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Costly Voting with Correlated Preferences

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Abstract

We consider a process of costly majority voting where people anticipate that others have similar preferences. This perceived consensus of opinion is the outcome of a fully rational Bayesian updating process where individuals consider their own tastes as draws from a population. We show that the correlation in preferences lowers expected turnout. In one extreme when preferences are independent, the amount of voting is excessive, while it is less than socially optimal when preferences are perfectly aligned. The intuition is that with correlation in preferences, votes have a positive externality on those who don't participate, which reduces incentives to participate. We study the effects of the public release of information ("polls") on participation levels. We find that polls raise expected turnout but reduce expected welfare. Finally, we discuss the merits of delegating voting authority to a committee. We show that a selfishly interested committee may improve welfare and that the optimal committee size is necessarily odd.

[1]We would like to thank Michelle Goeree, Tom Palfrey, and Martin Sefton for useful suggestions.

1. Introduction

It is well documented that people tend to use their own tastes and beliefs as information in guessing what others like and believe. For example, Ross, Greene, and House (1977) report a study where students were asked whether they would walk around campus wearing a sign. While those who agreed estimated 62% of the others would, those who refused thought 67% would refuse. Ross *et al.* (1977) termed this phenomenon the "false consensus effect:" people who engage in a given behavior estimate that behavior to be more common than people who engage in alternative behaviors.

Dawes (1990) first noted that the definition put forth by Ross *et al.* (1977) does not necessarily justify the term "false." When someone's decision is driven by their taste, which is seen as a draw from a population, then it is perfectly rational to use this decision the same way as any other random sample of size one. Only when the information about one's own taste is overweighed is the perceived consensus false. Engelmann and Strobel (2000) suggest the more cautious notion "consensus effect" to indicate that the perceived similarity in tastes and decisions may be the outcome of a fully rationally Bayesian updating process.[1]

The consensus effect appears to be a robust phenomenon and has been observed for a wide range of preferences and opinions (e.g. Mullen *et al.*, 1985). It seems especially strong when applied to political opinions. Brown (1982) reports the choices of 179 psychology students who had to indicate their preferred candidate in the 1980 US presidential election: Anderson, Carter, or Reagan. In addition, they had to estimate the percentage of students in the class believed to prefer each candidate. Supporters of all three candidates estimated significantly higher support for their own candidate compared to the predictions of the rest of the class.[2] A similar picture arises from an empirical study of the 1992 Constitutional Referendum in Canada (Baker *et al.*, 1995), where those planning to vote "Yes" predicted a significantly higher proportion of "Yes" votes than those planning to vote "No," and vice versa. The extent of the consensus effect in the Canadian referendum is noticeable especially because a substantial monetary reward was offered for accurate estimates.

[1] Engelmann and Strobel (2000) report experimental evidence that people do not tend to overweigh their own information when they are given representative information and are paid for their predictions.

[2] Anderson supporters estimated 30.4% of the class preferred their candidate, while Carter and Reagan supporters estimated that 24.4% and 22.0% favored Anderson. Similarly, Carter supporters predicted 38.3% would favor their candidate while Anderson and Reagan supporters estimated 34.1% and 33.6% favored Carter. More dramatic differences were observed for the estimates of support for Reagan: those favoring Reagan predicted this to be 44.5% while Anderson and Carter supporters' estimates were 35.4% and 37.9%.

One common explanation for the consensus effect is selective exposure: people tend to seek out those similar to themselves and draw their inferences from experiences with those they most often interact. This seems to apply to political opinions as people typically discuss politics with like-minded friends. While the occurrence of a consensus effect in voting processes has been well documented, its impact on voting decisions has not yet been studied. In this paper, we present a model of majority voting where decision-makers rationally anticipate that others have similar preferences. Our results provide the first analysis of costly voting with correlated preferences.

To avoid problems related to the Condorcet paradox we focus on situations where individuals choose between two alternatives. We assume the size of the electorate is commonly known but it is uncertain how many people favor either alternative. The best people can do to estimate the support for both alternatives is to use their own preference, which creates a consensus effect. In other words, individuals with opposite preferences will have conflicting views about which alternative is more likely to be favored by the majority.

Our approach contrasts with previous literature where common knowledge about how many people prefer either alternative is assumed. This seems restrictive and begs the question why a costly election should be held even though it is commonly known which alternative is favored. To some extent, the same critique applies to models where people know their own preference and the probability distribution over how many others prefer either alternative. For large electorates, the resulting randomness in support is immaterial and common knowledge of the distribution functions implies the majority-preferred alternative can be predicted perfectly.[3] The need for a costly election process can be justified, however, when there exist conflicting opinions about which alternative is favored.

In our model, voting is voluntary: individuals first choose whether or not to participate in the decision-making process, and those who participate then vote for one of the alternatives. We assume that participation is costly so that it may not be socially optimal for everyone to vote. Individuals' preferences reflect idiosyncratic tastes, i.e. our model is one of private values. In this case, voting against one's preferred alternative is strictly dominated by not

[3]See, however, Myatt (2002) who considers the case where individuals receive informative signals about support levels, which remain uncertain even in large electorates. In his model, voting is costless and there are more than two candidates. Myatt shows that multi-candidate support can arise in his model, in contrast with (the stricter version of) Duverger's Law (1954) that "with rational voters, multi-candidate contests under the plurality rule result in only two candidates getting any votes" (Palfrey, 1989). We should like to thank Tom Palfrey for pointing Myatt's (2002) paper out to us.

participating. Hence, the voting decision for those who participate is straightforward: one should vote sincerely for one's preferred alternative.

While the model is *ex ante* symmetric with respect to alternatives and individuals, voters' preferences are correlated. We show that this correlation reduces participation. One reason is that correlation in preferences lowers the benefits of participating since it lowers the chances of being pivotal. To see this, consider first the case when preferences are independent and suppose an individual believes that exactly one of the others will vote. Then the chance of being pivotal is one-half, i.e. the probability that the other voter has opposing preferences. If, however, preferences are perfectly correlated, the chance of being pivotal is zero when one other person votes.

In addition, when preferences are independent and both alternatives are equally likely to be favored, votes have no positive externality on those who don't participate, since any vote is equally likely to be in favor or against their preferred alternative. In contrast, with correlated preferences any vote is more likely to be in favor and thus creates a positive externality on those who don't vote. It is this positive externality that further reduces the incentives to participate. Note that irrespective of whether preferences are correlated, one person's decision to participate creates a negative externality for others who participate, since it reduces their chances of being pivotal.

These simple observations explain our results regarding equilibrium versus optimal voting. In the case of independent preferences, only a negative externality arises and voting is excessive as a result. The level of participation is such that expected welfare, as measured by the total payoff to the group, is zero. When the degree of correlation in preferences increases, however, the positive externality becomes stronger to the point where it overthrows the negative externality. Indeed, when preferences are perfectly aligned, participation levels are lower than what is socially optimal and expected welfare is positive.

We study the impact of public information release or "polls." When people use this public information to update their perceptions of others' preferences, it will correct the views of some and reinforce others. As a result, the public information will reduce the participation incentives of those who are reinforced in their belief that they belong to the majority. And it will stimulate participation of the minority group who realize they overestimated the support for their preferred alternative. We show that the net effect of polls is to raise expected turnout. However, the increase in participation levels due to polls is not welfare improving because the release of public information stimulates the "wrong" group to participate.

Of special interest is the limit case when the public release of information reduces all (perceived) correlation in preferences. In the resulting model, everyone knows that (i) others' preferences are independent of their own and that (ii) each other person favors alternative 1 with probability $p \geq 1/2$ and alternative 2 with probability $1-p$. This type of preference uncertainty is similar to the one studied by Ledyard (1981, 1984), Palfrey and Rosenthal (1985), and Börgers (2001). In Ledyard's (1984) model, each individual knows her own characteristics, i.e. her cost and "location," the spatial positions of both alternatives, and the distribution of others' costs and locations. Ledyard proves existence of a symmetric equilibrium, but the possibility of multiple symmetric equilibria is not studied.[4] Palfrey and Rosenthal (1985) consider two types, or groups, of individuals: within a group, individuals have identical preferences, but preferences are opposite between groups. They derive conditions for a (possibly non-unique) symmetric equilibrium and study its limit properties as the size of the electorate diverges.

We consider a more specific setup than that of Ledyard (1984) and Palfrey and Rosenthal (1985), which enables us to derive more detailed predictions regarding equilibrium and optimal behavior. Our model is closest to that of Börgers (2001). He proves existence of a unique symmetric equilibrium when preferences are independent and both alternatives are equally likely to be favored.[5] We extend Börgers' analysis to the case where one alternative is favored by a fraction $p \geq 1/2$ of the electorate. We show that, for some range of parameter values, there is a unique equilibrium in totally mixed strategies for which total expected turnout is *independent* of p. Moreover, the expected turnouts of both groups are *equal*, implying that the "smaller" group participates more frequently to offset the advantage of the "larger" group. The election therefore ends in a tie in expectation, with negative implications for expected welfare.

