Mathematical Logic and Natural Language: life at the border

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Abstract

This is a brief history of interfaces between logic and language in the 20th century, written by a non-historian. My aim is to show a complex and rich relationship, constantly evolving, which defies easy summaries or generalizations. Moreover, I want to make the reader aware of some grand patterns, that are sometimes lost while peering over type-theoretical formulas, ritualized example sentences, and the general business of exchanging broadsides with one's scientific neighbours.

1 Growing up together, and breaking up

Logic and grammar have been close historically, ever since Aristotle laid the formal foundations of both. This conjunction continued into the Middle Ages, with the work of the Scholastics, and one still finds it in the 17th century Logic of the Port-Royal. But at some stage in the 19th century, a break occurred. Gottlob Frege's *Begriffsschrift* of 1879 is an uncompromising departure from natural language, in favour of formal languages for logical purposes. To be sure, Frege – living in the Jena of Carl Zeiss – still draws the famous comparison between natural language and formal language as the *eye* versus the *microscope*. The former is much less precise, though in return, much more versatile than the latter. But soon after, with Russell, we find the highly critical 'Misleading Form Thesis': natural languages obfuscate true meanings by their systematically misleading forms of assertion. In the hands of Carnap and other people close to the Vienna Circle, this became a powerful weapon to be wielded against the philosophical establishment. Accordingly, the study of language and that of logic went separate ways in the first half of the 20th century. Linguistics produced De Saussure, and the subsequent structuralist school. Logic went into its 'mathematical turn', which resulted in the Golden Age with Hilbert, Gödel, Tarski, Turing, and so many others in the amazing twenties and thirties. This period also saw the consolidation of families of formal languages, that textbooks nowadays see as defining the field: first-order predicate logic, second-order logic, lambda calculus and type theories.

Of course, the mathematical turn also meant a *shift in agenda*. Logic had been a general study of methodology and reasoning in all its varieties (cf. the great works of Bolzano or Mill) – it now became a tool for analyzing mathematical proof and the foundations of the exact sciences.

2 Rapprochement

In the 1950s – another period of great intellectual ferment – the seeds were sown for a new rapprochement. First, there was the undeniable fact that natural language was proving rather resilient. Misleading as it was, it even maintained its position as the favoured medium of communication inside mathematics – where no one who mattered showed any inclination to adopt the more austere medicines prescribed by formal logicians. Inside philosophy, there was even a back-lash in the heartland of analytical philosophy. British 'natural language philosophers' like Ryle started mining the philosophical "gold in them hills", as Austin once phrased it: the systematic patterns in natural language that guide our thinking and practice. Eventually, this work led to such specific achievements as *speech act theory* (Searle, Grice) systematizing the various uses that people make of language.

Perhaps too much has been made of the opposition between this style of philosophizing and 'formal language philosophy'. The fact is that, also in what came to be called *philosophical logic*, natural language was on the rise. This was a period of advances in modal logic (Carnap, Hintikka, Kripke) and temporal logic (Prior) which derived their inspiration from a mix of linguistic observations about various modal expressions and verb tenses, and rigorous mathematical technique. All these things were not yet full-fledged collaborations between logic and linguistics, but they did fill Russell's gap. Indeed, some logicians in this period explicitly advocated natural language as an 'area of application' for mathematical logic – thinking in a somewhat 'colonizing mode'. Examples are Reichenbach, whose work on temporal expressions is still cited today, and Curry, who saw a new scope for his formal language systems, such as combinatory logic, in these broader fields. Finally, but this is only with hindsight, the 1950s also produced other trends that are highly relevant to logic–language interfaces as conceived today, such as the advent of *game theory*, and its penetration – up to a point – into logic and philosophy.

In The Netherlands, the 1950s are seen as a stagnant decade of the last century, where people devoted themselves to what is now seen as the boring pursuit of happiness inside their own families. The more one thinks about what actually happened scientifically and economically, the shallower this prejudice becomes.

