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ASTRONOMY AND ASTROPHYSICS

A Model for the Formaldehyde Maser near NGC 7538–IRS 1

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Summary. It is shown that the population of the 1_{10}–1_{11} transition of Formaldehyde can be inverted by the free-free radio continuum radiation of a nearby compact H II region. The H II region must be very compact with emission measures of 10^8–10^{10} cm^{-6} pc so that it is optically thick at cm wavelengths but still optically thin at millimeter wavelengths. A detailed model of the compact H II region NGC 7538–IRS 1 and its surrounding cloud is constructed to explain the observed 1_{10}–1_{11} brightness temperatures of about 10^6 K. The masing gas is in front of the H II region so that it amplifies the radio continuum radiation. To allow sufficiently rapid radiative pumping mm line photons must be able to escape from the cloud which requires a large velocity gradient in the gas. To create sufficient amplification molecules must be lined up in velocity over long pathlengths. Consistent with observations of CO, OH, and H110α lines we assume that NGC 7538–IRS 1 moves towards us through (or expands into) a molecular cloud with a velocity $V \simeq 10$ km s^{-1} and that the molecular gas in front of it is pushed sideways creating velocity gradients of $\sim V/R_{II}$ perpendicular to the line of sight and coherent pathlengths of $\sim R_{II}$ along the line of sight. With these assumptions an amplification factor of ~ 100 required to explain the observed brightness temperatures can be achieved if $n(H_2) \simeq 10^4$ cm^{-3}, $T_K \simeq 20$ K, and $x(H_2CO) \simeq 8\ 10^{-7}$. We show that those Formaldehyde abundances can be obtained in dark clouds. Our model also explains the observed maser spot sizes. The fact that the maser is saturated explains why both observed maser spots have about equal intensities.

Key words: Formaldehyde maser – compact H II region – molecular clouds

1. Introduction

Using the Westerbork Radio Synthesis Telescope at 6 cm Forster et al. (1980) observed two emission lines of the 1_{10}–1_{11} transition of Formaldehyde (H_2CO) at radial velocities of -57.6 and -59.8 km s^{-1} near the ultracompact H II region and infrared source NGC 7538–IRS 1. They found that the emission was spatially unresolved (source size $< 4''$) corresponding to a lower limit to the brightness temperature of the emission of 800 K.

More recently using the Very Large Array Rots et al. (1980) have resolved the source into two spots seperated by about 0".1. The emission spots have angular sizes smaller than 0".15 corresponding to linear dimensions of $\sim 8\ 10^{15}$ cm (at a distance of 3.5 kpc) and implying lower limits to the brightness temperatures of $\sim 5\ 10^5$ K. The small source sizes and the extremely high brightness temperatures strongly suggest a maser interpretation for the 6 cm H_2CO emission near NGC 7538–IRS 1.

In the direction of many dark clouds the 6 cm line of H_2CO is observed in absorption against the 2.7 K cosmic background radiation (Minn and Greenberg, 1973; Dieter, 1973). This anomalous absorption is best explained by a collisional pump model originally suggested by Townes and Cheung (1969) and later confirmed by accurate numerical calculations of H_2–H_2CO cross sections by Garrison et al. (1975). Since the circumstances in the molecular cloud in which NGC 7538–IRS 1 is embedded are presumably not very different from those in dark clouds the anomalous emission in the 6 cm H_2CO line will not be easily explained by collisional pumping.

Led by the fact that the level population of the H_2CO molecule is extremely sensitive to deviations from blackbody radiation at millimeter wavelengths (Thaddeus, 1972) and in view of the proximity of the ultra-compact H II region we propose in this paper that the H_2CO maser near NGC 7538–IRS 1 is pumped by the free-free continuum radiation of the H II region.

2. The Radiative Transfer Problem

We consider Formaldehyde molecules in a molecular Hydrogen cloud in the presence of a strong radio continuum radiation field. To find the H_2CO level populations we solve the equations of radiative transfer assuming that the thermal velocities of the molecules are much smaller than the large scale velocities in the cloud (cf. de Jong et al., 1975 and references therein).

Let $n_i(r)$ be the density of molecules in level i at distance r from the centre of a spherical cloud. In statistical equilibrium we have

$$\sum_j P_{jk} n_j = n_k \sum_j P_{kj} \quad (1)$$

where

$$P_{jk} = A_{jk}(\beta_{jk}(1 + H_{jk}) + D_{jk}) + C_{jk} \quad (j > k)$$
$$= A_{jk}(\beta_{jk} H_{jk} + D_{jk}) \frac{g_k}{g_j} + C_{jk} \quad (j < k). \quad (2)$$

The transition probabilities P_{jk} are expressed in terms of the spontaneous radiative transition probability A_{jk}, the collision induced transition probabilities C_{jk} and the net radiative transition rates $\beta_{jk} A_{jk}$ where β_{jk} is the so called escape probability (cf. de

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Table 1. Frequencies and spontaneous emission rates of rotational transitions in ortho-H_2CO

Transition	ν_{ij} (GHz)	A_{ij} (s^{-1})
1_{10}–1_{11}	4.83	$3.59\ 10^{-9}$
2_{12}–1_{11}	140.84	$5.34\ 10^{-5}$
2_{11}–1_{10}	150.50	$6.52\ 10^{-5}$
2_{11}–2_{12}	14.49	$3.23\ 10^{-8}$
3_{13}–2_{12}	211.23	$2.29\ 10^{-4}$
3_{12}–2_{11}	225.73	$2.79\ 10^{-4}$
3_{12}–3_{13}	28.98	$1.29\ 10^{-7}$
4_{14}–3_{13}	281.53	$5.93\ 10^{-4}$
4_{13}–3_{12}	300.84	$7.24\ 10^{-4}$
4_{13}–4_{14}	48.28	$3.59\ 10^{-7}$
5_{15}–4_{14}	355.71	$1.25\ 10^{-3}$
5_{14}–4_{13}	379.83	$1.52\ 10^{-3}$
5_{14}–5_{15}	72.41	$8.09\ 10^{-7}$
6_{16}–5_{15}	423.5	$2.17\ 10^{-3}$
6_{15}–5_{14}	452.5	$2.65\ 10^{-3}$
6_{15}–6_{16}	101.4	$1.59\ 10^{-6}$
7_{17}–6_{16}	494.0	$3.51\ 10^{-3}$
7_{16}–6_{15}	527.6	$4.28\ 10^{-3}$
7_{16}–7_{17}	135.0	$2.82\ 10^{-6}$

Jong et al., 1975)

$$\beta_{jk}(r)=\frac{1}{2}\int_{-1}^{+1}\beta_{jk}(r,\mu)\,d\mu=\frac{1}{2}\int_{-1}^{+1}\frac{1-\exp\{-\tau_{jk}(r,\mu)\}}{\tau_{jk}(r,\mu)}\,d\mu. \tag{3}$$

The optical depth

$$\tau_{jk}(r,\mu)=\frac{\lambda_{jk}^3 A_{jk}}{8\pi}\,\frac{r}{|1-\alpha\mu^2|v(r)}\left(n_k\frac{g_j}{g_k}-n_j\right) \tag{4}$$

is defined along a direction which makes an angle arc cos (μ) with the radius vector. The parameter α is related to the gradient of the large scale velocity field in the gas

$$\alpha=1-\frac{r}{v(r)}\,\frac{dv(r)}{dr}. \tag{5}$$

The quantities $\beta_{jk}H_{jk}$ and $D_{jk}(r)$ are the socalled photon occupation numbers of the jk line (Elitzur et al., 1976) of the blackbody background radiation field ($T_{bb}=2.7$ K)

$$H_{jk}=\left(\exp\left(\frac{hc}{\lambda_{jk}kT_{bb}}\right)-1\right)^{-1} \tag{6}$$

and of the free-free continuum radiation field

$$D_{jk}(r)=\frac{1-\exp\{-\tau_c(\lambda_{jk})\}}{\exp\left(\frac{hc}{\lambda_{jk}kT_e}\right)-1}\,W(r) \tag{7}$$

where

$$W(r)=\tfrac{1}{2}\int_{(1-(R_{II}/r)^2)^{1/2}}^{1}\beta(r,\mu)\,d\mu \tag{8}$$

is a factor with which the continuum radiation field at distance r from the centre of the H II region ($r>R_{II}$) is diluted. If the continuum radiation is not absorbed in the surrounding neutral gas ($\beta=1$) $W(r)$ reduces to the usual geometrical dilution factor.

The free-free continuum optical depth of an H II region with electron temperature $T_e=10^4$ K may be written (cf. Spitzer, 1978)

$$\tau_c(\lambda)=(\lambda/\lambda_0)^2\{1+0.311\ \log(\lambda)\} \tag{9}$$

where λ and λ_0 are expressed in cm. The wavelength λ_0 where the H II region becomes optically thick equals

$$\lambda_0=6.35\ 10^4\ E_m^{-0.5}\ \text{cm} \tag{10}$$

where

$$E_m=4/3\ n_e^2 R_{II} \tag{11}$$

is the emission measure of a spherical H II region.

The spontaneous radiative transition probabilities between the lowest 14 rotational levels of ortho-Formaldehyde, listed in Table 1, are calculated from the relation

$$A_{jk}=\frac{64\pi^4}{3h}\,E_{jk}^3\,\mu_{jk}^2 \tag{12}$$

where the wavenumbers E_{jk} of the lower transitions are taken from measurements of Nerf (1972) and Winnewisser et al. (1974) and of the higher transitions from Townes and Schawlow (1955). The matrix elements $(\mu_{jk}/\mu)^2$ are interpolated from tables of Schwendemann and Laurie (1958) for an asymmetry factor $\kappa=-0.961$ using a dipole moment of $\mu=2.34$ Debye (Townes and Schawlow, 1955). Hyperfine slitting of the rotational levels of H_2CO is neglected throughout our calculations.

The H_2–H_2CO collision rates are taken from Green et al. (1978) but multiplied by a factor 1.57 to correct for the difference in reduced mass of H_2 and He using a 80% H_2–20% He gas mixture.

3. Pump Mechanism

To illustrate the pump mechanism we approximate ortho-H_2CO by its four lowest rotational levels. Since we shall argue that the H_2CO maser is radiatively pumped by the radio continuum radiation of the H II region we consider only radiative transitions. Solving the equations of statistical equilibrium for this system we find

$$\frac{n_2}{n_1}=\frac{pP_{12}+P_{13}P_{34}P_{42}}{pP_{21}+P_{24}P_{43}P_{31}} \tag{13}$$

where $p=P_{31}(P_{42}+P_{43})+P_{42}P_{34}$ and the transition rates P_{ij} are defined by Eq. (2) (with $C_{ij}=0$). From Eq. (13) we derive the following condition for population inversion in the groundstate doublet

$$\frac{P_{34}P_{42}}{P_{43}P_{31}}\;\frac{P_{13}-q(1+P_{31}/P_{34})}{P_{24}+q}>1 \tag{14}$$

where $q=P_{21}-P_{12}=\beta_{21}A_{21}$. For illustrative purposes we further assume that all lines are optically thin ($\beta_{ij}=1$) and that the 2.7 K background radiation field can be neglected ($H_{ij}=0$) so that Eq. (2) reduce to

$$\begin{aligned} P_{jk}&=A_{jk}(1+D_{jk}) \qquad j>k \\ &=A_{jk}D_{jk}\frac{g_k}{g_j} \qquad j<k \end{aligned} \tag{15}$$

The only free parameters in Eqs. (14) and (15) are the wavelength λ_0 where the H II region becomes optically thick and the geometrical dilution factor W ($\beta_{ij}=1$) which depends on the distance of the masing gas to the H II region. Inserting molecular parameters (see Table 2) one easily verifies that $P_{24}\gg q$ and $P_{13}\gg q(1+P_{31}/P_{34})$ for $\lambda_0\ll 10^3$ cm and $W\gtrsim 0.1$ so that the population inversion condition may be written

$$\frac{F_{42}}{F_{31}F_{43}}>1 \tag{16}$$

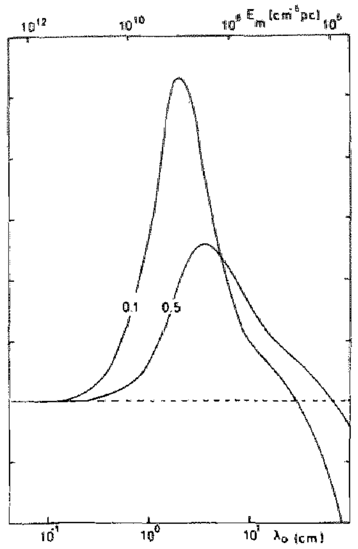

Fig. 1. Population ratios in the ground state doublet levels of 4-level ortho-Formaldehyde molecules near an H II region as a function of the radio continuum spectrum (λ_0 is the wavelength where the H II region becomes optically thick) and the distance to the H II region ($W=0.5$ corresponds to $r=R_{II}$ and $W=0.1$ to $r=5/3\ R_{II}$). Collisions and radiation trapping are neglected

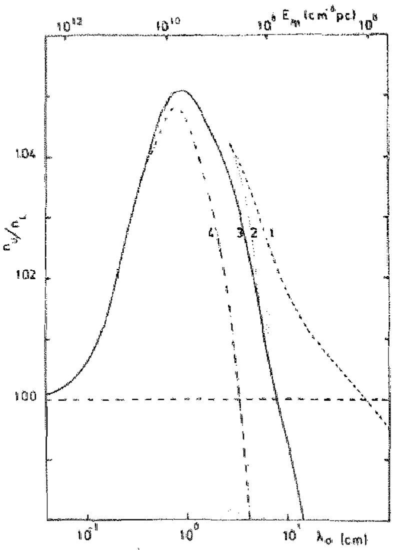

Fig. 2. Population ratios in the $J=1$ and $J=2$ doublet levels of 14-level ortho-Formaldehyde molecules near an H II region as a function of the radio continuum spectrum. Curves (1)–(3) are for the $1_{10}\rightarrow 1_{11}$ transition without collisions and radiation trapping (curve 1), with collisions but no radiation trapping (curve 2), with collisions and radiation trapping included (curve 3). Curve 4 gives the population ratio in the $2_{11}\rightarrow 2_{12}$ transition for the case that collisions and radiation trapping are included. The parameters adopted for this calculation are given in the text

Table 2. Parameters for NGC 7538 – IRS 1

		References
Electron temperature	$T_e=10^4$ K	
Total angular area	$\Delta\Omega=0.67$ arc s^2	Harris and Scott (1976)
Emission measure	$E_m=6\ 10^8$ cm^{-6} pc	Harris and Scott (1976)
Distance	$d=3.5$ kpc	Israel et al. (1973)
Radius	$R_{II}=2.4\ 10^{16}$ cm	
Mean electron density	$n_e=2\ 10^5$ cm^{-3}	
Turn-over wavelength	$\lambda_0=2.7$ cm	

where

$$F_{ij}=\frac{1+D_{ij}}{D_{ij}} \tag{17}$$

is the ratio of the downward to the upward radiative transition rates between levels i and j. If spontaneous radiative transitions dominate $F_{ij}\gg 1$ and if induced radiative transitions dominate $F_{ij}\rightarrow 1$. Condition (16) has a simple physical explanation. Population inversion of the ground state $J=1$ doublet of H_2CO occurs if the free-free continuum radiation field induces relatively more downward transitions from level 4 to level 2 than from level 3 to level 1 ($F_{42}>F_{31}$) while the populations of levels 4 and 3 are kept approximately equal by rapid radiative exchange ($F_{43}\simeq 1$). This is numerically illustrated in Fig. 1 where n_2/n_1 is plotted against λ_0 for two values of W. For $W=0.5$ the masing gas is situated at the boundary R_{II} of the H II region, for $W=0.1$ the inversion layer is located at a distance $5/3\ R_{II}$ from the centre of the H II region. If $W=0.5$ inversion occurs for 5 mm $\lesssim\lambda_0\lesssim$ 50 cm and if $W=0.1$ for 2 mm $\lesssim\lambda_0\lesssim$ 20 cm. From this illustrative example we conclude that the $J=1$ doublet of H_2CO can be inverted by the free-free radio continuum radiation of an H II region if it is very compact with emission measures in the range 10^8–10^{10} cm^{-6} pc.

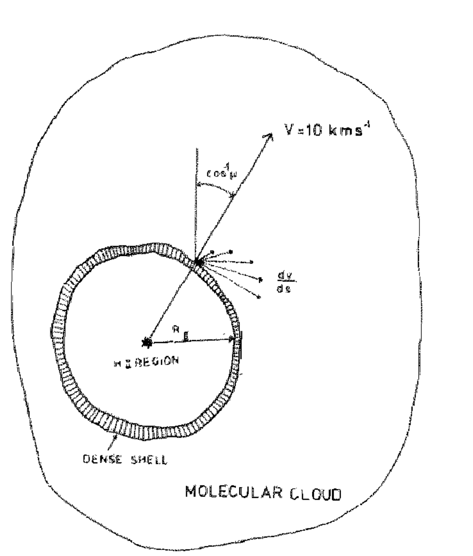

Fig. 3. Schematic picture of our model of the compact H II region NGC 7538–IRS 1 and its surrounding molecular cloud

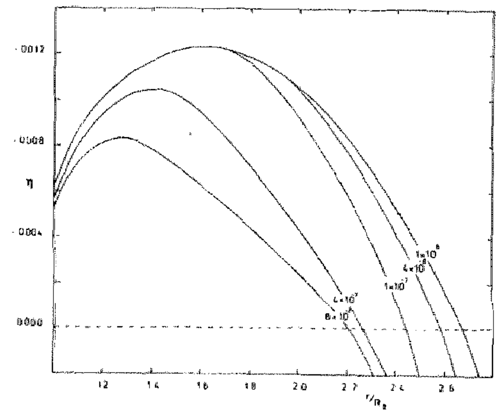

Fig. 4. The relative population inversion of the ground state doublet levels of ortho-Formaldehyde as a function of distance from the centre of NGC 7538–IRS 1 for different Formaldehyde abundances. The molecular gas has a density $n(H_2)=10^4$ cm^{-3} and a temperature $T_k=20$ K

In Fig. 2 we show the results of a more realistic calculation where we have included 14 levels of ortho-H_2CO and where the effects of collisions with H_2 molecules and of radiation trapping are also taken into account. The parameters characterizing this calculation are chosen as follows: a dilution factor $W=0.5$, a H_2 density $n(H_2)=3\ 10^4$ cm^{-3}, a gas kinetic temperature $T_k=20$ K, a Formaldehyde abundance $x(H_2CO)=n(H_2CO)/n(H_2)=4\ 10^{-7}$ and a velocity gradient $dv/dr=1280$ km s^{-1} pc^{-1} [$\alpha=0$, see Eq. (5)] corresponding to the observed line separation over the distance between the two maser spots.

From a comparison of Figs. 1 and 2 (curve 1) it is clear that including more Formaldehyde levels increases the population inversion of the ground state doublet and shifts the maximum inversion towards smaller values of λ_0 (more compact H II regions). Both effects are caused by the contributions to the pumping of the higher mm transitions in the H_2CO ladder. Including the effect of H_2 collisions narrows the range of λ_0 where population inversion occurs (curve 2). Taking in addition radiation trapping into account somewhat reduces the population inversion (curve 3). It also follows from Fig. 2 that for $\lambda_0 \lesssim 3$ cm both the 1_{10}–1_{11} transition (curve 3) and the 2_{11}–2_{12} transition (curve 4) are inverted while for 3 cm $\lesssim \lambda_0 \lesssim 10$ cm only the 1_{10}–1_{11} transition is inverted.

4. A Model for the Formaldehyde Maser near NGC 7538

We suppose that the H_2CO maser near NGC 7538 is located in dense cold molecular gas and that it is pumped by the free-free continuum radiation field of the compact H II region NGC 7538–IRS 1. Assuming spherical geometry one finds from its observed radio spectrum the physical parameters listed in Table 2. The free-free continuum optical depth becomes unity at $\lambda_0=2.7$ cm ($E_m=6\ 10^8$ cm^{-6} pc) so that we expect our radio continuum pump mechanism to operate (see Fig. 2).

The observed similarity of the 1720 MHz OH emission spectrum and the H_2CO emission spectrum suggests that both masers originate in the same gas cloud (cf. Forster et al., 1980). According to Elitzur (1976) the 1720 MHz maser is collision-dominated and it is most efficiently pumped for molecular Hydrogen densities in the range 10^3–10^5 cm^{-3} and gas temperatures in the range 15–200 K. This and the fact that the lowest rotational levels of H_2CO thermalize at densities $n(H_2)\gtrsim 3\ 10^5$ cm^{-3} suggest that both masers are located in a molecular cloud outside the thin dense shell of compressed gas at the interface of the H II region and the molecular cloud (Mathews and O'Dell, 1969). Because this shell is very thin ($\sim 0.01\ R_{II}$) and transparent to radio continuum radiation its presence is neglected in our model. The far-infrared radiation field generated in the shell and in the molecular cloud surrounding NGC 7538–IRS 1 (Willner, 1976; Werner et al., 1979) is also neglected because the free-free radio continuum radiation field dominates at all wavelengths of interest.

The H II region is optically thick at both 6 cm and at 18 cm so that the H_2CO and the OH lines must originate in molecular gas in front of the H II region and the observed high brightness temperatures are due to amplification of the radio continuum radiation of the H II region.

A comparison of the radial velocities of the H110α recombination line (-66 ± 5 km s^{-1}, Forster et al., 1980), of the H_2CO maser lines (-57.6 and -59.8 km s^{-1}, Forster et al., 1980) and of several other molecular lines (~ -57 km s^{-1}, CO observed by Wilson et al., 1974 and HCN, CN, HCO^+, and N_2H^+ by Turner and Thaddeus, 1977) indicates that NGC 7538–IRS 1 is moving through the molecular cloud with a velocity of about 10 km s^{-1} towards us.

Led by the observations discussed above we assume that the compact H II region NGC 7538–IRS 1 moves through or expands into a molecular cloud with a velocity $V\simeq 10$ km s^{-1}. We further assume that it moves towards us and that the molecular gas in front of the H II region is pushed sideways. The velocity field in the

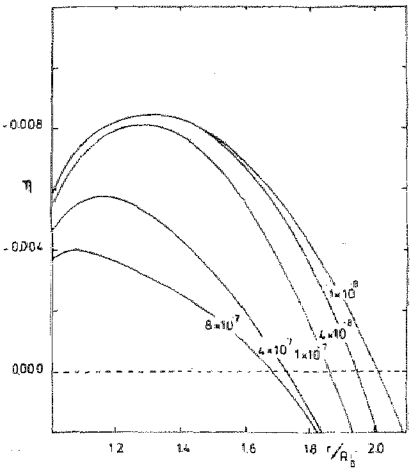

Fig. 5. Same as Fig. 4 for $n(H_2) = 3\ 10^4\ \mathrm{cm}^{-3}$

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gas may then be approximated by

$$\frac{dv}{ds} = \frac{V}{R_{II}}(1-\mu^2) \tag{18}$$

where μ is the cosine of the angle between line element ds and the line of sight. An attempt to give a pictorial representation of our model is shown in Fig. 3. The velocity gradient is zero (coherence) along the line of sight ($\mu=1$) and is maximal (V/R_{II}) perpendicular to the line of sight ($\mu=0$) in the direction in which the molecular gas is streaming away. The fact that the local approximation that we have used to solve the radiative transfer (cf. Sect. 2) breaks down in the radial direction does not invalidate our results because the solid angle over which $V(1-\mu^2) \lesssim v_{th}$ is small (~ 0.1 steradian), much smaller than the solid angle subtended by the H II region (cf. de Jong et al., 1975).

We have solved the line transfer problem for 14-level ortho-H_2CO molecules as outlined in Sect. 2 by putting $r = R_{II}$, $v(r) = V$ and $\alpha = 1$ in Eq. (4). In Fig. 4 we present the variation of the relative population inversion

$$\eta = (n_1 g_2/g_1 - n_2)/n(H_2CO) \tag{19}$$

with distance r from the centre of the H II region for several values of the H_2CO abundance $x(H_2CO) = n(H_2CO)/n(H_2)$. The results are obtained for $n(H_2) = 10^4\ \mathrm{cm}^{-3}$ and $T_k = 20$ K using the parameters of NGC 7538-IRS 1 listed in Table 2 to characterize the continuum radiation field. The relative population inversion decreases with increasing distance because the radio continuum pumping becomes less efficient. It decreases with increasing H_2CO abundance because optical depth effects in the millimeter lines reduce the pump rate. Figure 5 illustrates the effects of thermalization by collisions. It shows the results for the same parameters but for a molecular Hydrogen density $n(H_2) = 3\ 10^4\ \mathrm{cm}^{-3}$. Compared with Fig. 4 the population inversion and the thickness of the inversion layer are both smaller by about a factor 2.

The brightness temperature of the H_2CO line can be calculated from

$$T_b = T_c\{\exp(-\tau) - 1\} \tag{20}$$

where $T_c = 10^4$ K is the brightness temperature of the radio continuum radiation of the H II region and

$$\tau = 3.4\ 10^{-8}\ \bar{\eta} n(H_2CO)\frac{\Delta l}{v_{th}} \tag{21}$$

is the 6 cm H_2CO line optical depth [see Eq. (4)]. In Eq. (21) $\bar{\eta}$ is the relative population difference [Eq. (19)] averaged over the thickness of the inversion layer Δl and $V_{th} = 0.1\ \mathrm{km\ s}^{-1}$ is the thermal velocity of H_2CO molecules. To reproduce the observed brightness temperatures in the H_2CO maser lines of $\sim 10^6$ K a line optical depth $\tau \simeq -5$ is required. According to Fig. 4 this can be achieved for $n(H_2) = 10^4\ \mathrm{cm}^{-3}$ where we find $\bar{\eta} \simeq 0.006$ and $\Delta l \simeq 1.2\ R_{II}$ if $x(H_2CO) = 8\ 10^{-7}$.

To show that this Formaldehyde abundance can be obtained in dense dark clouds we now derive an analytic expression for the H_2CO abundance from a discussion of its chemical formation and destruction processes. For more details of dark cloud chemistry the reader is referred to the paper by de Jong et al. (1980).

The formation of H_2CO is initiated by the radiative association reaction

$$C^+ + H_2 \rightarrow CH_2^+ + h\nu \tag{a}$$

leading to CH_3^+ after collisions with H_2. At large densities $n(H_2) \gtrsim 10^4\ \mathrm{cm}^{-3}$) CH_3^+ is removed from the gas by

$$CH_3^+ + H_2 \rightarrow CH_5^+ + h\nu \tag{b}$$

rather than by recombination. CH_5^+ is predominantly destroyed by dissociative recombination leading to CH_3 and CH_4 upon recombination. CH_3 is destroyed by collisions with C^+ and H_3^+ and by the reaction

$$CH_3 + O \rightarrow H_2CO + H. \tag{c}$$

We assume that Formaldehyde is predominantly removed from the gas by collisions with HCO^+, H_3^+, and C^+. At large densities and for $A_v \gtrsim 2$ the formation of C^+ is initiated by cosmic ray ionization of He atoms followed by the reaction

$$He^+ + CO \rightarrow C^+ + O + He \tag{d}$$

and the C^+ ions are mainly destroyed by reaction (a) as long as $k_a > 1\ 10^{-9}\ n(O_2)/n(H_2)$.

From our models of molecular clouds (de Jong et al., 1980) we found that in dense dark clouds all Hydrogen is in H_2, that all Carbon is in the form of CO and that approximately 1/3 of the remaining Oxygen atoms are in the form of atomic Oxygen and 2/3 in the form of O_2. Assuming that electrons are predominantly produced by cosmic ray ionization of molecular Hydrogen and removed from the gas by dissociative recombination with molecular ions like HCO^+ and H_3^+ one finds

$$n(e) = 1.6\ 10^{-4}\left(\frac{\zeta}{5\ 10^{-18}\ \mathrm{s}^{-1}}\right)^{1/2}\left(\frac{n(H_2)}{10^4\ \mathrm{cm}^{-3}}\right)^{1/2}\ \mathrm{cm}^{-3} \tag{22}$$

for a gas temperature of $T = 20$ K where ζ is the cosmic ray ionization rate of H_2. Using a H_2CO destruction rate due to collisions with molecular ions of $2\ 10^{-9}\ n(e)\ \mathrm{s}^{-1}$ we finally obtain the following expression for the Formaldehyde abundance

$$\frac{n(H_2CO)}{n(H_2)} = 8\ 10^{-7}\left(\frac{\zeta}{5\ 10^{-18}\ \mathrm{s}^{-1}}\right)^{1/2}\left(\frac{10^4\ \mathrm{cm}^{-3}}{n(H_2)}\right)^{1/2} \tag{23}$$

where we have assumed that CH_3 is predominantly destroyed by reaction (c). This is the case as long as Carbon and Oxygen are not more depleted than about a factor 100 compared to their cosmical abundances. For our chemical scheme to work we need a reaction rate coefficient of reaction (a): $k_a \gtrsim 5\,10^{-15}$ cm^3 s^{-1} in good agreement with a theoretical calculation of Herbst et al. (1977).

Sofar we have neglected photodestruction of Formaldehyde. Because the extinction in the compressed shell between the shock front and the ionization front around the compact H II region is quite large the UV radiation of the O6 star exciting NGC 7538–IRS 1 is completely absorbed in the shell (Willner, 1976). The O7 star exciting the optical nebula NGC 7538 (cf. Israel et al., 1973) at a projected distance of about 4 pc from NGC 7538–IRS 1 is another possible source of UV radiation. From Kurucz (1979) we estimate that the radiation field of an O7 star with a radius of 10 $R_\odot$ at 4 pc distance is about 300 times as intense as the average interstellar radiation field at 1300 Å (Habing, 1968). For photodestruction to be negligible (less than 10^{-13} s^{-1}) we require that the maser is formed at depth $A_v \geqq 10$ into the molecular cloud (cf. de Jong et al., 1980).

We conclude that our model can explain the maser amplification required to reproduce the observed brightness temperatures. Our model predicts a maser spot size with typical dimensions $\sim 2\,R_{II}$ arc cos $\{(1-v_{th}/V)^{1/2}\} \simeq 5\,10^{15}$ cm consistent with the observed upper limit. From our calculations we further find that the induced emission rate in the maser transition is approximately equal to the rate of population exchange in the millimeter lines so that the maser is close to saturation. This explains why both maser features have about the same intensity. Our model does not account for the fact that there are two maser spots separated by about 0."1 ($5\,10^{15}$ cm) in position and by about 2 km s^{-1} in velocity. Special circumstances presumably due to gas dynamical effects (fluctuations in the gas density and/or the velocity field) are required to explain these facts.

Our model can also be used to predict a brightness temperature of the 2_{11}–2_{12} line of formaldehyde. From our numerical results we obtain an optical depth in that line of ~ -1 so that we predict a brightness temperature of $\sim 2\,10^4$ K consistent with the upper limit of $5\,10^4$ K that can be derived from the H_2CO 2 cm line observations of NGC 7538–IRS 1 by Evans et al. (1975) for a source size of 0."1.

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