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Chapter 4

Vertexing

Vertexing is a central issue of this analysis. As explained in Sec. 3.3, the momentum of a helix track is meaningful only at a given point along the helix. Because the invariant mass of two tracks depends directly on the value and orientation of each of the two track momenta, a rigorous approach to vertex reconstruction is taken. In this chapter, the mathematical formalism of vertexing will be described. The material presented here is described in greater detail in [28]. This formalism has been "coded" in the ZEUS vertexing package *kfvertex*. Also, tagging variables such as decay length, decay length significance, impact parameter and lifetime are defined.

4.1 Track parametrization

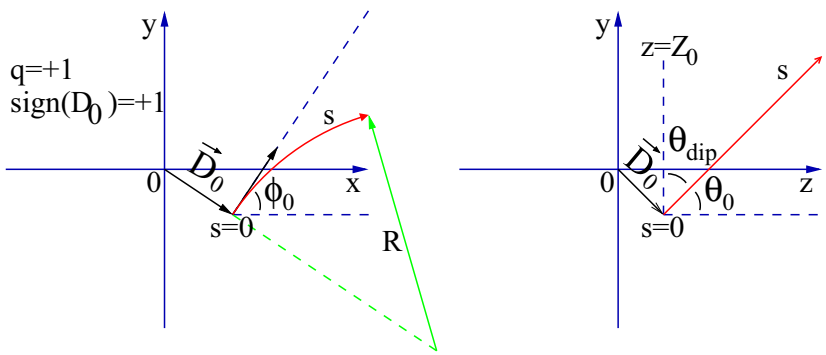

Figure 4.1. Projection of a helix track onto the XY plane (left) and YZ plane(right). The helix parameters are explicitly shown.

The trajectory of a charged particle inside an uniform magnetic field is a helix. The velocity $v_{\parallel}$ of the particle along the field lines determines the *step* of the helix;

the radius of the helix circle is proportional to $v_\perp$, the component of the particle velocity perpendicular to magnetic field lines. ZEUS employs a right handed coordinate system with colliding particles moving along the OZ axis, with the protons advancing in the positive direction. A uniform magnetic field of strength $B_z \simeq 1.43$ Tesla is aligned along the OZ axis: $\vec{B} = (0, 0, B_z)$.
The trajectory of each track in the global Cartesian (x, y, z) frame is parametrized by a 5-vector helix:

$$\vec{p} = (W, T, \phi_0, D_0, Z_0) \tag{4.1}$$

W is the signed curvature of the track: $W = q/R$, with q the charge and R the helix radius. The parameter T is defined as $tan(\theta_{dip})$ where θ_{dip} is the complement of the polar angle θ_0. The Cartesian vector $\vec{D_0}$ points to the point on the helix closest to the z-axis. The scalar D_0 is then the distance of closest approach to the z-axis and Z_0 the z coordinate of the helix at this point. Projecting the helix onto a circle in the xy plane, ϕ_0 is the angle that $\vec{t}$, the tangent of the (projected) track at the point of closest approach, makes with the x axis. D_0 is signed. The sign is defined by the sign of the cross-product $\vec{D_0} \times \vec{t}$. Fig. 4.1 shows the projection of a helix track onto the xy plane (left plot) and yz plane (right plot) for a positively charged particle.

One can now recover the position (x, y, z) of any point along the helix using the above parametrization:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \vec{D_0} + \begin{pmatrix} \frac{1}{W}\cos(\phi_0)\sin(Ws_\perp) + \frac{1}{W}\sin(\phi_0)[1 - \cos(Ws_\perp)] \\ \frac{1}{W}\sin(\phi_0)\sin(Ws_\perp) - \frac{1}{W}\cos(\phi_0)[1 - \cos(Ws_\perp)] \\ Ts_\perp \end{pmatrix} \tag{4.2}$$

Here, $s_\perp = s\sin(\theta)$ is the distance along the helix, measured from the point of closest approach.