3 Mathematical linguistics

Modern linguistics started with the work of Harris and his student Chomsky, whose *Syntactic Structures* (1957) revolutionized the field. It gave an exact analysis of grammars for natural languages, derived from inspirations in mathematical logic, which showed that, at least in principle, natural language has firm structures that can be studied by formal means. Incidentally, Chomsky's book was submitted by the logician Beth for publication in the well-known series *Studies in Logic* – and rejected by Heyting. Prophecy was not a gift of the intuitionists, inspired though they were otherwise. In the 1960s, this work generated the first significant interaction between mathematics and linguistics, which still exists today: the theory of *formal languages* and *automata*. This theory turned out to be rich in notions and results, and it received further impetus when it also turned out to apply to the various languages used in computer science. The field is still active today.

Much of this theory is about the recognizing power of grammars or automata arranged in hierarchies, and thus, it was geared toward *syntax*, a conscious focus of Chomsky's work, to be kept separate from semantics and pragmatics. The agenda was this: describe the well-formed expressions of natural syntax, which may vary in complexity across human languages, using the judgments of competent individual speakers as one's guide. Further issues of meaning and inference were assigned a role at a further 'deep-structure' level of 'logical form', whose status remained somewhat unclear, and subject to modification (it often contained 'everything that God had forbidden' for honest syntax). Mathematical linguistics in this syntactic sense does not have an obvious connection with mathematical logic proper, although there are some overlaps, e.g., in the study of decision methods. A more direct connection was the independent work on linguistic *categorial grammars*, pioneered around 1960 by Bar-Hillel and Lambek, which shows great analogies with lambda calculus and *type theories* as found in the foundations of mathematics.

Another noteworthy development in this period was again inside philosophical logic: the emergence of a mathematized *modal logic*, based on possible worlds semantics, which was turned into a powerful instrument for conceptual analysis by Hintikka, Kripke, Lewis, Stalnaker, and others.

4 The semantic breakthrough

These streams were brought together in one systematic approach to natural language around 1970 by Richard Montague. The time was ripe for this, and in fairness, similar enterprises attracted attention around the same time by Cresswell and Lewis, who have now lost the Race for Fame. 'Montague Grammar' provided a rich categorial grammar for analyzing natural language expressions in tandem with their semantic meanings expressed in a modal type theory, which captured category structure and intensionality. In particular, on this view, the 'Misleading Form Thesis' now evaporated, since natural language and formal language lived side-by-side. This was the first significant paradigm for 'logical analysis' of natural language, and its impact has been broad. In particular, it attracted a whole milieu of people who are not easily classified as either logicians or linguists, since two souls lived equally within their breasts. Incidentally, historians might wish to note that 1970 was also about the time when Kripke published 'Naming and Necessity', which was the breakthrough of the philosophical logic of the sixties into mainstream philosophy.

Montague's system had a number of noteworthy features not like Chomsky's. Indeed, it *changed the agenda*. To judge the correctness of a semantic theory, one cannot just go by intuitions of grammatical well-formedness by native speakers. Equally crucial evidence now comes in the forms of judgments concerning valid or invalid inference, different readings of a given sentence, etcetera. These are more ethereal matters, pertaining not just to the linguistic code, but also to the uses that people make of it. Thus, the word 'natural language' gets a wider meaning, including certain practices of reasoning and communication. Montague encapsulated his views in a much-quoted Thesis saying that

> *there are no significant differences in principle between*
> *natural languages and mathematical languages.*

This is Russell reversed.

Nevertheless, mathematical impacts of Montague's system were not immediate. The system could be viewed as a huge design for an abstract Universal Algebra – but there were few pointed questions. Montague himself emphasized the role of the principle of *Compositionality* in relating syntactic analysis to semantic meaning, and some theory on homomorphisms and partial algebras emerged. Perhaps the more active development along these lines came from a 'transfer' phenomenon. Just as with mathematical linguistics, the realm of Montague Grammar really extended beyond natural and mathematical languages into *computer science*, where the Thesis applies equally well. This was shown by van Emde Boas & Janssen, who drew surprising analogies between compositionality in both fields, plus various other phenomena, such as the intensional features of assignment and update in general. It is not entirely unfair to see the theory of Abstract Data Types as the general mathematics of Montague Grammar. One reason why there was little development at this level was the extreme generality of the framework, and the lack of fine-structure. To be sure, people have attempted to import some of the 'Chomsky fine-structure' into the Montagovian Machine, e.g. using 'semantic automata' for classifying quantifier meanings of different complexity. But no rich mathematics has emerged.