Finally, we discuss the merits of delegating voting authority to a smaller committee, which regularly occurs in smaller groups. While this creates an asymmetry between individuals, the advantage is that a certain level of participation can be enforced. We find that the optimal committee size is necessarily odd. The intuition is that when going from an odd to an even sized committee, the extra vote only matters when it creates a tie. However, a tie

[4]Ledyard (1984) also considers the case where the spatial positions of the alternatives are endogenized. He proves that the resulting equilibrium is optimal in the sense that candidates choose identical positions which maximize welfare and no one votes.

[5]This result sharply contrasts with the multiplicity of equilibria in participation games with equal and known group sizes, see Palfrey and Rosenthal (1983).

with an even number of voters implies that the alternative will be selected randomly, and the expected benefit of a randomly selected outcome is zero.

When preferences are perfectly aligned, the optimal committee size is obviously one. In the other extreme, when both alternatives are equally likely to be favored, the optimal committee size depends on the cost of participating, but is independent of the size of the group. The intuition is that all non-committee members are equally likely to agree or disagree with the committee's choice. As a result, total expected benefits are independent of the number of people not on the committee. When preferences are correlated but not perfectly so, the optimal committee size is odd, decreasing in the cost of participating and increasing in group size.

The paper is organized as follows. Section 2 explains the model and provides an equilibrium and welfare analysis of simple majority voting. In section 3, we study the effects of public information release. In section 4 we consider majority voting when it is commonly known that each individual favors one of the alternatives with probability $p \geq 1/2$. The merits of delegating voting power to a smaller committee are discussed in section 5. Section 6 concludes. Proofs of the propositions can be found in the Appendix.

2. Simple Majority Voting

There are $n \geq 2$ individuals labelled $i = 1, \ldots, n$. Individuals can choose to participate in the decision-making process at a cost, $c > 0$, which gives them the opportunity to vote for one of two alternatives, B (blue) or R (red). Individuals who do not wish to participate bear no costs and do not vote. The outcome of the voting process is determined by simple majority (with a random tie-breaking rule) and applies to all n individuals irrespective of whether they participated. Individuals earn \$1 if their preferred outcome wins and lose \$1 otherwise.

The model is *ex ante* symmetric both with respect to alternatives and individuals in the following sense. First, "nature" selects one of two possible states, 0 or 1, both of which are equally likely. The realization of this state is not observed by the electorate. If the state is 0, individuals favor alternative B with probability $p \geq 1/2$ and alternative R with probability $1 - p$.[6] Similarly, when the state is 1, individuals favor alternative R with probability p and

[6] By restricting $p \geq 1/2$, the 0-state makes a blue outcome more likely while the 1-state is more conducive

alternative B with probability $1-p$. Note that, independent of whether the realized state is 0 or 1, an individual with a preference for one color rationally anticipates that others are more likely to favor that same color. Indeed, a simple application of Bayes' rule shows that

$$P(\text{other prefers blue} \,|\, \text{I prefer blue}) \;=\; p^2 + (1-p)^2, \tag{2.1}$$

and the same equation holds for red. The right side of (2.1) is a convex function of p, which is minimized at $p = 1/2$ where its value is $1/2$. Hence, individuals believe others are more likely to prefer the same color whenever $p > 1/2$, while preferences are independent at $p = 1/2$.

In our setup, correlation of preferences arises via a fully rational Bayesian updating process. However, the behavior of rational voters in our model is identical to that of voters with possibly independent preferences who fall prey to the false consensus bias, i.e. erroneously assume that others favor the same color as per (2.1).

We next describe equilibrium behavior. Since our model is one of private values and voting is costly, voting against one's true preference is strictly dominated by not participating. Hence, those who participate vote sincerely. We assume that the decision to participate is independent of one's label and of one's preferred color as the model is *ex ante* symmetric. In other words, we restrict attention to symmetric Bayesian-Nash equilibria. Let γ denote the probability that an individual participates. This probability will vary with the cost of voting, c, and the size of the electorate, n. First note that with a finite number of people, there is always some benefit to voting (i.e. of being pivotal) even when all others participate. Hence, for very low costs $c < \underline{c}$, everyone participates with probability 1.[7] In contrast, when $c > \overline{c} = 1$, the equilibrium probability of voting is 0. To see this, suppose all individuals different from i decide not to participate. Then i's expected payoff of participating is $1-c$, while the expected payoff of not participating is $\frac{1}{2} \cdot (1) + \frac{1}{2} \cdot (-1) = 0$.

Henceforth, we consider the more interesting case $\underline{c} \leq c \leq \overline{c}$ so that both "staying out" and participating occur with strictly positive probability. In the terminology of Palfrey and Rosenthal (1983), we will consider only *totally mixed* equilibria in which no individual plays a pure strategy. Of course, individuals are only willing to mix between participating and not

to red outcomes. This link between states and colors plays no role in what follows, and nothing would change if it were reversed. This could be done, for instance, by considering values of p less than $1/2$. To avoid trivial duplication, we henceforth assume $p \geq 1/2$.

[7]For example, with an even number of people, the highest cost for which full participation occurs is $\underline{c} = \binom{n}{\frac{n}{2}} p^{\frac{n}{2}} (1-p)^{\frac{n}{2}}$, see Proposition 1.

participating if they are indifferent, which implies that the expected benefit of participating equals the cost. The benefit of voting is zero unless the vote makes a difference, i.e. when it is pivotal. There are two instances when individual i's vote is pivotal: either i's preferred color is one vote short, in which case i's vote creates a tie, or i's vote breaks a tie. In the former case, i's expected payoffs are raised from -\$1 to \$0 and in the latter case they increase from \$0 to \$1. So the expected benefit of being pivotal is \$1.

It remains to determine the probability of being pivotal, which depends on how many others participate. Suppose, without loss of generality, that i's preferred alternative is B and consider the case with an even number, $2k$, of other participants. Then i's vote can only be pivotal when k of the others prefer B and the remaining k prefer R. The probability of this event is

$$P_{piv}(2k) \;=\; \binom{2k}{k}\, P(k \text{ others prefer } B,\, k \text{ others prefer } R \,|\, i \text{ prefers } B),$$

and a straightforward application of Bayes' rule shows that this probability can be written as

$$P_{piv}(2k) \;=\; \binom{2k}{k}\, p^k(1-p)^k. \tag{2.2}$$

Similarly, if the number of other participants is odd, $2k+1$, then i's vote is pivotal only when k others vote for B and $k+1$ others vote for R. The probability of being pivotal in this case can be worked out as

$$P_{piv}(2k+1) \;=\; 2\binom{2k+1}{k}\, p^{k+1}(1-p)^{k+1} \;=\; \binom{2k+2}{k+1}\, p^{k+1}(1-p)^{k+1}. \tag{2.3}$$

Some important insights can be gleaned from (2.2) and (2.3). First, note that $p(1-p)$ for $p \geq 1/2$ is a concave function that is maximized at $p = 1/2$. This implies that the probability of being pivotal for a given number of other participants decreases with p. Indeed, in the extreme case of perfectly correlated preferences ($p = 1$), the probability of being pivotal is zero when $k \geq 1$ others participate. Second, comparing (2.2) and (2.3) shows that $P_{piv}(2k-1) = P_{piv}(2k)$ and $P_{piv}(2k+1) < P_{piv}(2k)$ for all $k \geq 1$.[8] Hence, the probability of being pivotal (weakly) decreases in the number of other participants.

In a symmetric Bayesian-Nash equilibrium where all individuals participate with probability γ the expected probability of being pivotal is simply $\sum_{k=0}^{n-1} P(k; n-1, \gamma) P_{piv}(k)$, with

[8] It is readily verified that $P_{piv}(2k+1) = \frac{k+1/2}{k+1}(4p(1-p))P_{piv}(2k) < P_{piv}(2k)$.

$P(k; n-1, \gamma)$ the usual binomial probability of observing k successes out of $n-1$ trials when the probability of each success is γ. Proposition 1 summarizes our findings so far, where $\lfloor x \rfloor$ denotes the integer part of x.

Proposition 1. *In the unique totally-mixed, symmetric Bayesian-Nash equilibrium, the probability of participating, γ^*, is determined by*

$$\sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^*)^k (1-\gamma^*)^{n-1-k} \binom{2\lfloor \frac{k+1}{2} \rfloor}{\lfloor \frac{k+1}{2} \rfloor} p^{\lfloor \frac{k+1}{2} \rfloor} (1-p)^{\lfloor \frac{k+1}{2} \rfloor} = c, \tag{2.4}$$

and those who participate vote sincerely for their preferred alternative.