The *trackfit* routine gives the best possible estimate of five track parameters and their uncertainties, based on a collection of hits inside the tracking detectors. A detailed description of how *trackfit* is implemented can be found in [22]. The estimated uncertainties of each of the five helix parameters are organized in a 5×5 covariance matrix V. Once reconstructed, a track is completely described by a 5-vector $\vec{p}$ in the helix parametrization and its covariance matrix V and can be further manipulated as such.

4.2 Kalman filter and vertex reconstruction

4.2.1 The mathematical formalism

One starts always with an estimate of the vertex[1]. Then, this estimate is compared to the information from a random[2] track in the event. If the track is consistent with coming from the vertex, the track information is added, yielding a new estimate. This procedure is then iterated for each track in the event and that leads to a final vertex estimate. This technique is called *filtering*. By *smoothing*, the momentum of each particle is recomputed at the final vertex position. A notation is defined:

x_k = estimate of the vertex position after *adding* k tracks to vertex

x^t = true vertex position

C_k = cov(x_k)

q_k = estimate of the momentum of particle k at x_k

D_k = cov(q_k)

E_k = cov(x_k,q_k)

m_k = measurement vector (five measured helix parameters)

v_k = measurement noise (disturbance in measuring m_k)

V_k = cov(v_k)

$G_k = V_k^{-1}$ = weightmatrix of particle k

The measurement equation is a map h of the true vertex position x^t and true momentum q_k^t of track k to the measured parameters m_k distorted by the noise v_k:

$$m_k = h_k(x^t, q_k^t) + v_k \tag{4.3}$$

[1] A vertex estimate can come from various sources. One good start could be the beam spot. In ZEUS, the *kfvertex* routine uses overlap points of track pairs, in the xy plane, as seeds for primary vertex finding.

[2] By random track it is understood here any track of a given event that passed certain quality cuts depending on the transverse momentum, number of MVD hits or CTD superlayers, η distribution, etc.

Kalman filtering can be applied only to linear measurement equations. Therefore, $h_k(x^t, q^t)$ is approximated by a first order Taylor expansion:

$$h_k(x^t, q_k^t) \approx h_k(x^{(0)}, q_k^{(0)}) + A_k(x^t - x_k^{(0)}) + B_k(q_k^t - q_k^{(0)}) = c_k^{(0)} + A_k x^t + B_k q_k^t \quad (4.4)$$

with

$$\begin{aligned} A_k &= \tfrac{\partial h_k}{\partial x^t}\Big|_{(x_k^{(0)}, q_k^{(0)})}, B_k = \tfrac{\partial h_k}{\partial q_k^t}\Big|_{(x_k^{(0)}, q_k^{(0)})} \\ c_k^{(0)} &= h_k(x^{(0)}, q_k^{(0)}) - A_k x_k^{(0)} - B_k q_k^{(0)} \end{aligned} \quad (4.5)$$

The vertex reconstruction proceeds as follows: choose a starting value x_0 for the vertex position and a covariance matrix. This can depend on information that one has before hand. A blind choice would be the origin of the coordinate system and a 3×3 covariance matrix with infinite quantities on the diagonal. A more fortunate choice is the beam spot and its standard deviations. Next, recompute an estimate x by adding the weighted information of track 1 [3]. This is done by the so called χ^2 *minimizing* method.

$$\begin{aligned} \chi^2_{KF}(x, q) &= (x - x_0)^T (C_0)^{-1} (x - x_0) + \\ &+ (m_1 - c_1^{(0)} - A_1 x - B_1 q)^T G_1 (m_1 - c_1^{(0)} - A_1 x - B_1 q) \end{aligned} \quad (4.6)$$