5 Blossoming: extensions, specializations, and alternatives

By the early 1980s, 'logical grammar' had become a recognized force, and its major implications and limitations taken in by a broader community. To some, this was the end of the road. Montague's system seemed a huge system of formal definitions, with little life in it – similar to e.g. Sneed's well-known formal apparatus from the same period (early 1970s) for analyzing scientific theories. Moreover, it was so powerful that success is guaranteed. Given some combinatorial tricks (the grammar has Turing machine power, the semantics is a full-fledged intensional type theory), any phenomenon can be described. So, you can formalize things – but then what?

5.1 Generalized quantifier theory

Then, a whole series of further developments took place, introducing fine-structure. A number of linguist–logician teams published breakthrough papers on specific linguistic categories of expression, especially *quantifiers* (Barwise & Cooper, Keenan & Stavi, Higginbotham & May). The aim of this work was to chart the variety of actual quantifier expressions found in natural languages. But there was also a more ambitious goal: to find *universal semantic laws* about the design of human languages that would explain why we find this vocabulary – preferably, in the form of some 'expressive completeness' theorem. This strand of work eventually became *generalized quantifier theory*, which turned out related in its methods to abstract model theory and the foundations of computation – where generalized quantifiers are equally important. By and large, no definitive system of semantic universals has emerged covering all of natural language, but the ideal is still alive.

5.2 Categorial grammar and linear logic

Another fine-structure theme was the renaissance of *categorial grammar*. This led to the study of weaker grammar systems inside Montague's system. But this time, there is a concomitant hierarchy of semantic expressiveness, obtainable via the 'Curry-Howard isomorphism', fine-tuning the combination of linguistic meanings using 'glue' available in weaker or stronger fragments of the full lambda calculus. The upshot was the insight that natural languages combine resources in an explicit manner, making the 'single occurrence' the unit of study – as had already been proposed by Lambek in 1958. This is like the basic idea in Girard's *linear logic*, a fine-structure development of classical proof theory, inspired by interests in the nature of computation and interaction. Modulo some conflicts of historical credit management, these traditions now form one strand of resource logics, that is still quite vigorous today. Categorial grammars have acquired many additional features these days (Moortgat, Morrill, Carpenter, Dalrymple), but they do form a kind of fine-structured proof theory conscious of both syntactic and semantic resources.

What categorial grammar does borrow from Montague grammar is the 'holistic' approach: all types are manipulated in the same setting. This is not always the best format for linguistic description. One can be legitimately interested in just one kind of phenomenon, say 'negation' or 'tense', and leave worries about the total combinatorics for later. Nevertheless, some puzzle fitting must occur in any logical system that is used in practice for handling significant parts of natural language, where everything interacts with everything. In this respect, *computational linguistics* has been another major influence, which has made itself felt over the 1980s and 1990s. But categories and types are not the only general technique for holistic system-building and computation. E.g., Fenstad et al. 1986 propose an alternative, non-derivational model for natural language analysis which is based on *constraint satisfaction* and equational logic. The major tasks of linguistics can be conceptualized in quite different ways!

5.3 Discourse representation theory

Other major developments in the 1980s involved more radical departures from Montague's system. To begin with, *discourse representation theory* (Heim, Kamp) replaced the simple logical match between syntactic form and semantic denotation by a *triangle* of three basic structures, involving one intermediate level:

> 'syntactic form', 'discourse representation',
> and only then the eventual 'logical semantics'.

This makes a lot of sense in the study of anaphora and temporal expressions, where coherent meanings are built up gradually as discourse unfolds. This is perhaps the most flexible semantic format today (cf. also ter Meulen 1995), providing Montague Grammar with a 'third dimension' – and it has attained a certain popularity in computational semantics. There is little systematic theory of all this. Kamp & Reyle gives a fair impression of the logical issues involved.