Uniqueness of the symmetric equilibrium follows since the probability of being pivotal falls with the number of other participants, k, and an increase in γ raises k in the sense of first-order stochastic dominance (see Lemma A1 in the Appendix). Hence, (2.4) cannot be satisfied for some γ and γ' with $\gamma > \gamma'$ or $\gamma' > \gamma$.

It is useful to clarify the relationship between our setup with symmetric costs and complete information and models that assume participation costs are private information and distributed according to some known distribution function $F(\cdot)$ (e.g. Ledyard, 1984, Palfrey and Rosenthal, 1985, and Börgers, 2001). Equilibrium behavior in the latter models can be characterized by a threshold cost, c^*: individuals with costs below c^* participate, while those with higher costs stay out. In contrast, in our model individuals mix between participating and staying out and the probability of voting is γ^* for all. As noted by Palfrey and Rosenthal (1985) there is a simple link between the two models. The fraction of the electorate that votes when costs are private, $F(c^*)$, is equal to γ^*, the fraction of voters in the model with symmetric and known costs. In other words, the necessary condition that determines the equilibrium threshold level c^* in the model with incomplete information is simply (2.4) with γ^* replaced by $F(c^*)$. In this sense, the results of this paper would not change if instead of symmetric and known costs we would assume that voters are privately informed about their participation costs.

A simple stochastic dominance argument can be applied to derive comparative statics predictions. Recall that the probability of being pivotal, $P_{piv}(k)$, decreases with the degree of correlation in preferences, p, for all k. Hence, to maintain equality (2.4), probability mass should shift to lower values of k when p increases. This implies that γ^* has to fall. Similarly, when the cost of participating rises, lower values of k should become more likely for (2.4) to hold, i.e. γ^* should decrease. Finally, an increase in the size of the electorate, n, raises the expected number of participants k, which has to be offset by a decrease in γ^* to maintain (2.4).

Proposition 2. *The equilibrium probability of participating, γ^*, is decreasing in the degree of correlation in preferences, p, the cost of participating, c, and the size of the electorate, n.*

What remains is to compare the equilibrium level of participation with the socially optimal level. Consider first the case of $n = 2$. The equilibrium level of participation that follows from (2.4) is given by

$$\gamma^* = \frac{1-c}{p^2 + (1-p)^2}.$$

To derive the socially optimal level, note that the sum of players' payoffs is positive only when individuals have the same preferences, which occurs with probability $p^2+(1-p)^2$. Both individuals then receive \$1 when at least one person participates while expected payoffs are zero if neither participates. Hence, expected welfare is $W = 2(\gamma^2 + 2\gamma(1-\gamma))(p^2 + (1-p)^2) - 2\gamma c$, and maximization of W with respect to γ yields the socially optimal level

$$\gamma^o = 1 - \frac{c/2}{p^2 + (1-p)^2}.$$

The equilibrium and socially optimal levels of participation are graphed in Figure 1 for the case of no correlation ($p = 1/2$) on the left and perfect correlation ($p = 1$) on the right. The left panel shows that voting is excessive when preferences are independent. The solid line representing equilibrium levels is above the dashed line representing optimal levels for all values of the cost parameter. It is readily verified that, in equilibrium, expected welfare is zero. In contrast, the probability of participating is too low when preferences are perfectly aligned (see the right panel of Figure 1). Expected welfare, however, is positive in this case. These findings are not restricted to the $n = 2$ case as the next proposition shows.

Proposition 3. *The equilibrium level of participation exceeds the socially optimal level if preferences are independent while the reverse is true if preferences are perfectly correlated. In equilibrium, expected welfare is zero when preferences are independent and positive when preferences are perfectly correlated.*

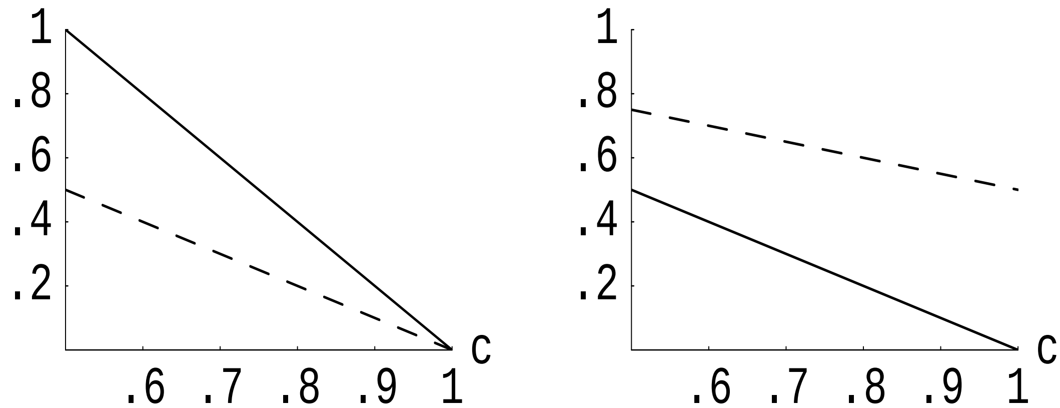

Figure 1: *Equilibrium (solid) and socially optimal (dashed) levels of participation as functions of the cost, c, when preferences are independent (left) and perfectly correlated (right).*

In the no-correlation case, the difference between optimal and equilibrium levels of participation can be understood as follows. The decision of one person to participate creates a negative externality for other participants since it reduces their chances of being pivotal. This negative externality is not incorporated enough by individuals, which is why they participate more than is socially optimal. When preferences are perfectly aligned, all individuals favor the same alternative. This situation is equivalent to a "volunteer's dilemma" game, where all individuals receive \$1 when at least one person makes the costly decision to participate. In other words, the decision of one person to participate results in a positive externality for non-participants, creating incentives to "free ride."[9]

We next explain why the equilibrium welfare level is zero when preferences are independent. Abbreviate the left side of (2.4) as $E_{k;n-1,\gamma^*}(P_{piv}(k))$.[10] If we multiply both sides of (2.4) by $n\gamma^*$ we can rewrite the resulting equation as[11]

$$E_{k;n,\gamma^*}(kP_{piv}(k-1)) \,=\, E_{k;n,\gamma^*}(kc).$$

[9]It is straightforward to show that the equilibrium level of participation for the volunteer's dilemma game is given by $\gamma^* = 1 - c^{\frac{1}{n-1}}$, while the socially optimal level is $\gamma^o = 1 - (c/n)^{\frac{1}{n-1}}$. To see how strong the incentives to free ride are, note that the probability that no one participates is $(1-\gamma^*)^n = c^{\frac{n}{n-1}}$. So even when the size of the electorate diverges to infinity, there is a chance $c > 0$ that the preferred alternative is not chosen.

[10]For an arbitrary function, $f(k)$, we have $E_{k;n,\gamma}(f(k)) = \sum_{k=0}^{n} P(k;n,\gamma)f(k)$.

[11]Here we use $n\gamma P(k-1;n-1,\gamma) = kP(k;n,\gamma)$ for all $k \geq 0$.

The left side is the expected sum of "selfish" individual benefits for those that vote, while the right side represents total expected costs for this group. Equilibrium expected welfare may be defined as

$$W \;=\; E_{k;n,\gamma^*}(W(k)) - E_{k;n,\gamma^*}(kc),$$

where $W(k)$ is the expected benefit of k people voting. When preferences are independent, votes have no positive externality on those who don't participate since any vote is equally likely to be in favor or against their preferred alternative. Hence, in this case, the expected benefit $W(k)$ measures only the benefits to those that participate: $W(k) = kP_{piv}(k-1)$.[12] Together with the above equations this shows that expected welfare is zero when preferences are independent.

3. Public Information Release

In many situations, potential voters possess more information than just knowledge of their own preferred alternative. For instance, when an academic department interviews candidates to fill a position, faculty members may discuss their impressions with colleagues. Likewise, in large elections, people often get an impression of others' preferences via polls. Here we study the effect of the public disclosure of information on the decision to participate. Intuitively, when information is released that makes state 0, say, more likely, then those that favor blue are reinforced in their opinion that others also favor blue. As a result, they will tend to participate less. In contrast, those that favor red adjust the likelihood that others favor the same color downwards, resulting in a higher propensity to participate. Below we determine the effects of public information disclosure on expected turnout and welfare.

Let $\mathcal{I}$ be a publicly observable signal, which provides information about the likelihood of the realized state $z = 0, 1$. For instance, when a poll is conducted among a thousand randomly selected people, $\mathcal{I}$ could be the percentage that favors a certain presidential candidate. Or, $\mathcal{I}$ could reveal a single colleague's impression of a particular job candidate. In both these examples, the public signal $\mathcal{I}$ provides some information about the state of the world, albeit of different precision or quality. More generally, we define the likelihood-ratio $\alpha \equiv P(\mathcal{I}\,|\,0)/P(\mathcal{I}\,|\,1)$. Without loss of generality we will focus on the case $\alpha \geq 1$, so that the 0 state is more likely. Note that when $\alpha = 1$, the signal $\mathcal{I}$ provides no information and

[12]See the proof of Proposition 3 in the Appendix for further details.

the model is identical to that of the previous section. When $\alpha \to \infty$, people know for sure that the realized state is 0 and that another's preferred choice is blue with probability p and red with probability $1 - p$.