The position x and the momentum q that minimize this χ^2_{KF} are the first guesses for x_1 and q_1 at the new vertex. One solves $\partial\chi^2_{KF}/\partial x = 0$ and $\partial\chi^2_{KF}/\partial q = 0$ and finds:

$$\begin{aligned} x_1 &= C_1 \left[(C_0)^{-1} x_0 + A_1^T G_k^B (m_1 - c_1^{(0)}) \right] \\ q_1 &= W_1 B_1^T G_1 (m_1 - c_1^{(0)} - A_1 x_1) \\ C_1 &= \left((C_0)^{-1} + A_1^T G_1^B A_1 \right)^{-1} \\ D_1 &= W_1 + Q_1 B_1^T G_1 A_1 C_1 A_1^T G_1 B_1 W_1 \\ E_1 &= -C_1 A_1^T G_1 B_1 W_1 \end{aligned} \quad (4.7)$$

[3] Note that the point $x^{(0)}$ around which we Taylor expand need not be the same as the vertex start value x_0. One choice is the point on the track closest to x_0.

with

$$W_1 = (B_1^T G_1 B_1)^{-1} \quad , \quad G_1^B = G_1 - G_1 B_1 W_1 B_1^T G_1$$
$$cov(x_1) = C_1 \ , \ cov(q_1) = D_1 \ , \ cov(x_1, q_1) = E_1$$

D_1 is called momentum covariance and E_1 cross-covariance.
The new (updated) vertex is located at x_1 with the covariance C_1. We can now add track 2 and so on. In this way, an iterative procedure adding track k will lead to the same answers as in Eq. (4.7) but with the lower index 1 replaced by k (and 0 by $k-1$). *Smoothing* track k to the new vertex changes its momentum:

$$q_k^s = W_k B_k^T G_k (m_k - c_k^{(0)} - A_k x_k) \tag{4.8}$$

where s upper index stands for *smoothed*. This step is essential for accurately determining the invariant mass at the vertex. After smoothing all tracks to the final vertex position, the covariance matrix Q for the correlation between the smoothed momenta is recovered. For instance, the cross-momentum correlation covariance matrix between tracks i and j after smoothing has the form:

$$Q_{i,j} = cov(q_i^s, q_k^s) = W_i B_i^T G_i A_i C_f A_j^T G_j B_j W_j \tag{4.9}$$

with C_f being the final vertex covariance matrix. Note that $Q_{i,j} = Q_{i,j}^T$.

So far, the formalism is valid for any track parametrization. In ZEUS, charged tracks parametrized as in Eq. 4.1 are vertexed in this way.

4.3 Expanding the tracking package

The *trackfit* routine can only reconstruct tracks that have crossed the detector. The *kfvertex* routine has been modified for this analysis such that new tracks can be constructed, associated to particles which decayed before reaching the MVD. A pseudotrack is defined as a composite object made from a vertex and several charged daughter tracks emerging from that vertex. The pseudotrack represents the mother particle which decayed at that vertex. A careful treatment of the errors involved leads to a completely determined object parametrized by a 5-vector in the helix parametrization and its covariance matrix. Furthermore, one could employ

the pseudotrack (a real particle) to search for new decay vertices at which the reconstructed mother particle had participated as a daughter/decay product.

The method will become more transparent if a specific parametrization is chosen. The main focus of this thesis is the neutral pseudotrack (neutral mother particle decaying in two charged tracks). The treatment of charged pseudotracks is similar [29](to be published).

4.3.1 Neutral Pseudotracks

As the curvature is not defined for neutral tracks moving in straight lines in a magnetic field, the parametrization from Eq. 4.1 presents a problem. Therefore, the following parameters are chosen for the measurement vector m :

$m(1) = p$	$(0, +\infty)$ absolute value of the total momentum
$m(2) = \phi$	$(-\pi, \pi)$ azimuth angle at DCA
$m(3) = \theta$	$(0, \pi)$ polar angle
$m(4) = dca$	$(-\infty, +\infty)$ distance of closest approach
$m(5) = z$	$(-\infty, +\infty)$ z at DCA

The geometrical momentum vector $\vec{q}$ is defined as follows:

$q(1) = p$	$(0, +\infty)$ absolute value of the total momentum
$q(2) = \phi$	$(-\pi, \pi)$ azimuth angle at DCA
$q(3) = \theta$	$(0, \pi)$ polar angle

The vector $\vec{p} = (p_x, p_y, p_z)$ is called the kinematical momentum of a particle.