5.4 Dynamic semantics and process logic

A concomitant, but separate new idea in this setting is that discourse representations are built up over time. Thus, we can conceive of semantic interpretation as a dynamic process, much like computational processes elsewhere. Perhaps not surprisingly, a similar much broader trend was observable by the late 1980s in philosophy and AI, witness the emergence of *belief revision theory* (Gärdenfors). At the interface of logic and language, *dynamic semantics* has been the most pregnant formulation of this interest (Groenendijk & Stokhof, Veltman, Visser) – and again it represents an agenda change. This time, the main phenomena to be explained are no longer well-formed sentences or meanings, but rather the key processes of natural language use: in particular communicative acts such as statements or questions, and their effects on the information states of speakers and hearers:

> *the essence of linguistic meaning is information change.*

The mathematical backdrop to dynamic semantics is *modal* and *dynamic logic*. By now, there is some sustained formal theory along these lines, exploiting analogies between linguistic processes and general process theory from computer science.

5.5 Situation semantics

Finally, the most radical contender in the 1980s was no doubt *situation semantics*, developed by Barwise, Perry, Etchemendy, and others. This contested all of Montague's major design choices, such as the use of type structure and modal logic, replacing them with 'situations' of various kinds, which can have a circular structure. Moreover, it shifted the agenda from 'jigsaw fitting' for meanings of particular expressions to broader questions concerning languages. It asked for a principled account of meaning in a physical environment – or its efficiency, as a balance between 'code' and implicit regularity. Many of these questions were inspired by Dretske's philosophy of knowledge and information, which itself borrows from Shannon's *information theory*, another product of those roaring 50s. These themes continued in later work by Perry, Barwise, and Seligman.

Situation semantics has not achieved the elimination of its competitors, the way it set out to do. But it did enrich the agenda of concerns, linking up the business of 'logic and language' with broader issues of information structure, which make equal sense in computer science, AI and cognitive science. In its modern formulations, there is often a strong emphasis on the mathematics of *non-well-founded sets* and *coalgebra* – because situations are inherently 'circular', as opposed to the type structures beloved by most semanticists.

6 Expansion: the impact of information technology

Around 1990, there was so much accumulated substance to all these developments that a number of researchers started the *European Association of Logic, Language and Information*. 'FoLLI' is still a major organisation at the interface of logic and language – but notice the 'information'. What must have become clear throughout the preceding history is that there is always a 'Dritter im Bund'. Virtually every significant topic at the interface of logic and linguistics has turned out to involve basic issues in computer science, or with a more felicitous name: *informatics* – in one way or another.

The above list is far from exhaustive in this respect. Not to clutter up our story too much, we have left out such very live joint concerns as 'context', knowledge representation, underspecification of information, ambiguity and combinatorial explosion, temporal semantics, or nonmonotonic reasoning.

6.1 Unifying trends

Given this variety, one of the trends in the 1990s has been one of unification, trying to find viewpoints that integrate between related trends inside or across these fields. E.g., there now exist powerful *modular methods* for combining many different semantic and syntactic theories (Muskens), as well as their dynamic uses. Another unifying trend has been the *modal logic* approach to *computational fine-structure* (van Benthem, de Rijke), seeing common patterns across dynamic semantics, categorial grammar, and AI techniques such as description logics. This again links up with mathematical research into decidable fragments of standard logics (Grädel).

The preceding two interests meet in a broader concern that is typical for the information-processing tasks of the 1990s. The separate logical interests mentioned above are often not in conflict. They just give parts of the total picture how humans use natural language, and try to provide lowest-complexity mechanisms accounting for these. But then there is a question of communication. How do these various logical devices manage to collaborate into one cognitive performance? These questions of *logical architecture* for combined systems have become gradually more prominent through the 1990s (Gabbay). We know that the complexity of the whole can be very hard to predict from that of the parts, involving the communication mechanisms between the modules in very delicate ways. But there is as yet no good mathematical theory giving us anything like a good grip.

An up-to-date attempt at describing and integrating the field was the 1997 *Handbook of Logic and Language*, which documents many of the strands of our story so far, at a more relaxed stage where partisan views of frameworks (a by-product of the 1980s) were retreating in favour of shared concerns and interaction.

6.2 More realism: bulk and statistics

Another trend has been toward more realism in describing what language users do. Typical examples are developments in *lexical semantics* (Winter and Zwart) and *natural logic* (Sanchez Valencia). These naturally interface with issues of concept formation and reasoning in cognitive psychology, which has studied performance rather than competence for a long time.