Before we present a general analysis consider again the case $n = 2$. Note that the presence of the public signal $\mathcal{I}$ alters (2.1), i.e. the chance that another person favors the same color. We now have

$$P(\text{other prefers blue} \,|\, \text{I prefer blue, public signal } \mathcal{I}) \;=\; \frac{\alpha p^2 + (1-p)^2}{\alpha p + (1-p)}, \tag{3.1}$$

while

$$P(\text{other prefers red} \,|\, \text{I prefer red, public signal } \mathcal{I}) \;=\; \frac{p^2 + \alpha(1-p)^2}{p + \alpha(1-p)}. \tag{3.2}$$

Since $p \geq 1/2$, an increase in the likelihood-ratio, α, raises (lowers) the probability that others favor blue (red) given that I prefer blue (red). Let $P(B \,|\, B,\mathcal{I})$ and $P(R \,|\, R,\mathcal{I})$ denote the probabilities in (3.1) and (3.2) respectively. Someone's vote is pivotal when the other person has opposite preferences (irrespective of whether the other participates) or when the other has the same preferences but doesn't participate. So the probability, γ_B, that an individual who favors blue will participate satisfies $P(B \,|\, B,\mathcal{I})(1-\gamma_B)+(1-P(B \,|\, B,\mathcal{I})) = c$, or

$$\gamma_B^* \;=\; \frac{1-c}{P(B \,|\, B,\mathcal{I})},$$

and similarly

$$\gamma_R^* \;=\; \frac{1-c}{P(R \,|\, R,\mathcal{I})}.$$

A more precise information signal that raises the likelihood of the 0 state therefore reduces the participation incentives for those that favor blue and increases the incentives for those that favor red.

It is interesting to compare the impact of the public information release on equilibrium versus socially optimal levels of participation. It is readily verified that the welfare maximizing levels of participation after the public signal $\mathcal{I}$ is released become

$$\gamma_B^o \;=\; 1 - \frac{c/2}{P(B \,|\, B,\mathcal{I})},$$

and

$$\gamma_R^o \;=\; 1 - \frac{c/2}{P(R \,|\, R,\mathcal{I})}.$$

Hence, the public information signal $\mathcal{I}$ affects equilibrium and optimal levels in an *opposite* manner. Information that makes blue more likely reduces the participation-incentives for those that favor blue, but it raises the value of a vote for blue (since more people benefit), which is why the socially optimal level of participation rises. Likewise, such information raises the equilibrium levels of participation for those that favor red, while their votes are welfare reducing since they lower the chance that the majority wins. For the $n = 2$ case, it is straightforward to compute the welfare level that results in equilibrium and show it declines with α.[13] In other words, *polls unambiguously reduce welfare.*

4. Independent Preferences with a Preferred Alternative

The analysis of polls becomes more complicated for general electorate sizes. We can, however, derive results for the interesting case $\alpha = \infty$, i.e. when the release of the public information signal, $\mathcal{I}$, resolves all uncertainty about the state. This would occur, for instance, when a poll is taken among a large number of people. In the resulting model, everyone knows that (i) others' preferences are independent of their own and that (ii) each other person favors B with probability $p \geq 1/2$ and R with probability $1 - p$. This model has previously been studied by Börgers (2001) for the special case of $p = 1/2$.

First, consider again the $n = 2$ case. The equilibrium participation probabilities given in section 3 limit to

$$\gamma_B^* = \frac{1-c}{p},$$

and

$$\gamma_R^* = \frac{1-c}{1-p}.$$

Hence, without correlation in preferences, the expected turnout of the two groups is the same: $2p\gamma_B^* = 2(1-p)\gamma_R^*$, and the election will end in a tie in expectation. The latter property generalizes to electorates of arbitrary size.

[13]When those that favor blue/red participate with probability γ_B^* and γ_R^* respectively, then welfare is given by

$$W = 2(1-c)\frac{(1-2p)^2(p^2+(1-p)^2)}{(\alpha p^2+(1-p)^2)(p^2+\alpha(1-p)^2)},$$

which decreases with α. Notice that welfare is 0, independent of α, when $p = 1/2$ (see also Proposition 3). In this case, a poll will not change the likelihood of either state $z = 0, 1$ and hence will not affect individuals' participation decisions.

Proposition 4. *Suppose it is commonly known that each individual favors B with probability $p \geq 1/2$ and R with probability $1-p$. In the unique totally-mixed Bayesian-Nash equilibrium, the probabilities of participating for those who prefer B and R are given by $\gamma_B^* = \gamma^*/(2p)$ and $\gamma_R^* = \gamma^*/(2(1-p))$ respectively, where γ^* satisfies*

$$\sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^*)^k (1-\gamma^*)^{n-1-k} \binom{2\lfloor \frac{k+1}{2} \rfloor}{\lfloor \frac{k+1}{2} \rfloor} \left(\tfrac{1}{2}\right)^{\lfloor \frac{k+1}{2} \rfloor} \left(\tfrac{1}{2}\right)^{\lfloor \frac{k+1}{2} \rfloor} = c, \tag{4.1}$$

and those who participate vote sincerely for their preferred alternative.

Recall that in the absence of any public information, expected turnout is equal to $n\gamma^*$ where γ^* satisfies (2.4). After the poll, total expected turnout is given by $n(p\gamma_B^* + (1-p)\gamma_R^*) = n\gamma^*$ where γ^* satisfies (4.1), which is (2.4) with p replaced by 1/2. Proposition 2 shows that γ^* is decreasing in p for $p \geq 1/2$, so expected turnout is higher after the poll.

Note, however, that it is the "minority color," red, that participates more frequently. Indeed, after the poll one should expect as many red as blue votes, even though only a $(1-p)$ minority of the population favors red. In contrast, in the absence of a poll those that favor blue or red participate with the same probability, and the minority color is only expected to receive a fraction $1-p$ of all votes. In other words, polls raise expected turnout (and, hence, expected costs) but lower the chance that the majority color wins.

Proposition 5. *The public release of information, which eliminates all correlation in preferences, raises expected turnout but lowers expected welfare to zero.*

The intuition for the decline in expected welfare is that in the absence of a poll, votes have a positive externality on those that don't participate. In other words, the expected benefit of k people voting is more than just the benefits for participants: $W(k) > kP_{piv}(k-1)$. Hence, expected welfare without a poll is positive:

$$W = E_{k;n,\gamma^*}(W(k)) - E_{k;n,\gamma^*}(kc) > E_{k;n,\gamma^*}(kP_{piv}(k-1)) - E_{k;n,\gamma^*}(kc) = 0.$$

After the poll, however, both types of votes are equally likely. Hence, for non-participants that favor B, for instance, the positive externality of a B vote is cancelled by the negative externality of an R vote. Now $W(k) = kP_{piv}(k-1)$ and expected welfare is zero.

5. Committees

In this section we study the effects of delegating voting power to a smaller group. For example, when positions at academic departments need filled, the faculty often appoint a recruiting committee to (pre)select candidates. We assume that those appointed to the committee are obliged to participate, e.g. a recruitment committee member has to be present at interviews and job talks and vote when the committee meets. Committee members are assumed to vote in their own selfish interest, however.[14]

It is easy to show that (selfish) committees can improve the overall welfare of the group. Indeed, denote the optimal committee size by k^*, where

$$k^* \in \operatorname{argmax}_k \{W(k) - kc\},$$

then expected welfare in the presence of an optimally-sized committee is simply given by $W_{committee} = W(k^*) - k^*c$. Without a committee the expected welfare of the group when all individuals follow their equilibrium strategies is

$$W = E_{k;n,\gamma^*}(W(k) - kc) \leq E_{k;n,\gamma^*}(W(k^*) - k^*c) = W_{committee}.$$

Below we determine the optimal size of the committee and study how it varies with participation costs, group size, and the degree of correlation in preferences.

When preferences are highly correlated, the committee can be smaller. Indeed, in the extreme case of perfectly aligned preferences, a one-person committee will suffice and the optimal committee size is independent of participation costs and independent of group size. The latter also holds when preferences are independent. The intuition is that all non-committee members are equally likely to agree or disagree with the committee's choice. As a result, total expected benefits are independent of the number of people not on the committee.

Proposition 6. *The optimal committee size is one when preferences are perfectly aligned. When preferences are independent and both alternatives are equally likely to be favored, the optimal committee size is odd, decreasing in participation costs, and independent of group size.*

[14] See, for instance, Persico (1999), Osborne, Rosenthal, and Turner (2000), and Börgers (2001) for alternative models.