Although the geometry of the neutral track does not depend on its momentum, its errors do. This happens essentially because the errors of the daughter track momenta translate into an error on the vertex position which will feed into the error of the pseudotrack. The calculation of the pseudotrack parameters is done in three steps:

(a) Determine the kinematical momentum of the mother from the fitted geometrical momenta of the daughters at the vertex

(b) Calculate the geometrical momentum of the mother particle from its kinematical momentum. This is necessary as it is part of the neutral track parametrization.

(c) Calculate the measurement parameters of the mother particle at the DCA as well as its covariance matrix.

In step (a), for each of the two charged daughters, one has:

$$p_x = p_t \cos(\phi) \ , \ p_y = p_t \sin(\phi) \ , \ p_z = p_t \cot(\theta) \tag{4.10}$$

So a mapping $\vec{p}_{a,b} = f(\vec{q}_{a,b})$ exists. The Jacobian propagation matrix is defined as $W_{a,b} = (\partial f / \partial q_{a,b})|_{q^{(0)}_{a,b}}$. With $\vec{p}_a$ and $\vec{p}_b$ calculated, the pseudotrack momentum is simply:

$$\vec{p}_{tot} = \vec{p}_a + \vec{p}_b \tag{4.11}$$

In step (b) the geometrical momentum of the mother particle is computed, starting from $\vec{p}_{tot}$. Using $\vec{p}_{tot} = (p_x, p_y, p_z)$, the mapping $\vec{q}_{tot} = g_n(p_{tot})$ takes the form:

$$\begin{aligned} p = \sqrt{p_x^2 + p_y^2 + p_z^2} \ , \quad \phi = \left(\frac{p_x}{\sqrt{p_x^2 + p_y^2}} \right) \ , \\ \theta = \arccos \left(\frac{p_x}{\sqrt{p_x^2 + p_y^2 + p_z^2}} \right) \end{aligned} \tag{4.12}$$

The Jacobian matrix W_n^- for the error propagation and its inverse are defined as:

$$W_n^- = \left. \frac{\partial \vec{g_n}}{\partial \vec{p}} \right|_{p^{(0)}_{tot}} = (W_n)^{-1} \tag{4.13}$$

Finally, all entries of the neutral track "measurement" vector m_n are known. Fig. 4.2 illustrates how parameters measured at the decay vertex P_0 are translated into a measurement vector given w.r.t. the point of closest approach. Using the notation of Fig. 4.2, it holds that:

$$\begin{aligned} m_n = h_n(x_0, y_0, z_0; p_0, \phi_0, \theta_0) \\ p = p_0 \ , \quad \phi = \phi_0 \ , \quad \theta = \theta_0 \ , \quad dca = r_\perp \sin(\xi) \\ z = z_0 - r_\perp \cos(\xi) \tfrac{1}{\tan(\theta)} \end{aligned} \tag{4.14}$$

where:

$$r_\perp = \sqrt{x_0^2 + y_0^2} \ , \ \phi_0' = \arccos \left(\frac{x_0}{r_\perp} \right) \ , \ \xi = \phi_0 - \phi_0' \tag{4.15}$$

The covariance matrix of the pseudotrack has to be calculated by a proper error propagation. Let us write down again the names and the physical meaning of each of the matrices involved:

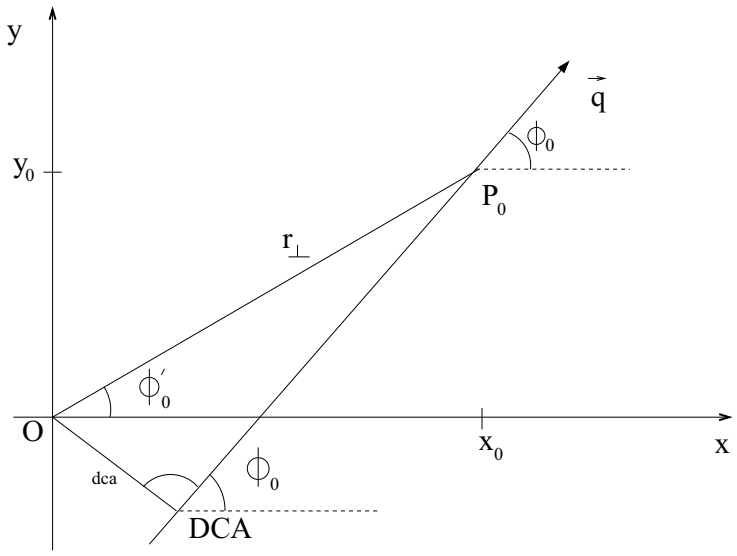

Figure 4.2. Pseudotrack projection on the xy plane. Parameters measured at decay vertex P_0 are expressed in Eq. 4.14, 4.15 in terms of parameters expressed at DCA.

$C_f = cov(x_f)$**,** covariance matrix of the smoothed vertex position

$E_i = cov(x_f, q_i^s)$**,** for daughter tracks

$Q_{i,j} = cov(q_i^s, q_j^s)$**,** correlation between tracks , $i \neq j$

$Q_{i,i} = cov(q_i^s, q_i^s) = D_i$**,** momentum covariance for each daughter track

A**,** B first order Taylor expansion matrices for the pseudotrack w.r.t. the point of closest approach, as given in Eq. 4.5.

We define:

$E = \sum_i E_i W_i^T$

$Q = \sum_{i,j} W_i Q_{i,j} W_j^T$

Then, the covariance matrix of the mother particle is:

$$C_{pseudo} = AC_fA^T + AE(W^{-1})^T B^T + BW^{-1}E^T A^T + BW^{-1}Q(W^{-1})^T B^T \quad (4.16)$$

The 5×5 covariance matrix C_{pseudo} is symmetric and positive definite.

4.3.2 The decay length significance

A neutral particle created at the (primary) vertex $\vec{v_1}$ decays later at the secondary vertex $\vec{v_2}$ into two oppositely charged daughter particles, as pictured in Fig. 4.3. The decay length vector is defined as:

$$\vec{L} = \vec{v_2} - \vec{v_1} \tag{4.17}$$

The decay length is then $L = |\vec{L}|$. Let $\vec{1_L}$ be a unit vector in the direction of $\vec{L}$: $|\vec{1_L}| = 1$. The error of the decay length is then computed by projecting the covariance matrices C_1 and C_2 of $\vec{v_1}$ and $\vec{v_2}$ respectively along the unit vector $\vec{1_L}$:

$$\Delta L = \sqrt{(\vec{1_L})^T \cdot C_1 \cdot \vec{1_L} + (\vec{1_L})^T \cdot C_2 \cdot \vec{1_L}} \tag{4.18}$$

Due to measurement errors, the momentum of the mother particle at the decay vertex, $\vec{p_M} = \vec{p_1} + \vec{p_2}$ does not always point in the same direction as the decay length vector $\vec{L}$. In order to separate poorly reconstructed decays in which the mother particle seems to be coming back towards the origin vertex $\vec{v_1}$, equivalent to $\vec{L} \cdot \vec{p_M} < 0$, from the more "physical" decays in which the mother particle moves away from $\vec{v_1}$, with $\vec{L} \cdot \vec{p_M} > 0$, the decay length is signed:

$$L \rightarrow Sign(\vec{L} \cdot \vec{p_M}) \cdot L \tag{4.19}$$

The decay length significance is the ratio of the signed decay length and its error:

$$\sigma_{D.L.} = Sign(\vec{L} \cdot \vec{p_M}) \frac{L}{\Delta L} \tag{4.20}$$

4.3.3 Impact parameter significance

The impact parameter d of the pseudotrack w.r.t. the primary vertex is a useful tool which discriminates between particles created at the interaction vertex or elsewhere. It is defined as the distance of closest approach of the pseudotrack momentum line to the primary vertex v_1, as shown in Fig. 4.3. The following holds:

$$\cos\gamma = \frac{\vec{L} \cdot \vec{p_M}}{|L|\,|p_M|} \tag{4.21}$$

with γ the angle opposing the impact parameter. Then, d is computed as:

$$d^2 = L^2 - \frac{(\vec{L} \cdot \vec{p_M})^2}{|p_M|^2} \tag{4.22}$$

where the relations $\sin\gamma = d/L$ and $\sin^2\gamma + \cos^2\gamma = 1$ were used.

If $\vec{1_d}$ is a unit vector pointing in the same direction as the impact parameter, then the impact parameter error is : $\Delta d = \sqrt{(\vec{1_d})^T \cdot C_1 \cdot \vec{1_d} + (\vec{1_d})^T \cdot C_2 \cdot \vec{1_d}}$ with C_1, C_2 covariance matrices of the primary and secondary vertices. The significance of the impact parameter is simply: $\sigma_d = d/\Delta d$ and is a positive quantity.

4.3.4 Lifetime

The lifetime of a particle that has the momentum $\vec{p}$, velocity $\beta = v/c$ and decay length $\vec{L}$ in the laboratory frame is:

$$c\tau = \frac{L}{\beta\gamma} = m\frac{L}{|p|} = m\frac{L_{xy}}{P_T} \tag{4.23}$$

where $\gamma = 1/\sqrt{1+\beta^2}$, P_T is the transverse momentum and L_{xy} the projection of the decay length vector $\vec{L}$ on the xy plane. The last equality exploits the fact that the ZEUS detector has a better resolution in the transverse plane than along the beam axis.

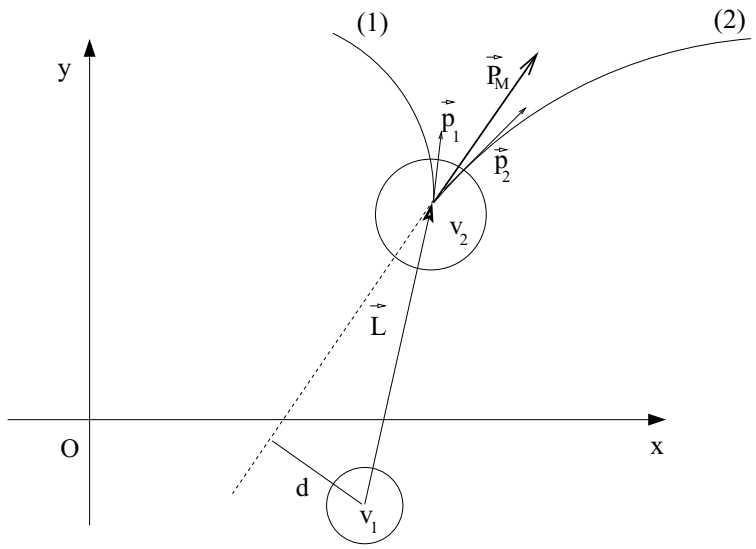

Figure 4.3. A neutral particle moves away from vertex v_1 and decays later on at vertex v_2. The particle momentum $\vec{p_M}$ at v_2, the decay length vector $\vec{L}$ and the impact parameter d of the particle w.r.t. to (creation) vertex v_1 are shown.