But there is a quite different aspect to both human performance and information technology, which may have drastic consequences beyond the usual frameworks. One feature of language use which has been neglected, by and large, in the logical tradition, is the fact that it is *repeated* very often. The usual emphasis on wellformedness or correctness judgments focuses on single linguistic episodes. But we use large amounts of sentences, all the time, in larger wholes, first in discourse, but over time, also far beyond. This results in *average behaviour*, which may have its own significant regularities. Some people think this is just the domain of computational linguistics, which provides enough statistics as a 'grease' for getting performance when logical theory does not quite match reality. But the situation is much more complex, and fundamental. Consider the situation in physics. Even when we know all mechanical laws concerning collisions between individual molecules, there are still significant emergent thermodynamic phenomena at the higher aggregation level of a gas, with its own laws. And similar phenomena occur inside logic itself! Around 1970, 'Lindström's Theorem' seemed to announce the 'end of history' concerning the meta-theory of first-order predicate logic. The latter is the strongest system satisfying the Compactness and Löwenheim-Skolem properties. But only two years later came the discovery of the first significant, and unexpected, *statistical* meta-property of first-order logic. This is the 'Zero-One Law' stating that, on finite models of increasing size, each first-order formula becomes either true or false with probability one. Emergent phenomena of this sort are now being found in various areas, such as automated theorem proving.

Similar emergent phenomena can happen in natural language. These are not just technological regularities. They also raise basic issues of architecture inside the brain. Not surprisingly then, computational linguists are now experimenting with grammars with significant statistical components – not as 'grease', but as models for memory of past performance, and other cognitive features (Pereira, Bod).

6.3 **Information handling**

More broadly than 'computational linguistics', much of the research at the logic/ language interface is now part of the information-processing phenomena found on the Internet, and related network environments. This, too, highlights new issues, that may have been present in earlier days – but that now acquire a new significance. One is the emphasis on *design of new languages*, blurring the border-line between 'natural language' as one unchangeable whole to be described, and specially designed 'formal languages'. Many semantic theories thereby get a new significance. They may not always be the best account of natural language, but they may suggest the design of useful new languages for certain tasks. Another aspect of information-processing in general is the broader sense of 'information' involved. The information on the web is partly textual, and partly graphical. Surely, language users avail themselves of both, and have little difficulty switching between these perspectives, integrating them where needed. Management of *mixed information*, including visual objects (diagrams, maps, pictures) is becoming an exciting logical topic these days. One can see this as 'pollution' of what should be separate issues, but information integration is really a basic process – on which one would like to have a deep grasp.

6.4 Social structure

A final noticeable trend in the current literature is toward *many-agent structure*. 'Natural language' can hardly be understood without realizing that it is a family of processes that involve several agents, involved in strategic interactive behaviour. A pioneer in this respect was Hintikka, with his program for a 'game-theoretical semantics' of natural language, providing a logical stiffening for Wittgenstein's more metaphorical 'language games'. Discourse involves many different procedures of communication or reasoning, and so does our social behaviour in general. Not surprisingly, this same trend can be observed in modern computer science, with its emphasis on information flow in many-agent networks. A modern mathematical connection here is with economic *game theory* (van Rooy, Baltag), which introduces strategies for language users, and mathematical equilibrium explanations for behaviour. This recent development is also a natural extension of dynamic semantics and dynamic logic into a more appropriate social setting.

7 'How it really happens': links with empirical cognitive science

The 1990s as described here brought us mainly the impact of computer science and information technology. But the story of logic and language continues. An interest in the performance aspects of *designed* information handling naturally suggests a parallel interest in *natural* information handling, and cognition in general. After all, natural language is one of our most striking features on this planet. This brings us to connections with *cognitive psychology* (always present in the deeper background of our history), and even *experimental neurocognition*.

Many of the above themes have a cognitive angle, which has never been taken seriously so far – except as a ploy for sugaring one's theory when debating with colleagues in logic or linguistics. But many of the claims made by logicians and linguists are susceptible to empirical investigation, as we know from cognitive psychology. Of course, these links are not always obvious, and experimentation with current logical theories will be difficult. But that is a challenge, not an objection. Even more dramatically, with modern brain imaging techniques, we can measure how the brain responds to linguistic or reasoning input, and thus, to many of the things that figured in our survey of themes in the 20th century. The repercussions of this will no doubt make themselves felt in the decade to come.