More generally, when preferences are correlated but not perfectly so, the optimal committee size will depend on the size of the group, n. In this case, the trade-off is that a larger committee better reflects the preferences of the group but it is also more costly. What remains true, however, is that the optimal committee size is always odd. The intuition is that when going from an odd to an even sized committee, the extra vote only matters when it creates a tie. However, a tie with an even number of voters implies that the alternative will be selected randomly, and the expected total benefit of a randomly selected outcome is always zero. Hence, there is no expected benefit from the additional vote.

Proposition 7. *In general, the optimal committee size is odd, decreasing in participation costs, and increasing in group size.*

To illustrate the welfare effects of committees, consider the case of perfectly aligned preferences. The total group payoff that results from a one-person committee is $n - c$. Contrast this with the group payoff that results from the symmetric equilibrium, where all individuals randomize between staying out and participating. Since the expected payoff of participating is $1 - c$, the group payoff must equal $n(1 - c)$ in equilibrium.[15] In other words, by delegating all voting power to a single person the group's payoff rises by $(n - 1)c$.

6. Conclusions

In this paper we consider voting behavior when individuals rationally anticipate that others have similar tastes. By introducing correlation in preferences we can analyze situations where individuals with opposite preferences have conflicting views about which alternative is more likely to be favored by the majority. Like Börgers (2001) we find that voting is excessive when preferences are independent and both alternatives are favored equally. We prove expected welfare is zero in this case. However, correlation in preferences reduces participation incentives because it lowers the chance that a vote is pivotal and because votes have a positive externality on those that don't participate. Expected turnout is lower as a result, and may be less than socially optimal. The electorate does benefit from the correlation in preferences, however, and expected welfare is strictly positive.

[15] Incidentally, this is the same group payoff as when all individuals decide to participate.

The (perceived) correlation in preferences becomes less when information is publicly released, e.g. when a poll is conducted. We show that polls reduce the participation incentives of those who are reinforced in their belief that they belong to the majority. And they stimulate participation of the minority group who realize they overestimated the support for their preferred alternative. The net effect of polls is to raise expected turnout. However, the increase in participation due to polls is welfare decreasing because the release of public information stimulates the "wrong" group to participate.

We also study the effects of "large polls" that eliminate all correlation in preferences. In the resulting model, it is common knowledge that each individual prefers one alternative with probability $p \geq 1/2$ and the other alternative with probability $1-p$. This type of preference uncertainty has previously been studied by Ledyard (1984) and Palfrey and Rosenthal (1985), but we present the first characterization of equilibrium and optimal behavior. Our results extend those of Börgers (2001) who considers only the case $p = 1/2$. We prove that there is a unique equilibrium in totally mixed strategies. In this equilibrium, expected turnout is independent of p, the expected size of the majority group. Moreover, expected turnout is the same across groups so the election ends in a tie in expectation. Our results seem roughly consistent with the empirical observation that one alternative may dominate the polls while the actual election is close. The welfare effects of large polls are unambiguously negative: they raise expected costs and lower the chance that the majority-preferred alternative wins.

In some contexts, a natural way to guarantee a certain level of participation is to delegate voting authority to a committee. We prove that committees can be welfare improving even when all committee members vote in their selfish interest. We show the optimal committee size is necessarily odd, decreasing in participation costs and increasing in group size.

One interesting extension of our model is to declare one of the alternatives the *status quo*, and count non-participation as a vote in favor of the status quo (see Börgers, 2001). The obvious benefit of a status quo is that those in favor save on the costs of participating. A drawback, however, is that those that oppose the status quo may free ride. With correlated preferences, a status quo rule may be especially beneficial. Suppose the chosen status quo coincides with the majority-preferred alternative. The minority group will overestimate their true size and free ride more as a result, which saves on participation costs. Hence, in this case, the majority-preferred alternative will be implemented at low costs. When the chosen status quo coincides with the minority-preferred alternative, the majority group will *underestimate* their true size.[16] As a consequence they will free ride less, making it more likely that the majority-preferred alternative will be implemented.

Another extension is to relax the assumption that values are private. For instance, a voter may favor a certain presidential candidate out of self interest but also be concerned about the quality of the candidates, which adds a common value element. In models of pure common value voting, where all individuals have identical preferences but different information, there are also positive externalities to voting. This creates incentives to free ride, and the voting process may do a poor job at aggregating the information present in the electorate (e.g. Fedderson and Pesendorfer, 1998). It is interesting to add some common-value elements to the private-values model of this paper, and reconsider the issue of information aggregation in this more general setting (see also Fedderson and Pesendorfer, 1997).

Finally, in a pure common-value setting, Persico (1999) studies the incentives for information acquisition in different voting mechanisms. In Persico's model, voters incur a cost before they receive their common-value information. Information acquisition may also play a role in a private-values model, e.g. voters may have to research presidential candidates before they know who best fits their needs. There are some obvious similarities between the costly-participation model discussed in this paper and one of information acquisition. In future work we plan to compare the incentives for information acquisition under different voting rules when both private and common values are present.

[16] Recall from (2.1) that the estimated fraction of people with the same preference is $p^2 + (1-p)^2$, which is less than the true fraction p for all $p \geq 1/2$.

A. Appendix

First, let us introduce some notation:

$$P(k;n,\gamma) \;=\; \binom{n}{k}\gamma^k(1-\gamma)^{n-k}$$

is the binomial probability of observing k successes out of n trials when the probability of each success is γ, and $F(k;n,\gamma) \equiv \sum_{x=0}^{k} P(x;n,\gamma)$ denotes the corresponding distribution.

Lemma A1. *$F(k;n,\gamma_1) < F(k;n,\gamma_2)$ for all $\gamma_1 > \gamma_2$ and $k < n$.*

Proof. Differentiating $F(k;n,\gamma)$ with respect to γ shows

$$\partial_\gamma F(k;n,\gamma) \;=\; -n\binom{n-1}{k}\gamma^k(1-\gamma)^{n-1-k}$$

which is strictly negative for all $k < n$. *Q.E.D.*

Lemma A2. *$F(k;n_1,\gamma) < F(k;n_2,\gamma)$ for all $n_1 > n_2$ and $k < n_1$.*

Proof. It suffices to show $F(k;n+1,\gamma) < F(k;n,\gamma)$ for all n and $k < n+1$. Note that $F(0;n+1,\gamma) < F(0;n,\gamma)$ as $(1-\gamma)^{n+1} < (1-\gamma)^n$. In addition, $F(n;n+1,\gamma) = (1-\gamma^{n+1}) < F(n;n+1,\gamma) = 1$. Hence, the distribution $F(k;n+1,\gamma)$ starts out below $F(k;n,\gamma)$ at $k=0$ and converges from below at $k = n+1$. We finish the proof by a standard induction argument. Suppose $F(l;n+1,\gamma) < F(l;n,\gamma)$ for all $l < k$, then

$$\begin{aligned}
F(k;n,\gamma) &= F(k-1;n,\gamma) + P(k;n,\gamma) \\
&> F(k-1;n+1,\gamma) + \Big(\frac{n+1-k}{(n+1)(1-\gamma)}\Big)P(k;n+1,\gamma) \\
&= F(k;n+1,\gamma) + \Big(\frac{(n+1)\gamma-k}{(n+1)(1-\gamma)}\Big)P(k;n+1,\gamma) \qquad \text{(A.1)}
\end{aligned}$$

Suppose, in contradiction, that $F(k;n+1,\gamma) \geq F(k;n,\gamma)$. From (A.1) this implies that $\gamma < k/(n+1)$ and a computation analogous to (A.1) shows that

$$F(k+1;n,\gamma) \;\leq\; F(k+1;n+1,\gamma) + \Big(\frac{(n+1)\gamma-k-1}{(n+1)(1-\gamma)}\Big)P(k+1;n+1,\gamma) < F(k+1;n+1,\gamma)$$

and, more generally, for $l > k+1$

$$F(l;n,\gamma) \;<\; F(l;n+1,\gamma) + \Big(\frac{(n+1)\gamma-l}{(n+1)(1-\gamma)}\Big)P(l;n+1,\gamma) < F(l;n+1,\gamma)$$

However, this contradicts the fact that $F(k;n+1,\gamma)$ converges to $F(k;n,\gamma)$ from below at $k = n+1$. Hence, $F(k;n+1,\gamma) < F(k;n,\gamma)$ and the induction argument is complete. *Q.E.D.*

Proposition 1. *In the unique totally-mixed, symmetric Bayesian-Nash equilibrium, the probability of participating, γ^*, is determined by*

$$\sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^*)^k (1-\gamma^*)^{n-1-k} \binom{2\lfloor \frac{k+1}{2} \rfloor}{\lfloor \frac{k+1}{2} \rfloor} p^{\lfloor \frac{k+1}{2} \rfloor} (1-p)^{\lfloor \frac{k+1}{2} \rfloor} = c, \tag{A.2}$$

and those who participate vote sincerely for their preferred alternative.

Proof. First, note that the probabilities of being pivotal in (2.2) and (2.3) can be combined as

$$P_{piv}(k) = \binom{2\lfloor \frac{k+1}{2} \rfloor}{\lfloor \frac{k+1}{2} \rfloor} p^{\lfloor \frac{k+1}{2} \rfloor} (1-p)^{\lfloor \frac{k+1}{2} \rfloor}, \tag{A.3}$$

and the expected probability of being pivotal can be written as the left side of (A.2). Uniqueness of the symmetric equilibrium follows since an increase in γ raises the expected number of other participants, in the sense of first-order stochastic dominance (Lemma A1). Since $P_{piv}(k)$ is decreasing in k, the expected benefit of being pivotal on the left side of (A.2) therefore falls with γ. Finally, the left side of (A.2) is equal to $\overline{c} = 1$ at $\gamma = 0$ and equal to $\underline{c}$ at $\gamma = 1$. Hence, for $\underline{c} \leq c \leq \overline{c}$ there is a unique γ that satisfies (A.2). *Q.E.D.*

Proposition 2. *The equilibrium probability of participating, γ^*, is decreasing in the degree of correlation in preferences, p, the cost of participating, c, and the size of the electorate, n.*

Proof. Note that the probability of being pivotal in (A.3) is decreasing in p for $p \geq 1/2$. Hence, when p increases, probability has to shifts towards lower values of k to maintain (A.2). By Lemma A1 this implies that γ^* has to fall. Similarly, when the cost of participation, c, rises, the left side of (A.2) has to increase, which by Lemma A1 means that γ^* should decrease. Finally, by Lemma A2, an increase in the size of the electorate, n, raises the number of participants in the sense of first degree stochastic dominance. Again γ^* has to fall to maintain (A.2). *Q.E.D.*

Proposition 3. *The equilibrium level of participation exceeds the socially optimal level if preferences are independent while the reverse is true if preferences are perfectly correlated. In equilibrium, expected welfare is zero when preferences are independent and positive when preferences are perfectly correlated.*

Proof. Expected welfare is simply measured by the total payoffs of the group minus total costs. If γ denotes the probability of participation, then expected welfare can be written as

$$W = \sum_{k=0}^{n} \binom{n}{k} \gamma^k (1-\gamma)^{n-k} W(k) - n\gamma c \tag{A.4}$$

where $W(k)$ is the expected group benefit when k individuals vote. Optimizing with respect to γ gives the following condition for the socially optimal level of participation, γ^o:

$$\sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^o)^k (1-\gamma^o)^{n-1-k} \Big(W(k+1) - W(k) \Big) = c. \tag{A.5}$$

We next determine $W(k)$. Suppose l people vote for R and $k-l$ vote for B. The group benefit depends on how many people prefer R and B. Consider those cases in which r individuals prefer R and $n-r$ prefer B, where $r \geq l$ and $n-r \geq k-l$. The group benefit is then $(n-2r)$ when $l < k-l$ so that B is the majority color, and the group benefit is $(2r-n)$ when $l > k-l$ so that R is the majority color.[17] We can combine the different cases by restricting $l = 0, \cdots, \lfloor \frac{k-1}{2} \rfloor$ and assign the minority group of l voters once to the R group and once to the B group. This way, $W(k)$ can be written as

$$\begin{aligned} W(k) &= \sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor} \sum_{r=l}^{n-k+l} \binom{k}{l} \binom{n-k}{r-l} p^{n-r}(1-p)^r (n-2r) \\ &- \sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor} \sum_{r=k-l}^{n-l} \binom{k}{l} \binom{n-k}{r+l-k} p^{n-r}(1-p)^r (n-2r) \end{aligned}$$

To understand the binomials that appear in these expressions, note, for instance, that in the top line we are drawing l people that prefer R from k voters and $r-l$ people that prefer R from $n-k$ non-voters. This can be done in $\binom{k}{l}\binom{n-k}{r-l}$ ways.

It is useful to redefine the summation variable $r \to r-l$ in the first line, and $r \to r-k+l$ in the second line. Then $W(k)$ becomes

$$\begin{aligned} W(k) &= \sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor} \sum_{r=0}^{n-k} \binom{k}{l} \binom{n-k}{r} p^{n-r-l}(1-p)^{r+l} (n-2r-2l) \\ &- \sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor} \sum_{r=0}^{n-k} \binom{k}{l} \binom{n-k}{r} p^{n-r-k+l}(1-p)^{k+r-l} (n-2k-2r+2l) \end{aligned}$$

The sum over r can now be done and the result is given by

$$\begin{aligned} W(k) &= \sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor} \binom{k}{l} \Big(p^l (1-p)^{k-l} + p^{k-l}(1-p)^l \Big)(k-2l) \\ &\quad + (n-k)(2p-1) \sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor} \binom{k}{l} \Big(p^{k-l}(1-p)^l - p^l (1-p)^{k-l} \Big) \end{aligned} \tag{A.6}$$

The top line represents the benefit of the k votes to those that participate, while the bottom line measures the positive externality of these votes for non-participants. To glean some further insight, it is useful to decompose the benefits of a vote to those that participate into a selfish benefit to the voter and a benefit to other voters. For this we need to distinguish whether an even or an odd number of people vote. It is readily verified that (A.6) is equal to

$$W_{even}(k) = k P_{piv}(k-1) + n(2p-1)^2 \sum_{l=0}^{\lfloor \frac{k}{2} \rfloor - 1} \binom{2l}{l} p^l (1-p)^l \tag{A.7}$$

[17] Note that the group payoff is zero when there are an equal number of B and R voters.

when k is even and

$$W_{odd}(k) \; = \; kP_{piv}(k-1) + (n-k)(2p-1)^2 P_{piv}(k-1) + n(2p-1)^2 \sum_{l=0}^{\lfloor \frac{k}{2} \rfloor - 1} \binom{2l}{l} p^l (1-p)^l \quad \text{(A.8)}$$

when k is odd. These results are quite intuitive. Consider the case of an even number of voters. Each voter receives a selfish benefit of \$1 when her vote changes the outcome of the election. The probability that this happens when $k-1$ others vote is $P_{piv}(k-1)$, with $P_{piv}(k-1)$ given by (A.3). The situation is symmetric with respect to all voters, so the sum of expected selfish benefits to all voters is $kP_{piv}(k-1)$. In addition, there are benefits for the group. Suppose the true state is 0, say, so that on average there are np people who favor B and $n(1-p)$ people that favor R. Those who favor B receive \$1 when there is a majority for B, in which case those that favor R receive $-$\$1. The benefit to the group can therefore be written as

$$n(2p-1) \sum_{l=0}^{\lfloor \frac{k-1}{2} \rfloor} \binom{k}{l} \Big(p^{k-l}(1-p)^l - p^l (1-p)^{k-l} \Big) \; = \; n(2p-1)^2 \sum_{l=0}^{\lfloor \frac{k}{2} \rfloor - 1} \binom{2l}{l} p^l (1-p)^l$$

This explains the expression for $W_{even}(k)$ given in (A.7). The result for odd k can be explained in a similar manner. The only difference is that with an even number of votes, a pivotal vote cannot have a benefit for non-participants since such a vote implies that the outcome will be selected at random. In contrast, with an odd number of voters, a pivotal vote can benefit the $n-k$ non-participants, which explains the extra term in (A.8).

We next derive expected welfare for the two extreme cases of no correlation and perfect correlation. We start with the former case, i.e. when $p = 1/2$. We have $W(k) = kP_{piv}(k-1)$ for all k. Using (A.3) for $p = 1/2$ it is readily verified that $W(k+1) - W(k) = P_{piv}(k)$ when k is even and $W(k+1) - W(k) = 0$ when k is odd. Let $W_{piv}(k) \equiv W(k+1) - W(k)$, then the socially optimal level of participation, γ^o, is determined by $\sum_{k=0}^{n-1} P(k; n-1, \gamma^o) W_{piv}(k) = c$. Compare this to the equation that determines the equilibrium level of participation, γ^*: $\sum_{k=0}^{n-1} P(k; n-1, \gamma^*) P_{piv}(k) = c$. Note that $W_{piv}(k) = P_{piv}(k)$ when k is even and $W_{piv}(k) < P_{piv}(k)$ when k is odd. Moreover, $W_{piv}(k)$ falls with k for those k for which it is positive. Hence, the distribution function $F(k; n-1, \gamma^o)$ should put more mass on low values of k than $F(k; n-1, \gamma^*)$, which implies that $\gamma^o < \gamma^*$ by Lemma A1. Since $W(k) = kP_{piv}(k-1)$ expected welfare in (A.4) becomes

$$\begin{aligned} W &= \sum_{k=0}^{n} \binom{n}{k} (\gamma^*)^k (1-\gamma^*)^{n-k} \, W(k) - n\gamma^* c \\ &= n\gamma^* \Big\{ \sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^*)^k (1-\gamma^*)^{n-k-1} \, P_{piv}(k) - c \Big\} \\ &= 0, \end{aligned}$$

by the definition of γ^*.

Now consider the case of perfectly aligned preferences, i.e. $p = 1$. From (A.6) we have $W(0) = 0$ and $W(k) = n$ for all $k > 0$, so $W_{piv}(k) = n$ for $k = 0$ and $W_{piv}(k) = 0$ otherwise. From $\sum_{k=0}^{n-1} P(k; n-1, \gamma^o) W_{piv}(k) = c$ we thus derive $\gamma^o = 1 - (c/n)^{\frac{1}{n-1}}$. Contrast this with the equation that defines γ^*: $\sum_{k=0}^{n-1} P(k; n-1, \gamma^*) P_{piv}(k) = c$, where $P_{piv}(k)$ follows from (A.3) with $p = 1$. It is readily verified that $P_{piv}(k) = 1$ for $k = 0$ and $P_{piv}(k) = 0$ otherwise. Hence, $\gamma^* = 1 - c^{\frac{1}{n-1}} < \gamma^o$. Using $W(0) = 0$ and $W(k) = n$ for all $k > 0$, and $\gamma^* = 1 - c^{\frac{1}{n-1}}$ it is straightforward to work our expected welfare in (A.4) as $W = n(1-c)$, which is positive for all $c < 1$. *Q.E.D.*

Proposition 4. *Suppose it is commonly known that each individual favors B with probability $p \geq 1/2$ and R with probability $1-p$. In the unique totally-mixed Bayesian-Nash equilibrium, the probabilities of participating for those who prefer B and R are given by $\gamma_B^* = \gamma^*/(2p)$ and $\gamma_R^* = \gamma^*/(2(1-p))$ respectively, where γ^* satisfies*

$$\sum_{k=0}^{n-1} \binom{n-1}{k} (\gamma^*)^k (1-\gamma^*)^{n-1-k} \binom{2\lfloor \frac{k+1}{2} \rfloor}{\lfloor \frac{k+1}{2} \rfloor} (\tfrac{1}{2})^{\lfloor \frac{k+1}{2} \rfloor} (\tfrac{1}{2})^{\lfloor \frac{k+1}{2} \rfloor} = c, \tag{A.9}$$

and those who participate vote sincerely for their preferred alternative.

Proof. First, take the point of view of someone who prefers B. Suppose k others also prefer B, while $n-k-1$ others favor R and a total of l others vote. If l is odd then a vote for B can be pivotal only when $(l+1)/2$ others vote for R and $(l-1)/2$ others vote for B. Similarly, when l is even, a vote for B is pivotal only when $l/2$ others vote for R and $l/2$ others vote for B. These cases can be combined by noting that when l others vote, a vote for B is pivotal only when $\lfloor \frac{l+1}{2} \rfloor$ vote for R and $\lfloor \frac{l}{2} \rfloor$ vote for B. Since there is a total of k others who favor B we necessarily have $k \geq \lfloor \frac{l}{2} \rfloor$. Likewise, we have $n-k-1 \geq \lfloor \frac{l+1}{2} \rfloor$ or $k \leq n-1-\lfloor \frac{l+1}{2} \rfloor$.

Let γ_B^* and γ_R^* denote the equilibrium probabilities of participating for B and R respectively. Equating the expected probability of being pivotal for a B-voter to the cost of voting yields:

$$\sum_{l=0}^{n-1} \sum_{k=\lfloor \frac{l}{2} \rfloor}^{n-1-\lfloor \frac{l+1}{2} \rfloor} \binom{n-1}{k} p^k (1-p)^{n-k-1} \times$$
$$\binom{k}{\lfloor \frac{l}{2} \rfloor} \binom{n-k-1}{\lfloor \frac{l+1}{2} \rfloor} (\gamma_B^*)^{\lfloor \frac{l}{2} \rfloor} (1-\gamma_B^*)^{k-\lfloor \frac{l}{2} \rfloor} (\gamma_R^*)^{\lfloor \frac{l+1}{2} \rfloor} (1-\gamma_R^*)^{n-k-1-\lfloor \frac{l+1}{2} \rfloor} = c.$$

It is useful to redefine the summation variable $k \to k - \lfloor \frac{l}{2} \rfloor$. After this redefinition, the binomial expressions can be worked out as

$$\frac{(n-1)!}{\lfloor \frac{l}{2} \rfloor! \, \lfloor \frac{l+1}{2} \rfloor! \, k! \, (n-k-l-1)!} = \binom{n-1}{l} \binom{l}{\lfloor \frac{l}{2} \rfloor} \binom{n-l-1}{k},$$

where we used $\lfloor \frac{l}{2} \rfloor + \lfloor \frac{l+1}{2} \rfloor = l$. Combining terms we can rewrite the pivotal equation for a B-voter as

$$\sum_{l=0}^{n-1} \binom{n-1}{l} \binom{l}{\lfloor \frac{l}{2} \rfloor} (p\gamma_B^*)^{\lfloor \frac{l}{2} \rfloor} ((1-p)\gamma_R^*)^{\lfloor \frac{l+1}{2} \rfloor} \times \\ \sum_{k=0}^{n-1-l} \binom{n-l-1}{k} (p(1-\gamma_B^*))^k ((1-p)(1-\gamma_R^*))^{n-l-1-k} = c.$$

The sum over k can now readily be done, yielding

$$(p(1-\gamma_B^*) + (1-p)(1-\gamma_R^*))^{n-1-l} = (1 - p\gamma_b^* - (1-p)\gamma_R^*)^{n-1-l},$$

so that the pivotal equation for a B-voter becomes

$$\sum_{l=0}^{n-1} \binom{n-1}{l} \binom{l}{\lfloor \frac{l}{2} \rfloor} (p\gamma_B^*)^{\lfloor \frac{l}{2} \rfloor} ((1-p)\gamma_R^*)^{\lfloor \frac{l+1}{2} \rfloor} (1 - p\gamma_B^* - (1-p)\gamma_R^*)^{n-1-l} = c. \tag{A.10}$$

Likewise, for an R-voter we have

$$\sum_{l=0}^{n-1} \binom{n-1}{l} \binom{l}{\lfloor \frac{l}{2} \rfloor} (p\gamma_B^*)^{\lfloor \frac{l+1}{2} \rfloor} ((1-p)\gamma_R^*)^{\lfloor \frac{l}{2} \rfloor} (1 - p\gamma_B^* - (1-p)\gamma_R^*)^{n-1-l} = c. \tag{A.11}$$

Notice that if we define $\gamma_B^* = \gamma^*/(2p)$ and $\gamma_R^* = \gamma^*/(2(1-p))$, the pivotal equations for B-voters and R-voters reduce to a single equation:

$$\sum_{l=0}^{n-1} \binom{n-1}{l} (\gamma^*)^l (1-\gamma^*)^{n-1-l} \binom{l}{\lfloor \frac{l}{2} \rfloor} \left(\frac{1}{2}\right)^l = c.$$

The expression in (A.9) now follows by noting that, for all l,

$$\binom{l}{\lfloor \frac{l}{2} \rfloor} \left(\frac{1}{2}\right)^l = \binom{2\lfloor \frac{l+1}{2} \rfloor}{\lfloor \frac{l+1}{2} \rfloor} (\tfrac{1}{2})^{\lfloor \frac{l+1}{2} \rfloor} (\tfrac{1}{2})^{\lfloor \frac{l+1}{2} \rfloor}$$

We next prove uniqueness of the totally-mixed equilibrium. Define $\gamma_1 = p\gamma_B^*$ and $\gamma_2 = (1-p)\gamma_R^*$ and suppose, in contradiction, there exists an equilibrium for which $\gamma_1 \neq \gamma_2$. Notice that the pivotal equations (A.10) and (A.11) for B and R voters can be written as

$$\sum_{l=0}^{n-1} \binom{n-1}{l} \binom{l}{\lfloor \frac{l}{2} \rfloor} \gamma_1^{\lfloor \frac{l}{2} \rfloor} \gamma_2^{\lfloor \frac{l+1}{2} \rfloor} (1-\gamma_1-\gamma_2)^{n-1-l} = c, \tag{A.12}$$

and

$$\sum_{l=0}^{n-1} \binom{n-1}{l} \binom{l}{\lfloor \frac{l}{2} \rfloor} \gamma_1^{\lfloor \frac{l+1}{2} \rfloor} \gamma_2^{\lfloor \frac{l}{2} \rfloor} (1-\gamma_1-\gamma_2)^{n-1-l} = c. \tag{A.13}$$

