478	period
2.50	0.244	chaos	6.40	0.276	chaos
2.90	0.300	chaos	6.90	0.339	chaos

Figure 5:

Figure 6: