Summary

This thesis is on modeling credit risk and credit derivatives. Credit risk is the risk that a debtor, or obligor, does not honor its payment obligations. This can be the risk that a customer does not pay its bills, or that a loan is not, or only partially, paid. Using credit derivatives one can transfer credit risk to a third party. Two of the best known credit derivatives are the ’credit default swap’ (CDS), which provides insurance against the loss due to the bankruptcy of a company, and the ’collateralized debt obligation’ (CDO), which covers a predescribed part of the default losses in a portfolio.

The first chapter of this thesis gives a brief introduction to credit risk and credit derivatives. First, the notion of credit risk is formalized, and some well known modeling techniques are discussed. Second, this chapter discusses a number of credit derivatives, and models to value these derivatives.

Chapters 2 and 3 focus on the mathematical modeling of the evolution of the cumulative loss process in a large portfolio. In Chapter 2 this process is modeled as a point process, or, more specifically, as a *Cox process*. The intensity of such a process is stochastic, and we assume it is driven by Brownian motion. We further assume that the intensity evolves according to the *Cox-Ingersoll-Ross equation*, also known as the square root process. In addition, we assume that the intensity is *not* observed. Using the theory of filtering with point process observations, we derive equations for the conditional moment generating function between jumps as well as at jump times of the loss process. These are subsequently solved, and by combining the solutions, a recursive expression for the conditional moment generating function is obtained. The chapter concludes with a discussion about possible extensions of the model.

In Chapter 3 we investigate how the loss process behaves when the size of the portfolio increases. We assume that the contributions of each company to the cumulative loss process are independent, and we furthermore assume that these amounts are independent of the time at which the company defaults. Under these, and a number of other 'mild', assumptions we derive a 'large deviations principle' for the path of the cumulative loss process. This principle allows one to obtain upper and lower bounds on how certain probabilities with respect to the loss process behave when the size of the portfolio increases. For two special cases, we present the exact form of the asymptotic behavior.

Chapters 4 and 5 are of a more practical nature, and they mainly focus on the valuation of CDO tranches. In Chapter 4 the prices of CDO tranches are investigated using a large collection of market data. We investigate if correlation moves can be directly explained from tranche prizes, after these have been corrected for general credit risk. Further, we study the *base correlation model* for the valuation of CDOs. By using, for each CDO tranche, different correlation values in the one-factor Gaussian copula model, it is possible to exactly match market prices. The problem with this approach is that, to value a nonstandard CDO tranche, one has to interpolate between these correlations. We compare three different interpolation techniques, and it turns out that the routine that takes losses in the portfolio, as well as tranche losses into account, yields the best performance.

Finally, Chapter 5 discusses some models for the valuation of CDO tranches. Using the same data set as in Chapter 4, we study the performance of four alternative models to the one-factor Gaussian copula. We consider the *student t-distribution* as well as the *normal inverse Gaussian distribution* as alternative to the normal distribution. Furthermore, we consider two models with random correlation. Firstly we consider a mixture of one-factor copulas, and secondly we let the correlation depend on the state of the economy. For each date in the data set, we calibrate the models to CDO tranche prizes. It turns out that the alternative models provide a much better fit to the market data. Based on the calibration errors, however, it is not possible to point out a model that significantly outperforms the other models. We conclude the chapter with an investigation of the optimal model parameters, and we find that the parameters for the models with stochastic correlation are more stable.