Without loss of generality assume $\gamma_1 < \gamma_2$. Taking the difference of (A.12) and (A.13) yields

$$\sum_{l=0}^{n-1} \binom{n-1}{l} \binom{l}{\lfloor \frac{l}{2} \rfloor} \gamma_1^{\lfloor \frac{l}{2} \rfloor} \gamma_2^{\lfloor \frac{l}{2} \rfloor} (\gamma_2-\gamma_1)^{\lfloor \frac{l+1}{2} \rfloor - \lfloor \frac{l}{2} \rfloor} (1-\gamma_1-\gamma_2)^{n-1-l} = 0,$$

a contradiction since $(\gamma_2-\gamma_1)^{\lfloor\frac{l+1}{2}\rfloor-\lfloor\frac{l}{2}\rfloor}=0$ for even l and $(\gamma_2-\gamma_1)^{\lfloor\frac{l+1}{2}\rfloor-\lfloor\frac{l}{2}\rfloor}>0$ for odd l. Hence, $\gamma_1=\gamma_2$. *Q.E.D.*

Proposition 5. *The public release of information, which eliminates all correlation in preferences, raises expected turnout but lowers expected welfare to zero.*

Proof. Before the poll, turnout is given by $n\gamma^*$ where γ^* is defined in (A.2). After the poll, turnout is $n\gamma^*$ with γ^* defined in (A.9), which is (A.2) with p replaced by 1/2. Recall from Proposition 2 that γ^* is decreasing in p, so turnout is higher after the poll.

To determine expected welfare, we closely follow the setup in the proof of Proposition 3. Suppose l people vote for R and $k-l$ vote for B, and consider those cases in which r individuals prefer R and $n-r$ prefer B, where $r\geq l$ and $n-r\geq k-l$. The group benefit is then $(n-2r)$ when $l<k-l$ so that B is the majority color, and the group benefit is $(2r-n)$ when $l>k-l$ so that R is the majority color. We can combine the different cases by restricting $l=0,\cdots,\lfloor\frac{k-1}{2}\rfloor$ and assign the minority group of l voters once to the R group and once to the B group. This way, expected welfare, W, can be written as

$$\begin{aligned}
W &= \sum_{k=0}^{n}\sum_{l=0}^{\lfloor\frac{k-1}{2}\rfloor}\sum_{r=l}^{n-k+l}\frac{n!}{l!(k-l)!(r-l)!(n-k-r+l)!}\times \\
&\qquad (1-\gamma_R)^{r-l}(1-\gamma_B)^{n-k-r+l}\gamma_R^l\gamma_B^{k-l}p^{n-r}(1-p)^r(n-2r) \\
&- \sum_{k=0}^{n}\sum_{l=0}^{\lfloor\frac{k-1}{2}\rfloor}\sum_{r=k-l}^{n-l}\frac{n!}{l!(k-l)!(r-k+l)!(n-r-l)!}\times \\
&\qquad (1-\gamma_R)^{r-k+l}(1-\gamma_B)^{n-r-l}\gamma_R^{k-l}\gamma_B^{l}p^{n-r}(1-p)^r(n-2r) \\
&- n(p\gamma_B+(1-p)\gamma_R)c.
\end{aligned}$$

Again it is useful to redefine the summation variable $r\to r-l$ in the first line, and $r\to r-k+l$ in the second line. Expected welfare can then be written as

$$\begin{aligned}
W &= \sum_{k=0}^{n}\sum_{l=0}^{\lfloor\frac{k-1}{2}\rfloor}\binom{n}{k}\binom{k}{l}((1-p)\gamma_R)^l(p\gamma_B)^{k-l}\times \\
&\qquad \sum_{r=0}^{n-k}\binom{n-k}{r}((1-p)(1-\gamma_R))^r(p(1-\gamma_B))^{n-k-r}(n-2r-2l) \\
&- \sum_{k=0}^{n}\sum_{l=0}^{\lfloor\frac{k-1}{2}\rfloor}\binom{n}{k}\binom{k}{l}(p\gamma_B)^l((1-p)\gamma_R)^{k-l}\times \\
&\qquad \sum_{r=0}^{n-k}\binom{n-k}{r}((1-p)(1-\gamma_R))^r(p(1-\gamma_B))^{n-k-r}(n-2k-2r+2l) \\
&- n(p\gamma_B+(1-p)\gamma_R)c.
\end{aligned}$$

The sum over r can now be done and the result is

$$W \;=\; \sum_{k=0}^{n}\binom{n}{k}(\gamma^*)^k(1-\gamma^*)^{n-k}\,\left(\frac{1}{2}\right)^{k-1}\sum_{l=0}^{\lfloor\frac{k-1}{2}\rfloor}\binom{k}{l}(k-2l)-n\gamma^*c,$$

were we used the fact that individuals participate according to the equilibrium probabilities $p\gamma_B^* = (1-p)\gamma_R^* = \gamma^*/2$, with γ^* given by Proposition 4. It is readily verified that

$$\left(\frac{1}{2}\right)^{k-1}\sum_{l=0}^{\lfloor\frac{k-1}{2}\rfloor}\binom{k}{l}(k-2l) \;=\; k\binom{2\lfloor\frac{k}{2}\rfloor}{\lfloor\frac{k}{2}\rfloor}(\tfrac{1}{2})^{\lfloor\frac{k}{2}\rfloor}(\tfrac{1}{2})^{\lfloor\frac{k}{2}\rfloor}$$

The right side is equal to $kP_{piv}(k-1)$, see (A.3) for the case $p = 1/2$. Hence, expected welfare when each individual favors B with probability p and R with probability $(1-p)$ is

$$W \;=\; n\gamma^*\Big\{\sum_{k=0}^{n-1}\binom{n-1}{k}(\gamma^*)^k(1-\gamma^*)^{n-k-1}\,P_{piv}(k)-c\Big\} = 0, \tag{A.14}$$

independent of p. *Q.E.D.*

Proposition 6. *The optimal committee size is one when preferences are perfectly aligned. When preferences are independent and both alternatives are equally likely to be favored, the optimal committee size is odd, decreasing in participation costs, and independent of group size.*

Proof. Adding another member to the committee is optimal if and only if the expected benefit of doing so, as measured by $W_{piv}(k) \equiv W(k+1) - W(k)$, exceeds the cost, c. Consider first the case of perfectly aligned preferences, i.e. $p = 1$. Recall from the proof of Proposition 3 that $W_{piv}(k)$ equals n for $k = 0$ and 0 otherwise. Hence, the optimal committee size is 1 (as long as $c < n$).

For the $p = 1/2$ case, recall that $W_{piv}(k) = (\frac{1}{2})^k\binom{k}{k/2}$ when k is even, and $W_{piv}(k) = 0$ when k is odd. Hence, the optimal committee size is always odd. It is given by $2k^* + 1$, where k^* is the largest integer such that

$$\left(\frac{1}{2}\right)^{2k^*}\binom{2k^*}{k^*} \geq 2c, \tag{A.15}$$

and $2k^* + 1 \leq n$. For low values of the cost parameter, k^* is equal to $\lfloor\frac{n-1}{2}\rfloor$ and for high values of the cost parameter k^* is equal to 0.[18] Since the left side of (A.15) decreases with k^*, there is a unique k^* for intermediate values of the cost parameter. Finally, when costs rise, the left side of (A.15) should increase, which implies that k^* has to fall. *Q.E.D.*

[18]The highest cost for which $k^* = \lfloor\frac{n-1}{2}\rfloor$ is $\left(\frac{1}{2}\right)^{2\lfloor\frac{n-1}{2}\rfloor}\binom{2\lfloor\frac{n-1}{2}\rfloor}{\lfloor\frac{n-1}{2}\rfloor}$ and the lowest cost for which $k^* = 0$ is 1/2.

Proposition 7. *In general, the optimal committee size is odd, decreasing in participation costs, and increasing in group size.*

Proof. From (A.6) it follows that

$$W(2k+1) - W(2k-1) = \binom{2k}{k} p^k (1-p)^k \left\{ n(2p-1)^2 - (4k+2)(p^2 + (1-p)^2 - \tfrac{1}{2}) + 1 \right\}, \tag{A.16}$$

and $W(2k) = W(2k-1)$. Hence, the optimal committee size is odd. The right side of (A.16) represents the marginal benefit of adding two members to the committee. Denote this marginal benefit by $MB(k;p,n)$. The optimal committee size is given by $2k^*+1$, where k^* is the largest integer such that $MB(k^*;p,n) \geq 2c$. Note that $MB(k;p,n)$ is decreasing in k and increasing in n. So when n falls, k^* has to fall as well in order to guarantee the inequality $MB(k^*;p,n) \geq 2c$. Similarly, when c rises, k^* has to fall to maintain the inequality $MB(k^*;p,n) \geq 2c$. *Q.E.D.*

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