8 References

Here are a few entry points to the vast literature behind the above story. A more extensive, though still light history of the logic and language interface since Antiquity can be found in L. T. F. GAMUT, 1991, *Logic, Language and Meaning*, Chicago University Press. This textbook provides a thorough basic introduction to most topics mentioned in this paper up to 1990. The Golden Age of mathematical logic, and its state of the art until the 1970s is documented in J. Barwise, ed., 1977, *Handbook of Mathematical Logic*, North-Holland, Amsterdam. The same was done for philosophical logic in the many-volume *Handbook of Philosophical Logic*, D. Gabbay & F. Guenthner, eds., Reidel, Dordrecht, 1983–1987, for which a new extended edition is in preparation. This amply documents not just philosophical logic proper, but also the natural language interface, and the incipient computational trends of the 1980s. The latter got their own documentation in the 1990s, including the *Handbook of Logic in Artificial Intelligence and Logic Programming*, D. Gabbay, T. Maibaum & J. Robinson, eds., Oxford University Press, 1991–1997. The latter has many language-related chapters, because the boundary with AI is thin. Of course, specific pioneering publications played an important historical role, such as N. Chomsky, 1957, *Syntactic Structures*, Mouton, Den Haag. A crucial, and still extremely readable reference at the birth of today's logic and language connection is the collection D. Davidson & G. Harman, eds., "Semantics of Natural Language", Reidel, Dordrecht, 1971. It contained many crucial pioneering papers, from Montague's logico-linguistic efforts all the way to Kripke's 'Naming and Necessity'. Another must is R.H. Thomason's seminal edition of Montague's collected papers, R. Montague, *Formal Philosophy*, 1975, Cornell University Press. An interesting alternative with similar scope is M. J. Cresswell, 1973, *Logics and Languages*, Methuen, London. A good reference on the first significant computer science connections is Th. Janssen's Ph.D. thesis from 1983, *Foundations and Applications of Montague Grammar*, Mathematical institute, University of Amsterdam. The major trends from the 1980s are well-documented in the chapters of the *Handbook of Logic and Language* (J. van Benthem & A. ter Meulen, eds., 1997, Elsevier Science Publishers, Amsterdam). Generalized quantifier theory is surveyed by Keenan & Westerståhl, categorial grammar by Moortgat, and also Buszkowski, discourse representation theory by Kamp & van Eijck, dynamic semantics by Muskens, van Benthem & Visser, and situation theory by Moss & Seligman. Moreover, game-theoretic semantics is presented by Hintikka & Sandu. These chapters also contain many side-links toward mathematics, philosophy, and computational issues. For the newer trends of the 1990s, one must look at journals like the *Journal of Logic, Language and Information*, *Linguistics and Philosophy, Language and Computation, Computational Linguistics*, in addition to proceedings of various conferences, such as the *Amsterdam Colloquia* or the *CSLI Workshops in Logic*, *Language and Computation.* A good place for taking the pulse of the field are the annual *ESSLI Summer Schools* of the European Association for Logic, Language and Information (FoLLI; http://www.folli.org/). In addition, here are just a few further references – without any pretence at completeness. Decidable fragments of standard logics with a typical modal slant (http://www-mgi.informatik.rwth-aachen.de/ Publications/pub/graedel/Gr-eatcs99.ps), logic combination (D. Gabbay, 1996, *Labelled Deductive Systems; principles and applications*, Oxford University Press; C. Areces, 2000, *Logic Engineering*, Ph.D. thesis, ILLC Amsterdam), natural logic (V. S. Valencia, 2001, *Natural Logic*, to appear with MIT Press), statistical language models (R. Bod, 1999, *Beyond Grammar, an experience-based theory of language*, CSLI Publications, Stanford), computational logic (http://turing.wins.uva. nl/~mdr/CALG/, http://www.ifcolog.org/), logic and game theory (J. van Benthem, 2000, "Logic in Games", electronic lecture notes, ILLC Amsterdam, cf. http://turing. wins.uva.nl/~johan/ Teaching/Phil.298.html,, or the more general gateway http://www. cwi.nl/~pauly/games.html), logic and experimental cognitive science (K. Stenning & M. van Lambalgen, 2000, 'Semantics as a Foundation for Psychology', http://www.hcrc.ed.ac.uk /~keith/Stenning_and_vanLambalgen/paper.pdf).