Downloaded from UvA-DARE, the institutional repository of the University of Amsterdam (UvA) http://dare.uva.nl/document/122470

File ID122470FilenameA: The rf-dressed potentialVersionFinal published version (publisher's pdf)

SOURCE (OR PART OF THE FOLLOWING SOURCE):	
Туре	Dissertation
Title	Bose-Einstein condensates in radio-frequency-dressed potentials on an atom chip
Author	J.J.P. van Es
Faculty	Faculty of Science
Year	2009
Pages	VIII, 131
ISBN	978-90-5776-186-7

FULL BIBLIOGRAPHIC DETAILS: http://dare.uva.nl/record/292198

Copyright

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use.

UvA-DARE is a service provided by the library of the University of Amsterdam (http://dare.uva.nl)

## $\mathbf{A}_{\mathsf{The}}$ rf-dressed potential

We calculate the potential that a neutral atom with a magnetic moment experiences in a time-dependent magnetic field  $\mathbf{B}(t)$ . The hamiltonian is

$$\dot{H} = -\hat{\boldsymbol{\mu}} \cdot \mathbf{B}(t), \tag{A.1}$$

the same as in the static case. The magnetic dipole moment operator  $\hat{\boldsymbol{\mu}}$  is proportional to the angular momentum operator  $\hat{\mathbf{J}}$  with a proportionality factor  $\mu_s$ 

$$\hat{\boldsymbol{\mu}} = \mu_s \hat{\mathbf{J}}.\tag{A.2}$$

At any fixed point in space the total field  $\mathbf{B}(t)$  can be divided in a static part  $\mathbf{B}_{s}$ and an oscillatory part  $\mathbf{B}_{rf}(t)$ . The z axis of the coordinate system we align with  $\mathbf{B}_{s}$  and take this as the quantization direction. The oscillatory field we assume has linear polarization and a sinusoidal time dependence with frequency  $\omega_{rf}$ . We align x along the oscillating field. We can thus write the field as

$$\mathbf{B} = \mathbf{B}_{s} + \mathbf{B}_{rf}(t) = (B_{s} + B_{rf,z}(t))\mathbf{e}_{z} + B_{rf,x}(t)\mathbf{e}_{x}.$$
 (A.3)

The time dependence of the oscillatory field  $\mathbf{B}_{rf}(t) = \mathbf{b}_{rf} \cos(\omega_{rf} t)$  can also be written in terms of complex exponentials

$$\mathbf{B}_{\rm rf}(t) = \frac{1}{2} \mathbf{b}_{\rm rf} \left( e^{i\omega_{\rm rf}t} + e^{-i\omega_{\rm rf}t} \right). \tag{A.4}$$

We write the wave function as  $|\psi\rangle = \sum_{m} c_{m} |m\rangle$  and take as orthonormal basis the eigenstates of  $\hat{J}_{z}$ :  $|m\rangle$  with eigenvalues:  $m = -J \dots J$ . We write the Schrödinger equation in matrix form

$$i\hbar\dot{c}_n(t) = \sum_m H_{nm}c_m(t). \tag{A.5}$$

The matrix elements  $H_{nm}$  are

$$H_{nm} = \left[-\mu_s B_{\rm s} - \mu_s B_{{\rm rf},z}(t)\right] m \delta_{nm} - \mu_s B_{{\rm rf},x}(t) \langle n|\hat{J}_x|m\rangle,\tag{A.6}$$

with

$$\langle n|\hat{J}_x|m\rangle = \frac{1}{2}\sqrt{j(j+1) - m(m+1)}\delta_{n,m+1} + \frac{1}{2}\sqrt{j(j+1) - m(m-1)}\delta_{n,m-1},$$
(A.7)

which directly follows from the ladder operators  $\hat{J}_{\pm} = \hat{J}_x \pm i \hat{J}_y$  which have the eigenvalues [167]:

$$\hat{J}_{+}|j \ m\rangle = \sqrt{j(j+1) - m(m+1)}|j \ m+1\rangle,$$
  
 $\hat{J}_{-}|j \ m\rangle = \sqrt{j(j+1) - m(m-1)}|j \ m-1\rangle.$ 

In Eq. (A.6) and Eq. (A.7)  $\delta_{nm}$  is the Kronecker delta symbol which equals 1 for n = m and 0 in all other cases.

We now apply the transformation

$$\tilde{c}_k(t) \equiv c_k(t)e^{-ik\omega_{\rm rf}t},\tag{A.8}$$

which can be interpreted as the transformation to the rotating frame of the precessing magnetic moment. Taking the derivative with respect to t and inserting Eq. (A.5) we find

$$i\hbar\dot{\tilde{c}}_k(t) = \sum_m \left( H_{km} e^{i(m-k)\omega_{\rm rf}t} + \delta_{km} m\hbar\omega_{\rm rf} \right) \tilde{c}_m(t), \tag{A.9}$$

which shows that the matrix elements in the rotating frame,  $\tilde{H}_{km}$ , are equal to

$$\tilde{H}_{km} = H_{km} e^{i(m-k)\omega_{\rm rf}t} + \delta_{km} m\hbar\omega_{\rm rf}.$$
(A.10)

We now calculate the matrix elements after transformation by inserting Eq. (A.6) into Eq. (A.10). For the time dependence of the oscillating field we use Eq. (A.4).

$$\tilde{H}_{km} = \mu_s m \left[ \frac{\hbar \omega_{\rm rf}}{\mu_s} - B_{\rm s} + \frac{b_{\rm rf,z}}{2} \left( e^{i\omega_{\rm rf}t} + e^{-i\omega_{\rm rf}t} \right) \right] \delta_{km} 
- \mu_s \frac{b_{\rm rf,x}}{2} \left[ \frac{1}{2} \sqrt{j(j+1) - m(m+1)} \left( 1 + e^{i2\omega_{\rm rf}t} \right) \delta_{k,m+1} \right] 
+ \frac{1}{2} \sqrt{j(j+1) - m(m-1)} \left( 1 + e^{-i2\omega_{\rm rf}t} \right) \delta_{k,m-1} .$$
(A.11)

Observe that this expression has static terms and terms that are oscillating at  $\pm \omega_{\rm rf} t$ and at  $\pm 2\omega_{\rm rf} t$ . We now apply the rotating wave approximation [108] which amounts to neglecting all time-dependent terms in Eq. (A.11), yielding

$$\tilde{H}_{km} = \mu_s m \left(\frac{\hbar\omega_{\rm rf}}{\mu_s} - B_{\rm s}\right) \delta_{km} - \mu_s \frac{b_{\rm rf,x}}{2} \left(\frac{1}{2}\sqrt{j(j+1) - m(m+1)}\delta_{k,m+1} + \frac{1}{2}\sqrt{j(j+1) - m(m-1)}\delta_{k,m-1}\right).$$
(A.12)

From comparison of Eq. (A.6) and Eq. (A.12) we see that Eq. (A.12) corresponds to a magnetic moment in a magnetic field

$$\tilde{\mathbf{B}} = \left(B_{\rm s} - \frac{\hbar\omega_{\rm rf}}{\mu_s}\right)\mathbf{e}_z + \frac{1}{2}b_{{\rm rf},x}\mathbf{e}_x,\tag{A.13}$$

and hence

$$|\tilde{\mathbf{B}}| = \sqrt{\left(B_{\rm s} - \frac{\hbar\omega_{\rm rf}}{\mu_s}\right)^2 + \left(\frac{b_{\rm rf,x}}{2}\right)^2}.$$
 (A.14)

Since  $\tilde{\mathbf{B}}$  is time independent it is easy to see that the magnetic moment in this field has eigenvalues  $\tilde{m}\mu_s|\tilde{\mathbf{B}}|$  with  $\tilde{m} = -J \dots J$ . If the atom moves slowly through the spatially-varying magnetic field, such that the adiabaticity criterion is satisfied we can write the position-dependent potential as

$$U = \tilde{m}\hbar\sqrt{\Delta^2 + \Omega^2},\tag{A.15}$$

where  $\Delta^2$  is called the resonance term with  $\Delta$  the local detuning of the rf frequency with respect to the Larmor frequency

$$\Delta = \omega_{\rm rf} - \omega_L = \omega_{\rm rf} - \frac{|g_F \mu_B|}{\hbar} |\mathbf{B}_{\rm s}(\mathbf{r})|.$$
(A.16)

The other term,  $\Omega^2$ , is referred to as the coupling term. The position-dependent Rabi frequency,  $\Omega$ , is given by the circularly polarized rf-field component referenced to the local direction of the static magnetic field. For a linearly polarized rf field the above shows that

$$\Omega = \frac{|g_F \mu_B|}{\hbar} \frac{|\mathbf{b}_{\rm rf} \times \mathbf{B}|}{2|\mathbf{B}|}.$$
(A.17)